
Financing Low-Carbon Transitions through Carbon Pricing and Green Bonds*

DIRK HEINE, WILLI SEMMLER, MARIANA MAZZUCATO, JOÃO PAULO BRAGA, MICHAEL FLAHERTY, ARKADY GEVORKYAN, ERIN HAYDE, AND SIAVASH RADPOUR

Dirk Heine, World Bank, Macroeconomics, Trade & Investment Global Practice, Global Macro and Debt; Université catholique de Louvain, Hoover Chair in Economic and Social Ethics, dheine@worldbank.org,
Willi Semmler, New School for Social Research, Department of Economics; University of Bielefeld, Faculty of Economics; International Institute for Applied Systems Analysis, Senior Researcher at IIASA, SemmlerW@newschool.edu
Mariana Mazzucato, University College London, Institute for Innovation and Public Purpose, m.mazzucato@ucl.ac.uk
João Paulo Braga, New School for Social Research, Department of Economics, bragj553@newschool.edu
Michael Flaherty, New School for Social Research, Department of Economics, flahertym@newschool.edu
Arkady Gevorkyan, Federal Reserve Bank of Cleveland, Surveillance and Macro, arkady.gevorkyan@clev.frb.org
Erin Hayde, World Bank, Macroeconomics, Trade & Investment Global Practice; New School for Social Research, Department of Economics, hayde690@newschool.edu
Siavash Radpour, New School for Social Research, Department of Economics, radps229@newschool.edu

Summary: To finance the transition to low-carbon economies required to mitigate climate change, countries are increasingly using a combination of carbon pricing and green bonds. This paper studies the reasoning behind such policy mixes and the economic interaction effects that result from these different policy instruments. We model these interactions using an intertemporal model, related to Sachs (2015), which proposes a burden sharing between current and future generations. The issuance of green bonds helps to enable immediate investment in climate change mitigation and adaptation, and the bonds would be repaid by future generations in such a way that those who benefit from reduced future environmental damage share in the burden of financing mitigation efforts undertaken today. We examine the effects of combining green bonds and carbon pricing in a three-phase model. We are using a numerical solution procedure which allows for finite-horizon solutions and phase changes. We show that green bonds perform better when they are combined with carbon

→ JEL classification: E2, H6, O2, Q5

→ Keywords: Carbon pricing; green bonds; intergenerational burden sharing; nonlinear model predictive control

* **Disclaimer:** The findings, interpretations, and conclusions expressed in this paper are entirely those of the authors. They do not necessarily represent the views of the Federal Reserve Bank of Cleveland or the Board of Governors of the Federal Reserve System, the International Monetary Fund, or the World Bank and its affiliated organizations, or those of the Executive Directors of the IMF or World Bank or the governments they represent.

Acknowledgement: This project was started when Willi Semmler and Dirk Heine were Visiting Scholars at the IMF. We want to thank Ian Parry, Manoj Atolia, Ottmar Edenhofer, Jeffrey Sachs, Prakash Loungani, Michael Mehling, Diego Rivetti and Sebastian Essl for encouragement and discussions. Willi Semmler also wants to acknowledge the productive discussions with colleagues at the International Institute for Applied Systems Analysis.

pricing. Our proposed policy option appears to be politically more feasible than a green transition based only on carbon pricing and is more prudent for debt sustainability than a green transition that relies overly on green bonds.

Zusammenfassung: Zur Finanzierung des Klimaschutzes verwenden Länder zunehmend eine Kombination von CO₂-Bepreisung und grünen Anleihen. Dieser Artikel analysiert die Gründe für einen solchen Politikmix sowie die Interaktionseffekte dieser Instrumente. Wir modellieren die Interaktionseffekte in einem intertemporalen Modell, das sich auf Sachs (2015) bezieht und eine Lastenverteilung zwischen gegenwärtigen und zukünftigen Generationen vorsieht. Die Ausgabe von grünen Anleihen trägt dazu bei, sofortige Investitionen in den Klimaschutz und die Anpassung an den Klimawandel zu ermöglichen, und die Anleihen würden von zukünftigen Generationen so zurückgezahlt, dass diejenigen, die von geringeren zukünftigen Umweltschäden profitieren, an der Last der Finanzierung der heute unternommenen Klimaschutzmaßnahmen beteiligt sind. Wir untersuchen die Auswirkungen der Kombination von grünen Anleihen und CO₂-Steuern in einem Drei-Phasen-Modell. Wir verwenden ein numerisches Lösungsverfahren, das endliche Lösungen und Phasenwechsel ermöglicht. Wir zeigen, dass grüne Anleihen besser abschneiden, wenn sie mit der CO₂-Besteuerung kombiniert werden. Unsere vorgeschlagene politische Option scheint politisch machbarer zu sein als ein grüner Übergang, der ausschließlich auf CO₂-Steuern basiert und für die Nachhaltigkeit der Verschuldung vorsichtiger ist als eine Klimapolitik, die zu sehr auf grüne Anleihen setzt.

I Introduction

Faced with climate change, countries use both carbon pricing and green bonds to finance their transition to low-carbon economies. This paper studies the economic interaction effects and the optimal combination of those two types of climate policies.

Carbon pricing, in the form of carbon taxes or emissions trading schemes, has been used since the 1990s as incentives for abating greenhouse gas (GHG) emissions, and since then spread to 46¹ jurisdictions, presently raising 43bn in revenues.² Green bonds (or climate bonds) represent a more recent development in the policy toolkit for financing climate change mitigation, adaptation or conservation of natural capital. First created by the European Investment Bank and the World Bank together with the Swedish SEB in 2007/2008, green bonds are today issued by government agencies, multilateral institutions, and private businesses. Despite their exponential uptake since 2011–12 (see figure 1 in section 2), both instruments are still far too small for containing climate change: less than five percent of emissions covered under explicit carbon pricing initiatives is currently priced at a level that would be consistent with the Paris Agreement;³ and the financing needs for the low-carbon climate-resilient transitions are estimated in trillions per year rather than the cumulative 500bn⁴ issued over the last decade. This paper therefore reviews the roles of both instruments, and their optimal combination in scaling up mitigation and adaptation.

1 Counting both countries that have already implemented carbon taxes or emissions trading systems and countries where implementation plans are far advanced, based on (World Bank 2019a).

2 Ibid.

3 Ibid.

4 World Bank (2018).

The paper is organized as follows. Section 2 reviews theory on the roles and interaction effects of carbon pricing and green bonds. In Section 3, we theoretically model these interaction effects for two low-carbon transition scenarios using a new numerical solution method, called nonlinear modelling predictive control (NMPC). Section 4 concludes. We find that there are strong synergies from combining green bonds with carbon pricing for sustainability outcomes, investor returns and debt sustainability. These synergies are stronger if carbon is priced through taxes than through emissions trading.

2 Combining Carbon Pricing and Green Bonds

The fundamental purpose of carbon pricing is to make consumers and producers of polluting goods take into account the costs imposed by this pollution on society as a whole. Carbon pricing policies, such as carbon taxes or emissions trading systems (ETS), can also be used in addition to green bonds to achieve both greater environmental effectiveness and lower overall cost of mitigation. Here, we review the efficiency benefits from including such carbon pricing in a joint policy with green bonds.

