

A Positive Theory of Monetary Policy in a Natural Rate Model*

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Abstract

A discretionary policymaker can create surprise inflation, which may reduce unemployment and raise government revenue. But when people understand the policymaker's objectives, these surprises can- not occur systematically. In equilibrium people form expectations rationally and the policymaker optimizes in each period, subject to the way that people form expectations. Then, we find that (1) the rates of monetary growth and inflation are excessive; (2) these rates depend on the slope of the Phillips curve, the natural unemployment rate, and other variables that affect the benefits and costs from inflation; (3) the monetary authority behaves countercyclically; and (4) unemployment is independent of monetary policy. Outcomes improve if rules commit future policy choices in the appropriate manner. The value of these commitments-which amount to long- term contracts between the government and the private sector- underlies the argument for rules over discretion.

The primary purpose of this paper is to develop a positive theory of monetary policy and inflation. On the one hand, the theory turns out to accord with two perceptions about the world in recent years:

1. Average rates of inflation and monetary growth are excessive relative to an efficiency criterion.
2. There is a tendency to pursue activist, countercyclical monetary policies.

Yet the model exhibits three other properties:

3. The unemployment rate – our proxy for real economic activity – is invariant with monetary policy (neglecting the familiar deadweight-loss aspect of inflation).

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4. The policymaker and the public all act rationally, subject to their environments.
5. The policymaker's objectives reflect the public's preferences.¹

Natural rate models with rational expectations – such as Sargent and Wallace (1975) – suggest that the systematic parts of monetary policy are irrelevant for real economic activity. Some empirical evidence on the real effects of monetary disturbances in the post-World War II United States (e.g., Barro 1977, 1981) is consistent with this result – in particular, there is some support for the proposition that anticipated monetary changes are neutral with respect to output, unemployment, and so on. On the other hand, these empirical studies and others indicated the presence of countercyclical monetary policy at least for the post-World War II United States – rises in the unemployment rate appear to generate subsequent expansions in monetary growth. Within the natural rate framework, it is difficult to reconcile this countercyclical monetary behavior with rationality of the policy-maker.² A principal object of our analysis is to achieve this reconciliation. The natural rate models that have appeared in the macroeconomics literature of the last decade share the characteristic that policy choice is over a class of prespecified monetary rules. With the policy rule predetermined, there is no scope for ongoing policymaking; discretionary policy choice is excluded *a priori*. If private agents can deduce the characteristics of the monetary process once it is implemented, it defines their expectations. Thus, the policy decision is made subject to the constraint that agents' expectations of future monetary policy will equal the realization. This framework allows the analysis to be reduced to a pair of single-agent decision problems, which can be considered independently. But, this approach cannot deal with the game-theoretic situation that arises when policy decisions are made on an ongoing basis. In our framework an equilibrium will include the following features:

- a) a decision rule for private agents, which determines their actions as a function of their current information,
- b) an expectations function, which determines the expectations of private agents as a function of their current information, and
- c) a policy rule, which specifies the behavior of policy instruments as a function of the policymaker's current information set.

The outcome is said to be a rational expectations equilibrium if, first, the decision rule specified in *a* is optimal for agents given their expectations as calculated under *b*; and second, it is optimal for the policymaker, whose actions are described by *c*, to perform in accordance with agents' expectations *b*, given that the policymaker recognizes the form

¹ The model that we consider is sufficiently simple to allow for unanimity about desirable governmental actions.

² Many people respond with a willingness to view public policy as irrational. Despite the obvious attractions of this viewpoint, it does leave us without a theory of systematic governmental behavior. An earlier attempted reconciliation with rationality (Barro 1977, p. 104) relied on public finance considerations associated with cyclical changes in the revenue obtained from printing money. This avenue appears to be quantitatively insufficient to explain the facts about countercyclical monetary response. However, the revenue motive for money creation is important in some extreme cases. See, e.g., Hercowitz (1981) for an analysis of monetary behavior and government spending during the German hyperinflation.

of the private decision rules under a . Faced by a maximizing policymaker, it would be unreasonable for agents to maintain expectations from which they know it will be in the policymaker's interest to deviate.

If policy is precommitted, the only reasonable expectations that agents can hold are those defined by the rule. But, if policy is sequentially chosen, the equality of policy expectations and realizations is a characteristic of equilibrium – not a prior constraint. We have to determine which expectations agents can reasonably expect to be realized.

We view the policymaker as attempting to maximize an objective that reflects “society's” preferences on inflation and unemployment. (Additional arguments for the preference function are mentioned later.) Although the equilibrium involves a path of unemployment that is invariant with policy, the rational policymaker adopts an activist rule. The extent of countercyclical response depends, among other things, on society's relative dislikes for inflation and unemployment. There is an apparent contradiction because the policymaker pursues an activist policy that ends up having no desirable effects – in fact, unemployment is unaltered but inflation ends up being excessive. This outcome reflects the assumed inability of the policymaker – that is, of the institutional apparatus that is set up to manage monetary affairs – to commit its course of future actions. This feature has been stressed in an important paper by Kydland and Prescott (1977). If commitment were feasible through legal arrangements or other procedures, the countercyclical aspect of monetary policy would disappear (and, abstracting from costs of erecting and maintaining institutions, everyone would be better off). When this type of advance restriction is precluded, so that the policymaker sets instruments at each date subject only to the initial conditions prevailing for that date (which do not include restraints on policy choices), the equilibrium may involve an activist form of policy. This solution conforms to optimal behavior of private agents subject to a rationally anticipated policy rule. It corresponds also to optimality for the policymaker each period, subject to agents' decision rules. Although an equilibrium obtains, the results are suboptimal, relative to outcomes where commitment is permitted. Given an environment where this type of policy commitment is absent – as appears to characterize the United States and other countries in recent years – the results constitute a positive theory of monetary growth and inflation.