2.1 Role of carbon pricing

“A well-designed carbon price is an indispensable part of a strategy for reducing emissions in an efficient way” (Stiglitz et al. 2017). A root motivation for cost internalization is that economic markets require all exchanges in the economy to be voluntary, between freely consenting trade partners. Third parties must not be forced to pay for external costs arising from transactions. Market economies are meant to reward those who create net value, rather than those who merely redistribute value in zero-sum or negative-sum games. When the production of a good causes pollution, the costs of that pollution must, therefore, be paid by those taking the decision to produce and consume the product, rather than by unrelated third parties. Otherwise, producers and consumers can forcibly redistribute welfare from those third parties to themselves. Without bearing the full costs of their actions, such producers and consumers have an incentive to carry out transactions even when those transactions cause net harm to society after factoring in the external costs borne by their victims. To safeguard the core principles of liberty and net value creation, economic agents must, therefore, bear the full costs of their own actions. Pricing carbon emissions contributes to this “cost internalization.” Unlike carbon pricing, green bonds do not provide the needed marginal incentives for private decision makers to optimally factor carbon costs into their decision making.

Carbon pricing, and in particular carbon taxation, also has advantages for mitigating climate change in countries with a risk of “government failure”. When a government issues green bonds to finance a mitigation project, it needs to have good information about both the social costs and benefits of the project. The government also needs to have the needed institutional capacity for project implementation, despite potential corruption risk or a scarcity of public officials with the required technical knowledge. With carbon taxation, by contrast, the government does not need to have detailed information about costs and benefits of mitigation projects, but only about the marginal social cost of emissions (Posner 1992). After the government fixes the rate of a carbon tax to match this cost, the private sector determines in a decentralized process which mitigation projects carry the greatest private benefits per tonne of carbon reduced. Carbon taxation is also relatively simple to implement in countries with high corruption risk or low institutional capacity (Bento et al. 2018, Liu

2013, Markandya et al. 2013). This is because the government does not need to be able to observe where fuels are burned in the economy in order to price these emissions efficiently; carbon prices can instead be imposed “upstream” on the carbon content of fuels. The number of pipelines, mine mouths, and ports at which fuels enter the economy is much lower than the number of chimneys or mitigation projects to oversee. The government can accordingly concentrate its supervision over a small number of officials who impose a carbon price at a few fuel entry points to the economy, and all subsequent activities using these fuels are covered by the climate policy. It is then private trade partners who pass the carbon price signal through the market, to remote regions, to informal activities, to all industries. Each private agent has an incentive to fully enforce the price signal with their transaction partners, given the private incentive to pass on a tax incident. The public policy thus benefits from voluntary private enforcers where it lacks public ones.

As a by-product of its environmental role, carbon taxation or emissions trading systems with auctioned permits generate public revenues. This way climate policy can be designed so that it benefits not only future generations but also current ones. Using carbon revenues to reduce other taxes (such as labour taxes or social security contributions) can create net benefits in the welfare of current generations, in particular in developing countries (Heine and Black 2019, Coste et al 2019, Bento et al 2018, Parry et al 2015). Carbon taxes are effective for domestic resource mobilization, e.g. to finance green transitions or budget consolidations. This is because all fuel-related carbon emissions can be taxed at a few upstream choke-points, and cover the informal sector, whereas direct taxes (the counterfactual revenue source in many countries) need to be collected from a vast number of individuals and struggle in covering the informal sector of economies.

2.2 Role of green bonds

In principle, the needed financing for a low-carbon transition could be met entirely from pricing externalities. The IMF estimates that the gap between present taxation of fossil fuels and the level of taxation justified by external costs amounts to more than 5 trillion USD per year,⁵ which is well in excess of estimated financing needs to contain global warming.⁶ However, there are efficiency, political and equity arguments to use a combination of policy instruments.

Despite its potential, the present level of carbon pricing is completely insufficient to meet mitigation needs. “Apart from some modest steps forward in a couple of countries, there is little evidence of better use of taxes on energy use to address the mounting global environmental and climate challenges. Instead, real tax rates are gradually eroded by inflation in most countries, suggesting indifference to the environmental efficacy of taxes” (OECD 2018). Unless a breakthrough in fiscal policy can be achieved,⁷ additional financing sources for mitigation, such as green

5 See Lagarde and Gaspar (2019) which is based on Coady et al (2019) and Parry et al (2014).

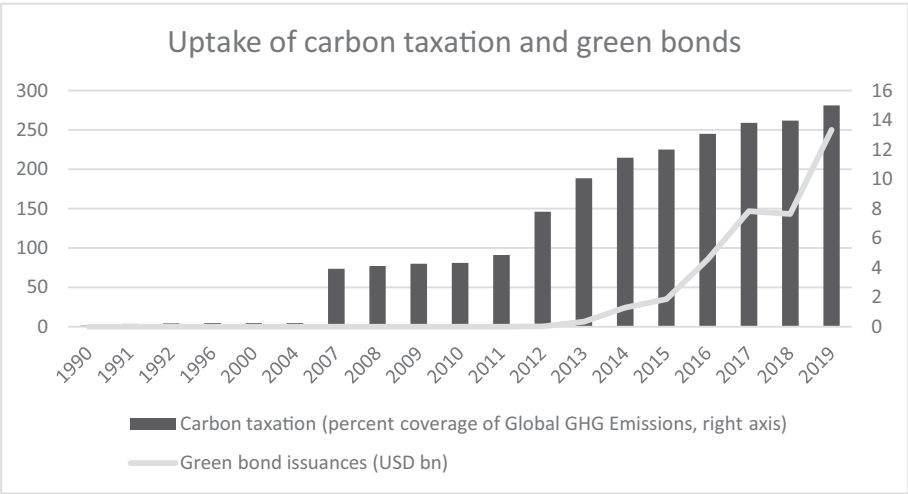
6 The International Energy Agency projects that limiting the temperature rise to 2 °C demands a tripling of annual low-carbon investment, to USD 790 billion by 2035 (IEA 2014). King et al. (2015) estimate “a minimum acceptable scale” for financing R&D in renewable energies at USD 1.5 trillion of the next decade.

7 Two recent unprecedented decisions do suggest that climate change is entering a new era of mainstreaming into fiscal policy: the IMF Board’s decision on carbon pricing and the inclusion of climate-fiscal policy in macroeconomic surveillance (IMF 2019), and the commitment by the new Coalition of Finance Ministers for Climate Action to jointly “work towards measures that result in effective carbon pricing” (World Bank 2019b).

bonds, are indispensable. Therefore, also Finance Ministers and Central Bank Governors that embrace carbon pricing equally call for stepping up green finance instruments.⁸

An efficiency case to go beyond carbon pricing arises due to the multiple market failures hampering a low-carbon transition (Acemoglu et al. 2012, Grubb et al. 2014). The green technology revolution will require investment across the innovation chain similar to that observed in the information and communication technology revolution (Mazzucato 2015, Norberg-Bohm 2000). Public investments must play a pivotal role in this change, to actively create low-carbon climate-resilient markets (Mazzucato 2016, Hallegatte et al. 2013). This “market-shaping” rather than “market-fixing” role for public finance is suggested both from evaluations of past large-scale technological transitions (Mazzucato 2013) and from assessments of the role of public finance in developing segments of the renewable energies market that have high potential returns but also high risk and where evidence suggests that public finance crowds in private finance and creates new markets (Mazzucato and Semieniuk 2017, 2018a). Green bonds offer a potential financing source for this public and private investment need.

Figure 1



Both carbon pricing policies and the issuance of green bonds have grown significantly since 1990 (Climate Bonds Initiative 2019, World Bank 2019a), but are still severely insufficient for containing climate change. Data for green bonds is available up until June 2019 and interpolated for the full year. From 2010 to 2018, the European Union (33.4%), China (14%), Multilateral Organizations (13.7%), and the United States (12%, excluding US municipalities bonds) were the world largest green bond's issuers (Bloomberg Green Bonds Database).

Green bonds can equally help finance adaptation to climate change impacts. A carbon tax provides mitigation incentives; it only indirectly reduces the vulnerability of the economy to climate change,

8 See the commitment of the new Coalition of Finance Ministers for Climate Action to “mobilize private sources of climate finance by facilitating investments and the development of a financial sector which supports climate mitigation and adaptation” (World Bank 2019b), the central banks’ actions in the Network for Greening the Financial System, and the European Commission’s Action Plan on Financing Sustainable Growth.

and it does not compensate victims. Addressing these additional challenges requires public and private expenditures. When deployed together, multiple policy levers can create a stable and consistent direction for long-term green investment and innovation (Mazzucato and McPherson, 2018, Mazzucato and Semieniuk, 2018b). Green bonds can cater to adaptation projects, although the vast majority is currently used for mitigation.⁹

In addition to economic efficiency, combining carbon pricing with green bonds is more politically feasible. The introduction of carbon pricing continues to be far too slow to meet the internationally-agreed objective of limiting climate change to well below 2 °C, and this delayed policy action causes significant economic harm. Stabilizing the climate later is dramatically more expensive than it would be earlier. If a sufficient level of carbon pricing is currently politically unattainable, it is better to use second-best policy instruments than to do too little too late. Where policymakers shy away from incurring short-term costs for long-term gains, bonds could make climate policy incentive-compatible by shifting some of the costs of today's climate policy into the future (Sachs 2015).

Such a shift reduces problems of intergenerational equity. Climate policy invariably involves intergenerational transfers; the mitigation of emissions today benefits future generations. The current generation may then wish to let future generations share in the cost of today's efforts. Green bonds, especially government issuances with longer maturities, are an instrument for achieving such burden-sharing when they are used to finance today's mitigation actions and are repaid by future generations. Such burden-sharing is Pareto superior to a business-as-usual scenario with insufficient mitigation today (Sachs 2015, Orlov et al. 2018).

Notwithstanding these potential benefits, the use of green bonds can be problematic in countries with existing high public or private debt. Debt levels are rising in many low-income countries (Essl et al. 2019), and in such circumstances it is optimal for climate policy to be financed by taxation or budget reallocation instead of deficit spending (Forni et al. 2019). However, the greatest mitigation effort should be borne by the largest carbon emitters, which are mostly high and middle-income countries that have already issued green bonds.¹⁰ And public debt levels are presently a smaller concern in that income group (Blanchard 2019), suggesting that for countries with sufficient fiscal space, green bonds might help improve the climate policy mix.

The relation of green bonds to debt levels also depends on their cost relative to traditional bonds. A critique has been that green bonds may be more expensive than conventional bonds and thus raise debt. But this finding may be driven by sample selection bias. To evaluate climate financing strategies, it is important to control for the types of investment projects and investor characteristics when comparing yields of green and vanilla bonds. The relevant point of comparison for a country which is considering the use of green bonds for financing climate projects is the yield of vanilla bonds that are used in financing these same climate projects, and not the yield of conventional bonds averaged across the broader market. With these controls, green bonds are paying lower yields (Flammer 2018; Baker et al. 2018; Karpf and Mandel, 2018, Partridge and Medda, 2018). So given

9 The Climate Bonds Initiative (2017) finds that only 5% of the volume leveraged through green bonds by April/2016 were invested in adaptation projects. The authors also analyse the Bloomberg Green Bonds Database and find that only 2% of the certified issuers (from 2010 to 2018) declared that the volume leveraged should be used only for adaptation projects. The expected use of proceeds of each certified green bond is published by a third-party report at the time of issuance.

10 Among the largest emitters, Russia was the last country to enter the green bond market in January 2019.

that a country is seeking to finance investments into its climate transition, green bonds appear cheaper than vanilla bonds. There is also evidence that the act of certifying a green bond through a third party reduces its yield (Bachelet et al. 2019) and that multilateral organizations and some governments are able to de-risk green bonds and reduce capital costs for green investments (Braga, Semmler & Grass, 2019).

Another motivation for public and private agents to issue green bonds is the diversification of their investor base. With recent investor declarations,¹¹ there is a perception that demand for green bonds currently exceeds supply,¹² and some evidence that green bonds helped sovereigns attract new investors who had not previously invested in those countries.¹³

Lastly, green bonds have a communications role. Many sovereigns and corporates have made long-term climate commitments. Issuing green bonds can help signal that these commitments are more than cheap talk. But so can (implicit) carbon pricing.

2.3 Interaction effects of carbon pricing and green bonds

When carbon pricing and green bonds are implemented jointly, they have interaction effects, which vary depending on the type of carbon pricing used. Two major options are available for implementing carbon pricing: ETS and taxes. Much literature analyses their respective advantages; here we focus only on these interaction effects.

Green bonds are more successful when climate change mitigation projects have higher private returns. These returns, rise with carbon pricing which creates a level playing field for low-carbon investments over high-carbon alternatives, by ensuring that the social cost of pollution is factored into private costs of production. Thus, a sufficiently high CO₂ price supports the successful market introduction of green bonds. Climate bonds are only partly an alternative to carbon pricing if they require carbon pricing for their market success. However, if carbon pricing takes off, we can expect green bonds to thrive as well.

The interaction effect between the value of green bonds and carbon prices is more ambiguous for ETS than for carbon taxes. An ETS puts a cap on emissions, and emissions leakage can occur when green bonds finance climate change mitigation projects for industries covered by the cap. Mitigation achieved through bonds reduces the scarcity of emissions permits under the cap, reducing

11 The United Nations "Principles for Responsible Investment", the "Climate Action 100+", and the "Investor Group on Climate Change" (IGCC) respectively have been adopted by 2200, 42 and 170 investors which own or manage USD 80, 33.6 and 26 trillion in assets.

12 A recent survey with 540 large investors reported that only 14% of them believed that current supply of ESG fixed income products is sufficient for the market, and that only 6.6% are satisfied with the current bond market penetration of ESG disclosures (RBC Global Assessment Management 2018).