We illustrate the results with a simple model, which comes from an example in Kydland and Prescott (1977, pp. 477–80). We augment their example along the lines detailed in Gordon (1980) to include a theory of expectations formation. People form their expectations by effectively solving the problem that the optimizing policymaker will face. The policymaker's problem is then conditioned on the expectations function of private agents. Ultimately, there are no systematic differences between expected and realized inflation. But this property emerges as part of the equilibrium rather than as a constraint on the policy problem.

I. The Model of Unemployment and Inflation

The unemployment rate U_t , which is a convenient proxy for the overall state of real activity, equals a “natural rate”, U_t^n , plus a term that depends negatively on contemporaneous unexpected inflation, $\pi_t - \pi_t^e$,

$$(1) \quad U_t = U_t^n - (\pi_t - \pi_t^e), \alpha > 0.$$

For convenience, we treat the “Phillips curve slope” parameter, α , as a constant.³ Given the relevant inflationary expectations, π_t^e , equation (1) is assumed to reflect the maximizing behavior of private agents on decentralized markets. The formulation of π_t^e is detailed below. Equation (1) could be reformulated without changing the main conclusion by expressing U_t as a reduced-form function of monetary shocks. The natural unemployment rate can shift over time due to autonomous real shocks, ϵ_t . A single disturbance is allowed to have a persisting influence on unemployment, output, etc. This behavior is modeled as

$$(2) \quad U_t^n = \lambda U_{t-1}^n + (1 - \lambda) \bar{U}^n + \epsilon_t, 0 \leq \lambda \leq 1,$$

where ϵ_t independently, identically distributed with zero mean. If $0 < \lambda < 1$ applies, then the realization for the shock ϵ_t affects future natural unemployment rates in the same direction. However, the effect dissipates gradually over time – equation (2) implies that the long-run mean of the natural unemployment rate is \bar{U}^n , a constant. For convenience, we assume that U_t in equation (1) depends only on contemporaneous unexpected inflation, $\pi_t - \pi_t^e$, and not on lagged values. These additional terms could be introduced without changing the main results (see below). Thus, the main thrust of our analysis is compatible with either monetary or real theories of business cycles.

The policymaker’s (and society’s) objective for each period is summarized by a cost, Z_t , which depends on that period’s values for the unemployment rate and inflation. We assume a simple quadratic form,

$$(3) \quad Z_t = a(U_t - kU_t^n)^2 + b(\pi_t)^2; a, b > 0, 0 \leq k \leq 1.$$

We do not consider any divergence across individuals in their assessments of relative costs for unemployment and inflation.

The first term in equation (3) indicates that costs rise with the departure of the unemployment rate from a target value, kU_t^n , which depends positively on the contemporaneous natural rate. In the absence of external effects, $k = 1$ would correspond to an efficiency criterion – that is, departures of U_t from U_t^n in either direction would be penalized. In the presence of unemployment compensa-

³ The prior expectation of inflation for period t could be distinguished from the expectation that is conditioned on partial information about current prices. This distinction arises in models (e.g., Lucas 1972, 1973; Barro 1976) in which people operate in localized markets with incomplete information about contemporaneous nominal aggregates. In this setting the Phillips curve slope coefficient, α , turns out to depend on the relative variances for general and market-specific shocks.

tion, income taxation, and the like, the natural unemployment rate will tend to exceed the efficient level – that is, privately chosen quantities of marketable output and employment will tend to be too low. The inequality $k < 1$ captures this possibility.⁴ Not surprisingly, we shall need some existing distortion in the economy – that is, $k < 1$ – in order to generate activist policy in our model. This result conforms with those stressed by Calvo (1978a).

Governmental decisions on taxes and transfers will generally influence the value of k . However, given that some government expenditures are to be carried out, it will generally be infeasible to select a fiscal policy that avoids all distortions and yields $k = 1$. We assume that the government's optimization on the fiscal side – which we do not analyze explicitly – results in a value of k that satisfies $0 < k < 1$. The choice of monetary policy is then carried out conditional on this value of k .

Equation (3) regards departures of π_t from zero as generating costs. Economists have not come up with convincing arguments to explain why inflation is very costly. However, direct costs of changing prices would fit most easily into our model. More generally, the form of the cost function could be changed to include a term in $(\pi_t - \bar{\pi}_t)^2$, where $\bar{\pi}_t$ might involve the optimal rate of taxation on cash balances. A later section expands the analysis to consider the revenue from money creation.

We assume that the policymaker controls an instrument – say, monetary growth, μ_t – which has a direct connection to inflation, π_t , in each period. This specification neglects any dynamic relation between inflation and monetary growth or a correlation between $(\pi_t - \mu_t)$ and the real disturbances, $\epsilon_t, \epsilon_{t-1}, \dots$. In effect, we pretend that the policymaker chooses π_t directly in each period. We discuss later what happens when we allow a separation between inflation and monetary growth.

The choice of π_t at each date is designed to minimize the expected present value of costs, as calculated at some starting date zero. That is,

$$(4) \quad E \left[\sum_{t=1}^{\infty} \frac{Z_t}{(1+r)^t} \right] \mid I_0,$$

where I_0 represents the initial state of information and r is a constant, exogenous real discount rate. It should be stressed that the policy-maker's objective conforms with society's preferences.

⁴ The target unemployment rate is $U_t^* = kU_t^n < U_t^n$. The formulation implies also that $\frac{\partial U_t^*}{\partial U_t^n} < 1$. The last condition, which we use for some conclusions, is more difficult to justify.

The determination of inflation and unemployment can be characterized as a game between the policymaker and a large number of private-sector agents. The structure of this game is as follows. The policymaker enters period t with the information set, I_{t-1} . The inflation rate, π_t , is set based on I_{t-1} in order to be consistent with the cost-minimization objective that we set out in expression (4). Simultaneously, each individual formulates expectations, π_t^e , for the policymaker's choice of inflation for period t . These expectations are based on the same information set, I_{t-1} , as that available to the policymaker. Most important, in forming inflationary expectations, people incorporate the knowledge that π_t will emerge from the policymaker's cost-minimization problem that we specify in equation (4). Finally, the choices for π_t and π_t^e , together with the random disturbance, ϵ_t , determine U_t and the cost, Z_t , in accordance with equations (1)–(3).

1. The Expectations Mechanism

In order to determine π_t^e , agents must consider the policymaker's optimization problem, which determines the choice of π_t . Suppose for the moment that the policymaker, when selecting π_t , treats π_t^e and all future values of inflationary expectations, π_{t+i}^e , as given. Variations in π_t affect unemployment through the usual Phillips curve mechanism in equation (1). As the model is set out, this effect would not carry forward to direct effects on future unemployment rates, although this channel of persistence could be incorporated. We assume that the current choice of inflation, π_t , also implies no direct constraints on future choices, π_{t+i}^e . Therefore, with current and future inflationary expectations held fixed, the determination of π_t involves only a one period trade-off between higher inflation and lower unemployment in accordance with the cost function of equation (3).

In the present framework the determination of π_t^e divorced from the particular realization of π_t . At the start of period t , agents form π_t^e by forecasting the policymaker's "best" action, contingent on the information set, I_{t-1} . The expectation, π_t^e , is not conditioned on π_t itself. Therefore, the policymaker faces a choice problem in which π_t^e is fixed while π_t is selected. Further, in formulating π_t^e , the private agents understand that the policymaker is in this position.

The connection between π_t and future inflationary expectations, π_{t+i}^e , is less clear. As noted before, the present model allows for no direct connection between π_t (even with π_t^e held fixed) and future "objective" characteristics of the economy. There is also no scope for learning over time about the economy's structure; in particular, π_t supplies no additional information about the objective or technology of the policymaker. Accordingly, we are inclined to search for an equilibrium in which π_{t+i}^e does not depend on "extraneous" past variables, such as π_t . However, the severing of a link between π_t and π_{t+i}^e eliminates some

possibly interesting equilibria in which the government can invest in its reputation – that is, in “credibility.” The nature of these solutions is discussed later. For present purposes we examine situations in which future expectations, π_{t+i}^e , are invariant with π_t .

Given that future values of U and π^e are independent of π_t , there is no channel for π_t to affect future costs, Z_{t+i} . Therefore, the objective posed in expression (4) reduces to the one-period problem of selecting π_t in order to minimize $E_{t-1} Z_t$.

In a solution to the model the public will view the policymaker as setting π_t in accordance with the information set, I_{t-1} , which is available at the start of period t . Suppose that people perceive this process as described by the reaction function, $h^e(I_{t-1})$.⁵ Therefore, inflationary expectations – formed on the basis of I_{t-1} – are given by⁶

$$(5) \quad \pi_t^e = h^e(I_{t-1}).$$

A solution to the model involves finding a function $h^e(\bullet)$, such that setting $\pi_t = h^e(I_{t-1})$ is a solution to the policymaker’s cost-minimization problem, given that $\pi_t^e = h^e(I_{t-1})$. Expecting inflation as specified by $h^e(\bullet)$ must not contradict the policymaker’s minimization of expected costs, as set out in equation (3). The previous discussion suggests that lagged values of inflation will not appear as parts of the solution, $h^e(\bullet)$. That is, we are looking for an equilibrium where $\frac{\partial \pi_t^e}{\partial \pi_{t-i}} = \frac{\partial h^e}{\partial \pi_{t-i}} = 0$ applies for all $i > 0$. We also look for a solution where the policymaker understands that π_t^e generated from equation (5).

The unemployment rate is determined from equation (1) after substituting for U_t^n from equation (2) and for π_t^e from equation (5) as

$$(6) \quad U_t = \lambda U_{t-1}^n + (1 - \lambda) \overline{U}^n + \epsilon_t - \alpha [\pi_t - h^e(I_{t-1})].$$

Costs for period t follow by substituting for U_t and π_t^e in equation (3) as

$$(7) \quad Z_t = a \left\{ (1 - k) [\lambda U_{t-1}^n + (1 - \lambda) \overline{U}^n + \epsilon_t] - \alpha [\pi_t - h^e(I_{t-1})] \right\}^2 + b (\pi_t)^2.$$

Given that inflationary expectations for period t are $\pi_t^e = h^e(I_{t-1})$, the policymaker selects π_t in order to minimize $E_{t-1} Z_t$, where Z_t appears in equa-

⁵ In the present setting the policymaker has no incentive to randomize policy choices – therefore, the reaction function ends up being purely deterministic.