13 Given that green bonds attract new social responsible investors and asset managers, several issuers report the diversification of their investors base as a benefit of this type of security (OECD 2017, GIZ 2018). We can also observe this benefit for some governments and sovereign bond issuers (Climate Bonds Initiative 2018). Poland, for instance, issued its first sovereign green bond in 2016 and reported that around 61% of the investors were green investors, most of them new to the Poland market. France issued a green bond on January 2017 and foreign investors accounted for 63% of the volume issued, while the non-resident holders have a share of only 55% in the total french government debt (Moody's 2018). At the local level, the State of Massachusetts in the US (World Bank 2015) and the Transport for London in the UK (Climate Bonds Initiative 2016) also attracted new green individual investors for their base. Green bonds appear to be attracting new long-term oriented investors that value environmental issues (Flammer 2018) and are a diversification opportunity for investors that currently rely on equity and energy assets, as those co-move weakly with the green bonds market (Reboredo 2018).

Figure 2



The use of carbon pricing (top) and green bonds (bottom) has evolved with similar spatial distributions. Countries are increasingly using both instruments jointly to finance their low-carbon transitions. Data sources: World Bank (2019a), Bloomberg Green Bonds Database.

the price of those permits and thereby allowing the displacement of emissions rather than their net reduction. To prevent this, the emissions cap should be tightened when green bonds are introduced, but those adjustments may be politically impossible precisely in situations where green

bonds are sought. If green bonds are introduced as a second-best policy to fill the gap left by political opposition to serious carbon pricing, the same political opposition may prevent an adjustment in emissions caps.

The introduction of green bonds might break the political gridlock by creating new vested interests. Green bond holders have an interest in tightening ETS caps. Current lobbying by industries to loosen emissions caps could then be counterbalanced by new lobbying from these investors. By that reading, the creation of green bonds could both weaken and strengthen ETS. With a carbon tax, these outcomes are clearer. As the tax rate is more stable irrespective of the deployment of green bonds, the risk that green bonds and carbon taxes will undermine each other diminishes.

Another interaction effect between green bonds and carbon prices works through price volatility. As for other bonds, green investment projects can more easily attract green bond financing if returns on investment are less volatile (Gevorkyan et al. 2017). As the returns for green investment projects depend on carbon prices, a more stable carbon price also creates a more stable return on investment and accordingly greater demand for green bonds. Therefore, green bonds generally work better alongside carbon taxes than emissions trading schemes, because carbon taxes lock in a stable carbon price whereas ETS prices around the world have been very volatile.

3 Three Scenarios for the Climate Policy Mix

In this section, we model options for financing the green transition with carbon taxation, green bonds or no policy. The solution procedure that we use in this modelling is called “nonlinear model predictive control” (NMPC). As described in Gruene et al. (2015), this procedure allows solving models with finite time decision horizons, avoids the information requirements of infinite horizon models, allows for limited-information agents, and permits changes of stages and regimes in model variants. It also resembles human decision-making based on short-term interests instead of optimization over the long run or infinite horizon.¹⁴

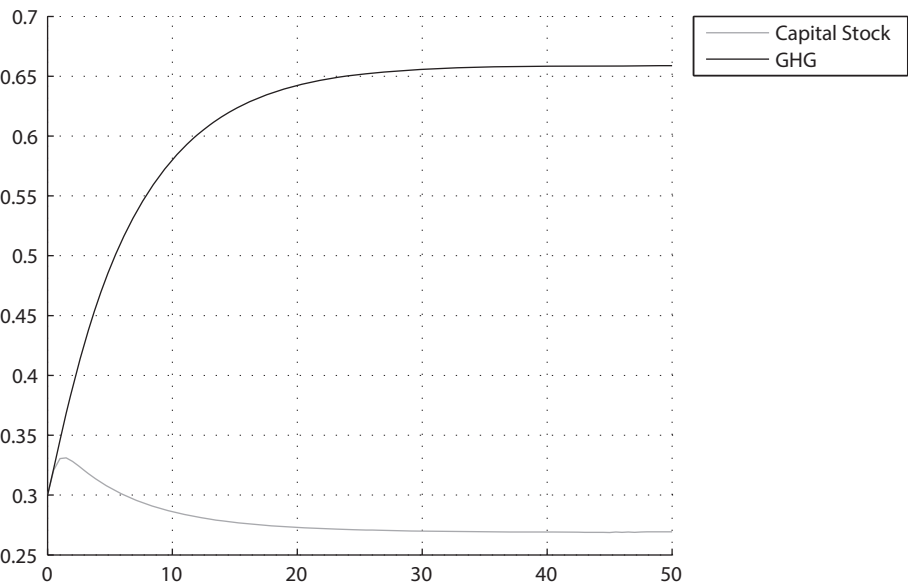
Scenario 1 presents business-as-usual (BAU).¹⁵ In this case, no attempt is made toward mitigation, and GHG levels increase indefinitely. Capital grows initially but thereafter declines as climate change damage is realized. This scenario is also modelled and calibrated in DICE (see Nordhaus 2008).

14 In our modelling procedure, we pursue simplified versions. We want to elaborate on transparent and simple scenarios and policy options, and thus we will abstain from detailed functional specifications and calibrations, as, for example, undertaken in the Nordhaus DICE model. For a detailed specification and calibration of the Nordhaus DICE model, and its solution using an advanced numerical solution, through NMPC, see Weller et al. (2015), Kellet et al (2018), and Nordhaus (2008), and Orlov et al (2018) who use GAMS. In our more stylized model variants, we have reduced the dimensions of the mode and simplified the CO₂ emissions process, the mitigation effort, and the damage function. In characterizing different policy options, we keep our model variant simple and transparent, and neglect the issue of participation rates of different countries. Details of such simplified model variants, with sparse calibration efforts, can be found in Smulders (1995), Hettich (2000), Greiner and Semmler (2008), Greiner et al. (2010), Orlov et al. (2018) and Flaherty et al (2017).

15 The solution procedure that we use in this modelling is nonlinear model predictive control (NMPC). As described in Gruene et al. (2015), this procedure allows solving models with finite time decision horizons, avoids the information requirements of infinite horizon models, allows for limited-information agents, and permits changes of stages and regimes in model variants. It also resembles human decision-making based on short-term interests instead of optimization over the long run or infinite horizon. We perceive the stages in scenarios 2 and 3 as separate regimes, in which thresholds for switching between the regimes are exogenous. We make several simplifying assumptions. Note that the switching time could also be a decision variable, but this approach is not pursued here. For a regime-switching model of this type, see Orlov et al. (2018). Another aspect not addressed in the model is a calculation of optimal levels of green bonds and

Figure 3

Simulation of scenario 1 (BAU).



Scenario 2 explores the effect of green bonds but does not include carbon taxation. In this scenario, investment in green infrastructure is immediately undertaken in the first stage and scheduled to be repaid by future generations in a second stage. While GHG levels drop, capital continues to accumulate, and a debt burden accumulates in the first time period, repaid by taxes in later time periods. Note that the two periods are modelled as consecutive stages, which do not overlap.