⁶ Because there are many private agents, they neglect any effect of their methods for formulating π_t^e on the policymaker’s choice of π_t .

tion (7). The first-order condition, $(\partial / \partial \pi_t)(E_{t-1} Z_t) = 0$, implies that the chosen inflation rate, denoted by $\widehat{\pi}_t$, satisfies the condition

$$(8) \quad \widehat{\pi}_t = \frac{a}{b} \alpha \left\{ -\alpha [\widehat{\pi}_t - h^e(I_{t-1})] + (1-k)[\lambda U_{t-1}^n + (1-\lambda)\overline{U}^n] \right\}.$$

The property, $E(\in_t I_{t-1}) = 0$, has been used here. The second-order condition for a minimum is satisfied.

Although the policymaker is not constrained to follow the anticipated rule, $h^e(I_{t-1})$, the public understands the nature of the policy-maker's optimization problem in each period. In particular, people understand that the actual choice, $\widehat{\pi}_t$, satisfies equation (8). Therefore, rationality entails using equation (8) in order to calculate $h^e(I_{t-1})$ in equation (5). Consistency requires $h^e(I_{t-1}) = \widehat{\pi}_t$. The unexpected inflation term, $\widehat{\pi}_t - h^e(I_{t-1})$, then cancels out in equation (8), which leads to the formula for the expectations function,

$$(9) \quad \begin{aligned} \pi_t^e &= h^e(I_{t-1}) = \frac{a}{b} \alpha (1-k) [\lambda U_{t-1}^n + (1-\lambda)\overline{U}^n] \\ &= \frac{a}{b} \alpha (1-k) E_{t-1} U_t^n. \end{aligned}$$

2. Equilibrium Policy

By the construction of the problem, a policymaker who faces the expectations given in equation (9) will be motivated from the first-order condition of equation (8) to choose an inflation rate, $\widehat{\pi}_t$, that coincides with π_t^e . That is, the equilibrium involves

$$(10) \quad \widehat{\pi}_t = \frac{a}{b} \alpha (1-k) E_{t-1} U_t^n = \pi_t^e.$$

Since $\widehat{\pi}_t = \pi_t^e$, $U_t = U_t^n$ applies also as part of the equilibrium.

Equation (10) provides an equilibrium (Nash equilibrium) in the following sense. Given the public's equilibrium perceptions, $\pi_t^e = h^e(\bullet)$, minimization of $E_{t-1} Z_t$ (for a given value of π_t^e) induces the policymaker to choose $\widehat{\pi}_t = h^e(\bullet)$ in each period.⁷ Expectations are rational and individuals optimize subject to these expectations (as summarized in eqq. [1] and [2]).

In order to provide perspective on the present framework, consider an alternative manner in which the policymaker's choice problem could have been for-

⁷ Note that no equilibrium exists if the policymaker gives no direct weight to inflation – i.e., if $b = 0$. More generally, we require that the marginal cost of inflation, π_t , be positive at a point where $\pi_t = \pi_t^e$.

mulated. Policy could have been viewed as the once-and-for-all choice of reaction function, $h(\bullet)$, so that $\pi_t^e = h^e(\bullet) = h(\bullet)$ holds automatically in every period for all choices of $h(\bullet)$. This perspective applies, for example, to the analysis of macro policy in Sargent and Wallace (1975). In their setting the choice of the function, $h(\bullet)$, affects not only π_t but also π_t^e in each period. The independence of π_t^e from π_t is broken in the context of a once-and-for-all selection of policy functions. The condition $\pi_t - \pi_t^e = \pi_t - h^e(I_{t-1}) = 0$ is then a constraint on the policy problem, which can be substituted into equation (7). In particular, with π_t^e guaranteed to move one-to-one with changes in π_t , the policymaker must regard unemployment, $U_t = U_t^n$, as invariant with $h(\bullet)$. Given the simple objective from equation (3), which penalizes departures of π_t from zero, the choice of $h(\bullet)$ that minimizes EZ_t , for all periods is a variant of the constant growth rate rule,⁸

$$(11) \quad \pi_t^* = h(I_{t-1}) = 0.$$

Note that $U_t = U_t^n$ obtains again as part of this solution.

Given the public's perceptions, $\pi_t^e = h^e(I_{t-1})$, U_t depends on the term, $\pi_t - \pi_t^e = \pi_t - h^e(I_{t-1})$. People have observed (Taylor 1975; Friedman 1979) that the policymaker can fool the public and reduce unemployment ("temporarily") by setting $\pi_t > \pi_t^e = h^e(I_{t-1})$ in period t . This possibility is ruled out in the case where policy amounts to a once-and-for-all binding choice of $h(\bullet)$. However, there may be no mechanism in place to constrain the policymaker to stick to the rule, $h(I_{t-1})$, as time evolves. This consideration leads to the setup for policy choice that we assumed before – namely, for given initial conditions in each period, including the expectations mechanism, $\pi_t^e = h^e(I_{t-1})$, set π_t in order to minimize $E_{t-1}Z_t$. The policymaker is not required to select an inflation rate that equals the given expected inflation rate. However, people also realize that the policymaker has the power to fool them at each date. Since the formation of expectations takes this potential for deception into account, a full equilibrium will ultimately involve $\pi_t = \pi_t^e$. The crucial point is that – unlike for a once-and-for-all choice of policy rules – the policymaker does not regard $\pi_t = \pi_t^e$ as occurring automatically for all possible choices of π_t . For this reason the (noncooperative) equilibrium does not correspond to equation (11).

Compare the equilibrium solution, $\widehat{\pi}_t$ from equation (10), with the choice, $\pi_t^* = 0$, which arises from a once-and-for-all selection of policy rules. The equilibrium solution delivers the same unemployment rate and a higher rate of inflation at each date. Therefore, the equilibrium cost, \widehat{Z}_t , exceeds that, Z_t^* which

⁸ If Z_t in eq. (3) depended on $(\pi_t - \overline{\pi}_t)^2$, $\pi_t^* = \overline{\pi}_t$ would emerge.

would arise under the rule. (Note that, with U_t the same in both cases, costs end up depending only on the path of the inflation rate.) Of course, this conclusion neglects any costs of setting up or operating the different institutional environments. Notably, the costs of enforcing commitments are excluded. With this cost neglected, the present type of result provides a normative argument (and positive theory?) for policy rules – that is, for commitment on future choices of π_t . We highlight these aspects of our results in a later section.

It may be useful to demonstrate directly that $\pi_t = 0$ is not an equilibrium for the case where the policymaker optimizes subject to given expectations in each period. Conjecture that $\pi_t^e = h^e(I_{t-1}) = 0$ holds. In this case the choice of $\pi_t > 0$ would reduce unemployment for period t . A trade-off arises between reduced costs of unemployment and increased costs from inflation. The balancing of these costs determines the chosen inflation rate, as shown in equation (8). Under the assumed conditions (marginal cost of inflation is zero at $\pi_t = 0$ and marginal benefit from reduced unemployment is positive when $U_t = U_t^n$), the selected inflation rate will be positive. However, since people understand this policy choice, the result $\pi_t > 0$ is inconsistent with the conjecture that $\pi_t^e = 0$. Zero inflation is not a reasonable expectation for individuals to hold.

An analogous argument can be used to find the positive rate of inflation that does provide an equilibrium. If a small positive value for π_t^e had been conjectured, the policymaker would still have been motivated to select $\pi_t > \pi_t^e$, which would be inconsistent with equilibrium. The equilibrium obtains when π_t^e is sufficiently high, so that $\pi_t = \pi_t^e$, is the policymaker's best choice, given this value of π_t^e . At this point the policymaker retains the option of choosing $\pi_t > \pi_t^e$ (or $\pi_t < \pi_t^e$) so as to accomplish a trade-off between lower unemployment and higher inflation (or vice versa). However, the level of π_t^e is sufficiently high so that the marginal cost of inflation just balances the marginal gain from reducing unemployment.⁹ The inflation rate that corresponds to this equilibrium condition is the one from equation (10).

The rules-type equilibrium, as in equation (11), is often referred to as the optimal, but time-inconsistent, solution (see, e.g., Kydland and Prescott 1977, p. 480). The term “time-inconsistent” refers to the policymaker's incentives to deviate from the rule when private agents expect it to be followed. On the other

⁹ Consider the more general case where $Z_t = Z(U_t - kU_t^n, \pi_t)$ and $U_t = U_t^n - f(\pi_t - \pi_t^e)$. The first-order condition entails $f' = (\partial Z / \partial \pi_t) / [\partial Z / \partial (U_t - kU_t^n)]$. This expression is evaluated in equilibrium at $\pi_t = \pi_t^e$ and $U_t = U_t^n$. An equilibrium will be found if $\partial Z / \partial \pi_t$ rises sufficiently with π_t (as in the quadratic case considered in the text) or if f' declines sufficiently with π_t . Given the last condition, it is no longer essential that inflation involve increasing marginal costs – $\frac{\partial^2 Z}{\partial \pi_t^2} > 0$.

hand, the discretionary equilibrium, as in equation (10), is often called the sub-optimal, but time-consistent, solution. This terminology is deceptive in that it suggests that these decision rules represent alternative solutions to the same problem. Though the objective function and decision rules of private agents are identical, the problems differ in the opportunity sets of the policymaker.

In one case, constraints on future policy actions are infeasible, by assumption. In the other case, rules are enforceable, so that the policymaker can commit the course of future policy (and thus of expectations). In the former case the time-inconsistent solution is not an equilibrium, given the problem faced by the policymaker. In the latter case, the incentives to deviate from the rule are irrelevant, since commitments are assumed to be binding. Thus, the time-inconsistency of the optimal solution is either irrelevant – when commitments are feasible – or else this solution does not solve the problem actually faced by the policymaker.

3. *Properties of the Discretionary Equilibrium*

Assume now that the policymaker cannot make binding commitments, so that optimization occurs period by period, as we have been assuming for most of our analysis. Under this discretionary regime, the solution for $\hat{\pi}_t$, in equation (10) constitutes a positive theory of inflation and monetary growth. The major implications are as follows:

1. The average inflation rate exceeds the value (zero in this model) that would be optimal if policy rules were feasible. Therefore, an exogenous shift from a regime that involved some commitment on nominal values – such as a gold standard or possibly a system with fixed exchange rates – to one without such restraints would produce a rise in the average rates of inflation and monetary growth.

2. Within a discretionary regime, the rate of inflation rises if the policymaker attaches greater benefits to unexpected inflation. One change that generates this outcome is an increase in the long-run average of the natural unemployment rate, U^n . In fact, the natural unemployment rate rose significantly in the United States over the last 10–15 years.

3. The benefits from surprise inflation depend on the gap between the natural unemployment rate and the target rate. In the model this gap reflects distortions, such as income taxation, which deter work effort and production. An increase in these distortions shows up as a decrease in the parameter k , which leads to a higher rate of inflation. One source of this change is the growth of government. Thus, more government is inflationary in the model.

4. An adverse shock to the unemployment rate (i.e., $\epsilon_t > 0$) tends to persist over time. Then, as in the case of an increase in U^n , the benefits from inflation shocks increase. Thus the rational policymaker behaves countercyclically, in the sense that inflation and monetary growth react positively to increases in unemployment. In a larger model it would be possible to distinguish the countercyclical response of monetary growth from that of inflation. However, these two variables are directly linked in the present model. See the discussion below.