Scenario 3 introduces a carbon tax in addition to green bonds. The second stage of this scenario is similar to that of the second scenario, as carbon tax revenues fall in the shift to a greener economy. The third scenario (green bonds with complimentary carbon tax) reduces GHG emissions and allows for a higher steady-state level of capital, while minimizing debt repayment by the second generation, as bond repayment is phased in earlier. In this scenario, the transition to a low-carbon economy is accelerated in consecutive stages.

Initial values and model parameters at each stage are presented in Appendix B. We run the simulation for each scenario separately, but the results for scenarios 2 and 3 are presented together to facilitate a comparison of the two policy choices.

In the business-as-usual scenario, there is a brief period of economic growth, propped up through the continued use of carbon-intensive processes. After a short period however, the economic

tax rates as well as the endogenous regime-switching from one stage to another required to achieve emissions mitigation in the most welfare-improving manner. The model formulae and specifications are detailed in Appendix A.

damages caused from the accumulated greenhouse gases lead to economic losses and a low steady-state capital accumulation. Greenhouse gas accumulations level off at a high level, as no transition is achieved.

Scenario 2 has two distinct phases: the green transition of the economy and the sustainable economy. During the greening phase, economic growth is accompanied by the accumulation of debt and the diminishing of greenhouse gas levels. This represents the transition from carbon-based fuels and production processes towards sustainable practices. Low-carbon infrastructure and technology requires investment and as it becomes more prevalent, greenhouse gases begin to dissipate from the atmosphere. In the absence of carbon taxation, the transition is paid by issuing debt. In the sustainable economy, economic production produces negligible emissions, and the debt accumulated during the transition period is repaid. Over the course of the two phases, capital accumulation is maintained at elevated levels, greenhouse gases shrink to negligible levels, and debt accumulates for a time and is subsequently repaid entirely.

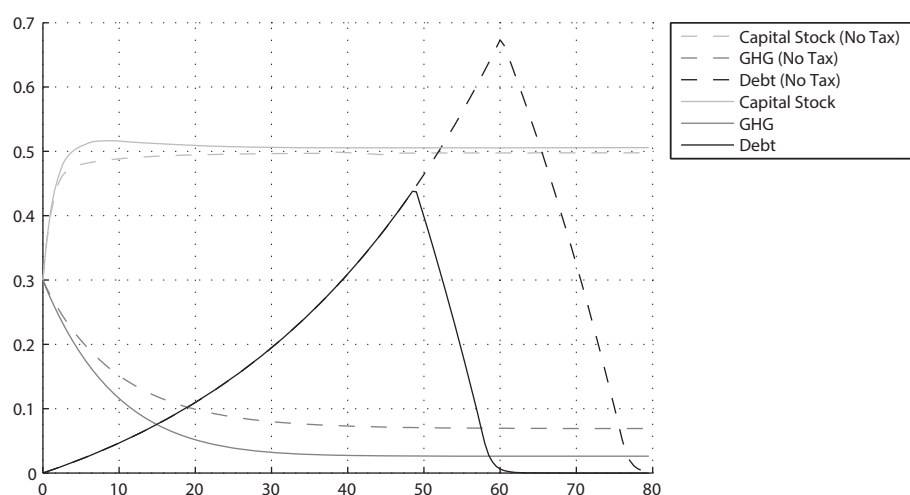
Scenario 3 mirrors the dynamics of Scenario 2 very closely, with the inclusion of a complimentary carbon tax policy. Inclusion of the carbon tax disincentivizes the use of carbon-intensive fuels, so the transition towards sustainability is accelerated. Greenhouse gases settle at a lower level than Scenario 2, capital accumulation is able to hit a higher steady-state, and debt repayment is achieved earlier. Overall, a comparison of the three scenarios shows that scenario 1, with no mitigation effort undertaken, is the most costly and scenario 3, with a mix of policies, produces the best welfare outcomes.

4 Conclusion

In the Paris Agreement, all countries committed to transitioning towards low-carbon, climate-resilient economies. Several policy instruments have been proposed to finance this transition, including green bonds and carbon pricing. Often these instruments are perceived as alternative choices, but this paper finds there are important gains from deploying them jointly, provided countries have sufficient fiscal space. Debt levels are rising in many low-income countries (Essl et al. 2019), and in such circumstances it is preferable for climate policy to be financed by taxation or budget reallocation instead of deficit spending (Forni et al. 2019). However, for advanced economies, Blanchard (2019) observes that sovereign debt is, in contrast to corporate debt, not rising that much, so there may be space for pursuing climate policies by green bonds and carbon pricing. Carbon pricing improves the performance of green bonds, which in turn improve inter-generational equity, political feasibility, and help address multiple market failures. Yet, not all carbon pricing is the same: the synergies with green bonds are greater for carbon taxation than for emissions trading.

In most current impact assessment models, the burden of enacting mitigation and adaptation policies falls on current generations. We set up an intertemporal model, related to Sachs (2015), that proposes burden sharing between current and future generations, by allowing for both carbon taxation and green bonds as funding sources for a low-carbon transition. The issuance of green bonds helps to improve intergenerational equity by funding immediate investment in climate mitigation, with bonds repaid by future generations which thereby participate in financing the reduced climate damage from which they will benefit. We examine three scenarios using a new numerical procedure called NMPC that allows for finite horizon solutions and phase changes. We

Figure 4

Simulations of scenarios 2 and 3

show that the combination of carbon pricing and green bonds improves environmental effectiveness, capital accumulation and debt sustainability. The main conclusion is that, in the financing of low-carbon transitions, a mix of policies should be used that includes green bonds and carbon pricing, which seems to be politically more feasible, speeds up the transition, and exhibits greater intergenerational fairness.

References

- Acemoglu, D., P. Aghion, L. Bursztyn, and D. Hemous (2012): The Environment and Directed Technical Change, *American Economic Review*, 102(1), 131–166.
- Bachelet, M. J., L. Becchetti, and S. Manfredonia (2019): The Green Bonds Premium Puzzle: The Role of Issuer Characteristics and Third-Party Verification. *Sustainability*, 11(4), 10–98.
- Baker, M., D. Bergstresser, G. Serafeim, and J. Wurgler (2018): Financing the response to climate change: The pricing and ownership of US green bonds. Working Paper. 25194. National Bureau of Economic Research.
- Bartlett, N., E. Whittington, S. Reuvers, and K. Thoday (2016): Driving Low-Carbon Growth through Business and Investor Action. Paper presented at Seizing the Global Opportunity: Partnerships for Better Growth and a Better Climate, New Climate Economy, Washington D.C.
- Bento, A., M. Jacobsen, and A. Liu (2018): Environmental Policy in the Presence of an Informal Sector. *Journal of Environmental Economics and Management*, 90, 61–77.
- Blanchard, O. (2019): Public Debt: Fiscal and Welfare Costs in a Time of Low Interest Rates, Peterson Institute of International Economics, Policy Brief 19–2.