5. The mean rate of inflation and the extent of countercyclical response rise with α – the Phillips curve slope parameter in equation (1) – and the relative value of the cost coefficients, a/b , attached to unemployment versus inflation. In particular, if inflation is not very costly – as many economists have argued – then the parameter b is small and we wind up having a lot of inflation.

Some of the results listed above are the sorts of normative implications for aggregate demand policy that are delivered by Keynesian models in which policymakers can exploit a systematic (possibly dynamic) trade-off between inflation and unemployment. However, in the present model:

6. Unemployment, $U_t = U_t^n$, is invariant with the systematic parts of inflation and monetary growth.¹⁰ In this sense policy ends up with no effect on real economic activity.

Some people have argued that policymakers do not face a “cruel choice” between inflation and unemployment in a natural rate environment. This argument is misleading in a context where monetary institutions do not allow for policy choice to be committed. Although $U_t = U_t^n$ emerges in equilibrium – that is, unemployment is invariant with policy in this sense – policymakers do optimize in each period subject to the appropriate givens, which include the formation of expectations. Given these expectations, the choice of π_t does influence the unemployment rate “right now” – that is, for date t . The social trade-off between unemployment and inflation, as expressed by the preference ratio, a/b , is central to the policymaker’s decision.¹¹ No cruel choice arises, and $\pi_t = 0$ follows only if the policymaker can commit future actions. Within the present model, this outcome is infeasible. Counseling stable prices (or constant and small rates of monetary growth) in this environment is analogous to advising firms to produce more output with given inputs. Policymakers in a discretion-

¹⁰ Formally, changes in the parameters a , b , α or k – which alter $E_{t-1} \widehat{\pi}_t$ for all dates t in eq. (10) – have no significance for the time path of unemployment.

¹¹ We are tempted to say that setting $\pi_t < \widehat{\pi}_t$ in eq. (10) would deliver $U_t > U_t^n$. (As an analogue, a firm that ends up in equilibrium with an ordinary rate of return would end up with below normal rates of return if it did not strive to maximize profits at all times.) However, the choice of $\pi_t < \widehat{\pi}_t$ is inconsistent with the prescribed form of the policymaker’s objective.

any regime really are finding the optimal policy, subject to the applicable constraints, when they determine a countercyclical monetary reaction with positive average rates of inflation.

II. Extensions to the Model

1. *Monetary Growth as the Policy Instrument*

In Barro and Gordon (1981) we developed a simple model to treat monetary growth, rather than inflation, as the instrument of policy. We allowed for control errors in the supply of money, as well as stochastic shifts to velocity.

In equilibrium we found that the discretionary policymaker would set the monetary instrument in order to equate the mean of the inflation rate to the value determined in equation (10). However, the actual inflation rate differs from its mean because of shocks to money supply and velocity. Therefore, surprises in money or in velocity lead to unexpected inflation, $\pi_t - \pi_t^e$, which affects unemployment through the mechanism of the Phillips curve in equation (1). So, the unemployment rate does not always equal the natural rate in this model. In particular, positive shocks to money or velocity reduce the unemployment rate.

We found also that some disturbances would generate divergent reactions of monetary growth and inflation. The differences involve the behavior of real money demanded, which responds to changes in output (i.e., in the unemployment rate) and to shifts in the expected rate of inflation.

2. *Persisting Effects of Nominal Shocks*

We can modify the model to allow the effects of inflation shocks to persist over time – that is, we can change equation (1) to allow U_t to depend on current and lagged values of $(\pi - \pi^e)$. This extension complicates the policymaker's first-order condition in equation (8) to include effects from a distributed lead of prospective values of unemployment and inflation. Ultimately, the equilibrium is altered in that expected future values of U and π appear as influences on $\widehat{\pi}_t$ in equation (10).

Our basic analysis is compatible with either monetary or real disturbances as the impulses underlying the business cycle. Both of the shocks mentioned in the previous section (monetary control errors and velocity shocks) are potential nominal sources of such disturbances, but systematic monetary policy is not. Without some informational asymmetry, the policymaker is, in equilibrium, incapable of counteracting the real effects of exogenous disturbances, whatever their source.

III. Revenue from Money Creation and Depreciation of Public Debt

An important element in our model is the negative effect of unexpected inflation on the unemployment rate. Because the policymaker likes a lower unemployment rate, he attaches a benefit to positive inflation surprises. In finding the discretionary equilibrium, the crucial item is this benefit from unexpected inflation – the underlying Phillips curve does not matter, *per se*. In fact, there are other reasons for the policymaker to value unexpected inflation. These include the revenue from money creation and the inverse effect of inflation on the real value of public debt.

Surprise inflation constitutes an unanticipated capital levy on holdings of the government's nominal liabilities. As with other capital levies, this form of tax – when not foreseen – can raise revenue at little deadweight loss. Therefore, from the standpoint of public finance, the policymaker would attach some benefits to surprise inflation. Further, we can identify some variables that influence the extent of these benefits. These include: (1) the deadweight losses associated with other methods of taxation; (2) the volume of government expenditure, since a greater share of output absorbed by the government is likely to raise the marginal deadweight loss from conventional taxes; (3) the extent of temporary government spending, as in wartime, which may have an especially strong effect on the marginal cost of alternative taxes; (4) the position of the money-demand function (a higher level makes surprise inflation more rewarding); and (5) the outstanding real quantity of nominally denominated public debt.

Since people understand the attractions of *ex post* capital levies, they will attempt to forecast the policymaker's incentives to exploit such situations. Therefore, as in the case of our example about the Phillips curve, we find that systematic surprises to inflation cannot arise in equilibrium. In an equilibrium the inflation rate is sufficiently high so that the marginal cost of inflation balances the marginal benefit from a hypothetical unit of surprise inflation.¹² Whereas before the benefit involved reductions in unemployment, we now have that the benefit concerns increased governmental revenues. Therefore, any items that people know about in advance and that shift around the benefits from those revenues will end up raising the equilibrium rate of inflation. So, from our examples mentioned before, we find that the inflation rate rises with an increase in government spending, especially during wartime. A higher outstanding quantity of real public debt also raises the equilibrium inflation rate.

¹² In Calvo's (1978*b*) model of inflationary finance, the policymaker attaches no cost to inflation. Therefore, there is no discretionary equilibrium with a finite inflation rate under rational expectations. The details of the case where inflation is viewed as costly are worked out in Barro (1983).

Unlike in our simple model of the Phillips curve, we find that realizations of unexpected inflation occur when there are unanticipated changes in the benefits from governmental revenues.¹³ For example, when an unpredicted war starts, the policymaker will exercise some of his power to depreciate the real value of money and bonds. From an *ex ante* standpoint, this possibility is balanced by the more favorable returns on money and bonds during peacetime. In particular, the nominal interest rate paid on government bonds provides a satisfactory distribution of real returns, given the dependence of these returns on conditions of war or peace, and so forth.

The significance of rules is similar to that in our previous model. For the case of public debt, the indexation of returns for inflation is a simple form of rule. Indexation eliminates the government's power, *ex post*, to use inflation to depreciate the real value of its debts. From our perspective, we predict that the implementation of an indexing rule lowers the equilibrium growth rates of prices and money. However, this conclusion holds unambiguously only if the costs from inflation do not change. If the existence of the government's indexed bonds reduces the costs attached to inflation – that is, the *b*-coefficient in the cost function from equation (3) – then an opposing force emerges.

IV. Reputational Equilibria

A different form of equilibrium may emerge in which the policymaker forgoes short-term gains for the sake of maintaining a long-term “reputation.” Consider again the initial setting where costs depend on unemployment and inflation, as in equation (3). The “rules equilibrium” generates $U_t = U_t^n$ and $\pi_t = 0$, while the noncooperative, period-by-period solution yields the inferior outcome, $U_t = U_t^n$ and $\pi_t = \hat{\pi}_t > 0$.

Another possible form of solution, which has been discussed in the related game theory literature (e.g., in Friedman 1971), takes the following form. Private agents anticipate the cooperative result,¹⁴ $\pi_t = 0$, unless they have seen something else. Once observing a different value for inflation, agents henceforth expect the noncooperative policy, $\pi_t = \hat{\pi}_t$.¹⁵ Confronted by this behavior, the

¹³ We would get this type of result in our earlier model if the unemployment rate depended on unanticipated inflation, $\pi_t - E_{t-1}\pi_t$, where the forecast, $E_{t-1}\pi_t$, is formed before all data from period $t-1$ are available.

¹⁴ The result is not fully cooperative because of the underlying externality, which makes the natural unemployment rate “too high”.

¹⁵ The reaction can be modified so that $\pi_t^e = \hat{\pi}_t$ applies only for a finite time period. However, a shorter “punishment interval” makes it more difficult to induce the policymaker to opt for the cooperative result.

policymaker has two options: first, $\pi_1 = \widehat{\pi}_1$ can be chosen in period one. In conjunction with the initial expectation, $\pi_1^e = 0$, the choice of $\pi_1 = \widehat{\pi}_1$ generates a favorable first-period trade-off between low unemployment, $U_1 < U_1^n$, and high inflation. For the first period the policymaker prefers this outcome to the rules solution, where $U_1 < U_1^n$ and $\pi_1 = 0$. In subsequent periods individuals would set $\pi_t^e = \widehat{\pi}_1$. Therefore, the policymaker selects $\pi_t = \widehat{\pi}_t$ as the best possible response, given expectations. In other words the noncooperative equilibrium, $U_t = U_t^n$ and $\pi_t = \widehat{\pi}_t$, arises from period 2 onward.

The policymaker's second option is to set $\pi_t = 0$ in each period. Since, $\pi_t^e = 0$ is sustained under this policy, the cooperative solution, $U_t = U_t^n$ and $\pi_t = 0$, obtains in all periods. Under this option the policymaker forgoes the hypothetical short-run gain in order to sustain credibility and thereby enjoy the benefits of future cooperative outcomes.

From the policymaker's viewpoint, the central new feature is the linkage between current policy choices and subsequent inflationary expectations. In particular, the policymaker knows that $\pi_t^e = 0$ will apply only if $\pi_{t-i} = 0$ has been set at all previous dates. Whether the reputational equilibrium will arise depends on the policymaker's weighing of the benefits from the two possible modes of behavior. In particular, it will not arise if the hypothetical one-period benefit from low unemployment outweighs the present value of the losses from higher inflation in future periods. A high discount rate makes this outcome more likely.¹⁶

There are many features that can cause the reputational equilibrium to break down. First, any known finite horizon for the game rules out these types of equilibria. The cooperative solution is clearly unsustainable in the final period – working backward, period by period, this breakdown can be shown to be transmitted to all earlier periods.¹⁷ However, if the game ends only probabilistically, the reputational equilibrium might be sustainable. A higher probability of termination effectively raises the discount rate that applies to outcomes in future

¹⁶ The form of behavior described under the first option cannot arise in equilibrium in the present model. If this option were attractive for the policymaker, private agents would anticipate this outcome. In that case $\pi_t^e = 0$ would not be maintained. The noncooperative solution, $U_t = U_t^n$ and $\pi_t = \widehat{\pi}_t$, would then arise for all periods, including the first. However, there will always exist some intermediate values of π_t , where $0 \leq \pi_t < \widehat{\pi}_t$, such that a cooperative solution based on π_t would be sustainable. Assuming an infinite horizon for the problem (see below), a sufficiently high value of π_t within this interval must make option two preferable to option one. However, the admissible range for π_t would depend on the realizations for U_t^n and other variables.