- Braga, J. P., W. Semmler, and D. Grass (2019): De-risking of Green Investments: An Empirics of the Green Bond Market and a Dynamic Model. Working Paper. The New School for Social Research.
- Climate Bonds Initiative (2016): Case Study: Transport for London (TfL) Green Bond. Available at <https://www.climatebonds.net/files/files/Case%20study%20-%20TfL%20final.pdf>.
- Climate Bonds Initiative (2017): Post-issuance Reporting in the Green Bond Market. Available at https://www.climatebonds.net/files/files/ClimateBonds_Green_Bonds_Post_Issuance_Use_of_Proceeds_Report_.
- Climate Bonds Initiative (2018): Sovereign Green Bonds Briefing. Available at https://www.climatebonds.net/files/files/Sovereign_Briefing2017.pdf.
- Climate Bonds Initiative (2019): Green Bonds Market 2019. Available at <http://climatebonds.net>.
- Coste, A., M. Cali, N. Cantore, and D. Heine (2019): Staying Competitive: Productivity Effects of Environmental Taxes, in: Pigato, M. (ed.), *Fiscal Policies for Development and Climate Action*. Washington D.C.: World Bank Group.
- Essl, S., S. Kilic Celik, P. A. Kirby, and A. Proite (2019): Debt in Low-Income Countries: Evolution, Implications, and Remedies. World Bank Policy Research Working Paper 8794.
- Flaherty, M., A. Gevorkyan, S. Radpour, and W. Semmler (2017): Financing Climate Policies through Climate Bonds: A Three-Stage Model and Empirics. *Research in International Business and Finance*, 42(C), 468–479.
- Flammer, C. (2018): Corporate Green Bonds. Working Paper. Boston University.
- Forni, L., M. Catalano, and E. Pezzolla (2019): Increasing Resilience: Fiscal Policy for Climate Adaptation, in: Pigato, M. (ed.), *Fiscal Policies for Development and Climate Action*. Washington D.C.: World Bank Group.
- Gevorkyan, A., M. Flaherty, S. Radpour, and W. Semmler (2017): Financing Low Carbon Transitions Through Carbon Pricing and Green Bonds – Theory and Empirics. Manuscript, New School for Social Research.
- GIZ (2018): Green Bonds – Ecosystem, Issuance Process and Case Studies. Consultation Draft. Bonn: Deutsche Gesellschaft für Internationale Zusammenarbeit.
- Greiner, A., L. Gruene, and W. Semmler (2010): Growth and Climate Change: Threshold and Multiple Equilibria, in: *Dynamic Systems, Economic Growth and Environment*. Oxford University Press, New York.
- Greiner, A., and W. Semmler (2008): *Global Environment, Natural Resources and Economic Growth*. Oxford University Press, New York.
- Grubb, M., J. Hourcade, and K. Neuhoﬀ (2014): *Planetary Economics*. Abingdon, Routledge.
- Gruene, L., W. Semmler, and M. Stier (2015): Using Nonlinear Model Predictive Control for Solving Dynamic Decisions Problems in Economics, *Journal of Economic Dynamic and Control*, 60, 112–133.
- Hallegatte, S., M. Fay, and A. Vogt-Schilb (2013): Green Industrial Policies: When and How, World Bank Policy Research Working Paper 6677.
- Heine, D., and S. Black (2019): Benefits beyond Climate: Environmental Tax Reforms, in: Pigato, M. (ed.), *Fiscal Policies for Development and Climate Action*. Washington D.C.: World Bank Group, 1–64.
- Hettich, S. (2000): *Economic Growth and Environment Policy*. Cheltenham, UK: Edward Elgar.
- International Energy Agency (2014): *World Energy Investment Outlook*, Paris: IEA.
- International Monetary Fund (2019): IMF Executive Board Reviews Fiscal Policies for Paris Climate Strategies, Press Release 19/136.

- Karpf, A., and A. Mandel (2018): The changing value of the ‘green’ label on the US municipal bond market. *Nature Climate Change* 8(2), 161.
- Kellett, C., S. Weller, T. Faulwasser, L. Grüne, and W. Semmler (2019): Feedback, dynamics, and optimal control in climate economics, *Annual Reviews of Control*.
- King, D., J. Browne, R. Layard, G. O’Donnell, M. Rees, N. Stern, and A. Turner (2015): *A Global Apollo Programme to Combat Climate Change*, London: LSE Center for Economic Performance.
- Lagarde, C., and V. Gaspar (2019): Getting Real on Meeting Paris Climate Change Commitments. IMF Blog. Available at <https://blogs.imf.org/2019/05/03/getting-real-on-meeting-paris-climate-change-commitments/>.
- Liu, A. (2013): Tax Evasion and Optimal Environmental Taxes. *Journal of Environmental Economics and Management*, 66(3), 656–670.
- Markandya, A., M. González-Eguino, and M. Escapa (2013): From shadow to green: Linking environmental fiscal reforms and the informal economy. *Energy Economics*, 40, S108–S118.
- Marron, D., and A. Morris (2016): *How to Use Carbon Tax Revenues*. Washington D.C.: Tax Policy Center.
- Mazzucato, M. (2013): *The Entrepreneurial State: Debunking Public vs. Private Sector Myths*. London: Anthem Press.
- Mazzucato, M. (2015): The Green Entrepreneurial State, in: Scoones, I., Leach, M., and Newell, P. (eds.), *The Politics of Green Transitions*. Abingdon: Routledge.
- Mazzucato, M. (2016): From Market Fixing to Market-Creating: A New Framework for Innovation Policy. *Industry and Innovation*, 23 (2), 140–156.
- Mazzucato, M., and M. McPherson, (2018): *The Green New Deal: A bold, mission-oriented approach*. UCL Institute for Innovation and Public Purpose Working Paper IIPP PB 04.
- Mazzucato, M., and G. Semieniuk (2017): Public financing of innovation: new questions, *Oxford Review of Economic Policy*, 33(1), pp. 24–48.
- Mazzucato, M., and G. Semieniuk (2018a): Financing renewable energy: Who is financing what and why it matters, *Technological Forecasting and Social Change*, 127, 8–22.
- Mazzucato, M., and G. Semieniuk, (2018b): Bridging the gap: The role of innovation policy and market creation, in: Christensen et al. (eds.), *Emissions Gap Report 2018*, Nairobi: United Nations Environment Programme.
- Moody’s (2018): Sovereign green bond market on course for critical mass, but challenges remain. *Sector in Depth: Sovereign and Supranational*. Press Release 09/07/2018.
- Norberg-Bohm, V. (2000): Creating Incentives for Environmentally Enhancing Technological Change: Lessons From 30 Years of US Energy Technology Policy. *Technological Forecasting and Social Change*, 65(2), 125–148.
- Nordhaus, W. (2008): *A Question of Balance*. New Haven: Yale University Press.
- Organisation for Economic Cooperation and Development (2017): *Mobilising Bond Markets for a Low-Carbon Transition, Green Finance and Investment*. Paris: OECD Publishing.
- Organisation for Economic Cooperation and Development (2018): *Taxing Energy Use 2018: Companion to the Taxing Energy Use Database*. Paris: OECD Publishing.
- Orlov, S., E. Rovenskaya, J. Ptaschunder, and W. Semmler (2018): *Green Bonds, Transition to a Low-carbon Economy, and Intergenerational Fairness – Evidence from the DICE Model*. IIASA Working Paper, Laxenburg, Austria.
- Parry, I., D. Heine, S. Li, and E. Lis (2014): *Getting Energy Prices Right: From Principle to Practice*. Washington, D.C.: International Monetary Fund.
- Parry, I., C. Veung, and D. Heine (2015): How Much Carbon Pricing Is in Countries’ Own Interests? The Critical Role of Co-Benefits. *Climate Change Economics*, 6(4), 1550019.

- Partridge, C., and F. Medda (2018): Green Premium in the Primary and Secondary US Municipal Bond Markets. London: QASER Laboratory, University College London.
- Posner, R. (1992): *Economic Analysis of Law*. 4th ed. Chicago: Chicago University Press.
- RBC Global assessment Management (2018): Responsible Investing Survey. Available at: <https://www.rbcgam.com/en/ca/about-us/responsible-investment/our-latest-independent-research>.
- Reboredo, J. C. (2018): Green bond and financial markets: Co-movement, diversification and price spillover effects. *Energy Economics*, 74, 38–50.
- Sachs, J. (2015): Climate Change and Intergenerational Well-Being, in: Bernard, L. & W. Semmler (ed.), *The Oxford Handbook of the Macroeconomics of Global Warming*, Oxford: Oxford University Press, 248–259.
- Smulders, S. (1995): Entropy, Environment and Endogenous Growth. *International Tax and Public Finance*, 2(2), 319–340.
- Stiglitz, J., N. Stern, M. Duan, O. Edenhofer, G. Giraud, G. Heal, E. Lèbre la Rovere, A. Morris, E. Moyer, M. Pangestu, P. Shukla, Y. Sokona, and H. Winkler (2017): Report of the High-Level Commission on Carbon Prices. Washington D.C.: World Bank Group.
- Wagner, G., T. Kaberger, S. Olai, M. Oppenheimer, K. Rittenhouse, and T. Sterner (2015): Energy policy: Push renewables to spur carbon pricing. *Nature News*, 525(7567), 27.
- Weller, S., S. Hafeez, and C. Kellett (2015): A receding horizon control approach to estimating the social cost of carbon in the presence of emissions and temperature uncertainty. *IEEE 54th Annual Conference on Decision and Control (CDC)*, Osaka.
- World Bank (2015): Green Bonds Are Changing Investor Expectations & Making Sustainable Investing Easier. Available at <https://www.worldbank.org/en/news/feature/2015/01/22/green-bonds-changing-investor-expectations-three-trends>.
- World Bank (2018): Green Bond Impact Report, Washington D.C.: World Bank Treasury.
- World Bank (2019a): State and Trends of Carbon Pricing 2019. Washington D.C.: World Bank.
- World Bank (2019b): Finance Ministers Join Forces to Raise Climate Ambition. Press Release 2019/152/CCG.

A Models and Assumptions

A.1 Scenario 1: Business As Usual

The baseline model, implicit in the Nordhaus DICE model and also employed in Flaherty et al. (2016), contains a policy maker that optimizes consumption over a finite period to maximize utility over a continuous time horizon. The utility function and budget constraint are similar to those in standard models, with the addition of greenhouse gas (GHG) accumulation, M , and damage caused by the resulting climate change, D . Future consumption is discounted at rate ρ in a logarithmic utility function.¹⁶

$$\text{Max}_C \int_{t=0}^N e^{-\rho t} \ln(C) dt \quad (1)$$

subject to

$$\dot{K} = D \cdot Y - C - (\delta + n)K \quad (2)$$

and

$$\dot{M} = \beta E - \mu M \quad (3)$$

where $K(0) = K_0$, and $M(0) = M_0$. The production is defined as:

$$Y = K^\alpha \quad (4)$$

with $\alpha \in (0, 1)$.

Emissions,¹⁷ are given by the function¹⁸

$$E = \left(\frac{aK}{A_0} \right)^\gamma \quad (5)$$

where A_0 represents an exogenous mitigation effort¹⁹ that exists even in the absence of any conscious mitigation action and γ affects emissions growth, such as emissions mitigation that occurs through remaining technological progress in the absence of policy.

The stock of GHG emissions has a damaging effect on production. Accumulation of these emissions

¹⁶We make a simplification to study stylized scenarios and stages. Instead of standard preferences, one could allow for heterogeneous agents, habit formation, or penalty functions, instead of simple log preferences. In Eq. (2), we have no bond holding. This baseline model thus avoids a discussion on the relationship of the above type of model to the Ricardian Equivalence Theorem. This relationship is analysed in Orlov et al. (2018).

¹⁷Following Greiner et al. (2010) with the addition of abatement efforts.

¹⁸Note that our specification of the emission equation (5) is similar to the DICE model, where emissions arise with GDP at a declining rate (Weller et al. 2015, and Kellet et al. 2019). Since our γ is set at 1, and the capital-output ratio is roughly 3 to 4 in most economies, our formulation of emissions resembles that of the DICE model.

¹⁹Note that in the DICE model, mitigation effort is multiplicatively applied to GDP. Mitigation effort, a control variable in the DICE model, affects a fraction of GDP, and the remaining fraction is then “brown” GDP. In our formulation, we let mitigation affect capital stock, which in our case is the polluting variable. As to the calibration of GHG emissions per \$1,000 of the capital stock, the DICE measure (Weller et al. 2015, and Kellet et al. 2019), can directly applied to our case, since we assume a constant capital-output ratio.

results in a damage function, $D(\cdot)$, to adversely affect output, Y . We assume a damage function, which is often used in integrated assessment models,²⁰

$$D(\cdot) = (a_1 \cdot M^2 + 1)^{-\psi} \quad (6)$$

with²¹ $a_1 > 0$, $\psi > 0$.

A.2 Scenario 2: Financing the Green Economy through Green Bonds

In scenario 2, GHG mitigation is funded through the sale of green bonds in the first stage; accumulated debt is repaid in the second stage. Despite this addition, the basic characteristics of the model remain the same as the cost of abatement is factored into both the capital stock equation and emissions function, and a state equation is introduced to account for the accumulation of public debt. Households choose consumption in order to maximize welfare over a finite time horizon.

$$\text{Max}_C \int_{t=0}^N e^{-\rho t} \ln(C) dt \quad (7)$$

subject to

$$\dot{K} = D \cdot Y - C - (\delta + n)K \quad (8)$$

$$\dot{M} = \beta E - \mu M \quad (9)$$

$$E = \left(\frac{aK}{bA + A_0} \right)^\gamma \quad (10)$$

A represents abatement or mitigation efforts²² financed by green bonds (during the first stage) which are repaid in the second stage. Since the abatement effort in the capital stock equation is reimbursed by the bonds issued, it does not appear as a cost. The debt dynamics equation for the first stage is:

$$\dot{B} = r \cdot B + A \quad (11)$$

In this stage, public debt (B) is a function of the cost of abatement efforts (A), interest rate (r) and initial public debt ($B(0)$). Green bonds are issued until time T , at which point GHG mitigation has reduced emissions to a lower equilibrium point (compared to the first model) as a result of abatement efforts and

²⁰See Greiner et al. (2010). In the DICE model, temperature is used instead of the stock of GHG, M .

²¹Note that the value of Eq. (5) will decline with rising GHG emissions, and is multiplied with GDP; see Eq. (8); the non-damaged part of GDP will fall with rising GHG emissions. Note also that even though temperature is usually modelled as driving the damages, we use GHG as the driving variable to avoid an additional state variable.

²²Note that subsequently we will take $b = 5$.

effectively reduced climate impacts on production.²³

The second stage of the model consists of the repayment of green bonds through taxation, facilitated by the extra output gained from higher capital stock accumulated as a result of lower emissions levels, and preserving the green economy by continued abatement efforts. In the absence of notable climate damage, consumption is reduced by only the taxation required to pay down public debt and keep abatement efforts in place.

$$Max_C \int_{t=0}^N e^{-\rho t} \ln(C) dt \quad (12)$$

subject to

$$\dot{K} = Y(1 - \tau) - C - (\delta + n)K \quad (13)$$

and

$$\dot{B} = r \cdot B - \tau Y \quad (14)$$

where τ is an income tax rate to repay accumulated debt and maintain the abatement effort at its equilibrium level.

A.3 Scenario 3: Supporting the Green Economy through Carbon Taxes and Green Bonds

Scenario 3 is similar to scenario 2, but includes an additional carbon tax in the capital stock equation during the first stage. The first stage equations for this scenario are:

$$Max_C \int_{t=0}^N e^{-\rho t} \ln(C) dt \quad (15)$$

$$\dot{K} = D \cdot Y - C - \chi Y - (\delta + n)K \quad (16)$$

$$\dot{M} = \beta E - \mu M \quad (17)$$

$$E = \left(\frac{aK}{b(A + \chi Y) + A_0} \right)^\gamma \quad (18)$$

where χ is the carbon tax rate levied on remaining emissions-intensive output. The carbon tax rate is set using an arc-tangent function:

²³We make a distinction between the optimization time horizon, N , and the length of the stage T . The switching of the stages occurs at period T , which may differ between the first stage (bond-financed mitigation policy) and the second stage (repayment of bonds).

$$\chi = b_1 \frac{2}{\pi} \arctan(b_2 M^2 - 0.01) \quad (19)$$

Since our model does not explicitly differentiate between low-carbon and carbon-intensive capital, we can only tax the capital or its output as a whole.²⁴ This is done with a changing (decreasing) tax rate to take into account that some capital stock may already represent green capital. The suggested carbon tax is applied as a semi-flat tax rate on output as long as the emissions level is higher than the desired amount. Revenue from the carbon tax is spent on abatement efforts.

The second stage of the model is the same as the second stage of scenario 2:

$$\text{Max}_C \int_{t=0}^N e^{-\rho t} \ln(C) dt \quad (20)$$

subject to

$$\dot{K} = Y(1 - \tau) - C - (\delta + n)K \quad (21)$$

and

$$\dot{B} = r \cdot B - \tau Y \quad (22)$$

After repayment of green bonds, the total tax revenue, τY , is equal to the cost of abatement efforts at the equilibrium, $A + \chi Y$, with a small χ . Although the carbon tax rate is higher for the first generation, which faces higher levels of emissions and related damages, future generations, which pay almost nothing in carbon taxes and lose less output to climate damage, have to repay the green bonds and maintain the level of abatement efforts by paying a small carbon tax.

In this third scenario, too, the time periods of switching to the second stage, being in the first stage, as well as the duration of the second stage when the bonds are repaid, is exogenously defined. Only measured small tolerances for GHG emissions, as well as bond repayments, determine the switching. The switching times are not a choice variable.²⁵

B Parameters and Assumptions

B.1 Scenario 1: Business as Usual

We chose initial values of both capital stock and GHG levels below their equilibrium values to let the system grow initially. However, as a result of the damage from accumulating GHG, the final equilibrium value for the capital stock is below the initial value of 0.3. This decline in the equilibrium value of capital

²⁴A similar nonlinearity is observable in the DICE model, in which the non-polluted share of GDP is determined by mitigation efforts as well as the time-varying carbon intensity of output (Weller et al. 2015 and Kellet et al. 2019).

²⁵For a model with endogenous switching time, see Orlov et al. (2018).

Parameter	Definition	Scenario 1	Scenario 2	Scenario 3
n	Population growth rate	0.02	0.02	0.02
α	Output elasticity of capital	0.18	0.18	0.18
β	Share of emission added to GHG	0.49	0.49	0.49
γ	Capital elasticity of emission	1	1	1
δ	Depreciation rate	0.075	0.075	0.075
μ	Constant decay rate	0.1	0.1	0.1
ρ	Discount rate	0.03	0.03	0.03
ψ	Exponential damage factor	1	1	1
A	Abatement effort	0	0.005	0.005
A_0	Constant abatement parameter	0.0012	0.0012	0.0012
a_1	Quadratic damage parameter	0.5	0.5	0.5
a	GHG emission scaling factor	0.00035	0.00035	0.00035
b_1	Maximum Carbon Tax Rate	0	0	0.05
b_2	SGS sensitivity of Carbon Tax Rate	0	0	10
r	Interest rate	0	0.03	0.03
τ	Debt repayment tax rate (second stage)	0	0.055	0.055
K_0	Initial capital stock	0.3	0.3	0.62
M_0	Initial GHG level	0.3	0.3	0.03
B_0	Initial debt	0	0	0

Table 1: Simulation Parameters and Initial Values

stock (and therefore output and consumption, as discussed in Section 3.1) is what justifies abatement efforts and taxing future generations for abatement efforts previously undertaken. The simulation results of this stage are shown in Figure 3.

B.2 Scenarios 2 and 3: Climate Policies

Figure 4 shows the effects of two policy sets introduced in scenarios 2 and 3. In the first stage of scenario 2 (represented in Figure 3 by dashed lines), the government sells green bonds and accumulates debt to finance mitigation efforts. This policy reduces the emissions level and the equilibrium level of capital stock increases compared to scenario 1. The second stage initiates when changes in both the emissions level and capital stock become smaller than $\varepsilon = 0.0001$. The main objective of this stage is to reduce the emissions level, and therefore we do not need to optimize the length of the period or when to switch to the third phase.²⁶

Compared to scenario 2, the carbon taxation included in scenario 3 (presented by solid lines in the figure), further reduces the emissions level. The increased taxation initially reduce investment, and the capital stock grows slightly more slowly than in scenario 3 (the difference is too small to be shown in the graph); it starts to take over the investment of scenario 3 as soon as emissions levels decline. While the rapid decline in the emissions level and higher taxes in scenario 3 have almost no effect on the equilibrium level

²⁶This can be modelled with a different software (Orlov et al. 2018).

of capital stock; the main difference in the two scenarios is in adjustment speed. Additional financing in model 3 accelerates the mitigation process, and the emissions level reaches its equilibrium level more rapidly than in scenario 2. Faster adjustment means less accumulated debt, less accumulated interest, and faster repayment.