¹⁷ Some attempts to avoid this conclusion in analogous contexts have been explored in, e.g., Radner (1979) and Kreps and Wilson (1980).

periods. This higher discount rate lowers the benefits from long-term reputation (low inflation) relative to those from short-run gains (low unemployment). Accordingly, while a finite expected horizon for the game does not make the reputational equilibrium impossible, it does make it more difficult to maintain.

Second, at least the simple form of cooperation is lost if option 1 becomes preferable to option 2 during any period. In the present example, a run-up in the natural unemployment rate might make the hypothetical short-run benefit from reduced unemployment exceed the present value of losses from higher future inflation.

Third, in a context of partial information, agents may have difficulty verifying the underlying monetary policy. Some form of stochastic decision rule would have to be implemented. Policymakers would have a corresponding incentive to cheat – such situations would be characterized by claims that inflation or monetary growth was not caused by past governmental actions. Similarly, policymakers would desire to proclaim the end of a previous regime that involved excessive inflation in order to restore matters to the “first period” in which $\pi_t^e = 0$ was based on trust rather than on performance.

The essential problem is the lack of an objective link between current actions, π_t , and future expectations, π_{t+i}^e . An enforced rule ties actual and anticipated values together. In this sense the reputational equilibrium amounts to a fragile approximation to the rules equilibrium. Despite the apparent difficulties with sustaining reputational equilibria, casual observation suggests that reputational forces, unreinforced by formal rules, can generate satisfactory outcomes in some areas. Further investigation seems warranted into the factors that allow reputational equilibria to be sustained.¹⁸

V. Rules versus Discretion Once Again

The presence or absence of precommitment is the most important distinction between rules and discretion. However, it is useful to consider two other points that have arisen in the previous literature.

1. Policy is described by a once-and-for-all choice of reaction function, $h(I_{t-1})$ but discretion allows I_{t-1} to encompass a larger set of arguments than does a rule. This viewpoint makes rules look like pointless constraints on the options of the policymaker. From this perspective, rules are defensible only if the policymaker is incompetent or nontrustworthy, in the sense of using an inappropriate objective. However, it may be true that complicated rules cannot be adequately monitored and enforced. Then, we may need to consider

¹⁸ See Barro and Gordon (1983) for our further work in this area.

the operating characteristics of simple rules, which allow only for limited contingencies.

2. Ignorance about the workings of the economy favors a simple rule for policy. While this outcome is possible, the conclusion is not general. It is readily imaginable that uncertainty about variables or about model structure would magnify the number of factors to which feedback was justified.

The important dimension of a rule is its capacity to restrict the manner in which future policy choices will be made. In many private arrangements, as with governmental policies, efficiency requires the potential for advance commitments – that is, for contractual obligations. Kydland and Prescott (1977) describe numerous areas of public policy in which formal or implicit prior restraints on future actions are important, including patents, flood plain projects, and energy investments. Other areas include repudiation of national debt and taxation of capital income generally. Actual methods for framing governmental policies seem to be successful to different degrees in each case.

In the unemployment-inflation example, the outcome is suboptimal relative to that generated by a policy rule, if we disregard the costs of erecting and enforcing the rules. The “optimal” solution, $\pi_t = 0$ and $U_t = U_t^n$, is then attainable through a (costlessly operating) mechanism that restricts future governmental actions on inflation. Under a discretionary regime, the policymaker faces an unemployment-inflation trade-off at each date and performs accordingly. The policymaker does as well for the public as possible within an environment where commitments – that is, long-term contracts with the public – are precluded. Rather than rules being less flexible than discretion, the situation is reversed. Discretion amounts to disallowing a set of long-term arrangements between the policymaker and the public. Purely discretionary policies are the subset of rules that involve no guarantees about the government’s future behavior.¹⁹

¹⁹ If the desirability of commitments on monetary growth and inflation is accepted, there are numerous procedures within the present model that can generate outcomes that are equivalent to those produced by a once-and-for-all choice of rules. Discretion, e.g., could be maintained, but the parameters of the policymaker’s preferences could be artificially manipulated in order to generate a noncooperative solution where $\widehat{\pi}_t = 0$. This result follows if the policymaker gives infinite weight to inflation ($b = \infty$), gives zero weight to unemployment ($a = 0$), or regards the natural unemployment rate as optimal ($k = 1$). In the context of discretionary policy, outcomes may improve if there is a divergence in preferences between the principal (society) and its agent (the policy-maker).

VI. Monetary Institutions and Policy Choice

The spirit of this paper is to characterize monetary growth and inflation as reflections of optimal public policy within a given institutional setup. Under a discretionary regime, the policymaker performs optimally subject to an assumed inability to commit future actions. The framework assumes rationality in terms of the day-to-day actions that are carried out repeatedly within the given institutional mode. The intention here is to model the regular behavior of a monetary authority, such as the Federal Reserve. Excessive inflation, apparently unrewarding countercyclical policy response, and reactions of monetary growth and inflation to other exogenous influences can be viewed as products of rational calculation under a regime where long-term commitments are precluded.

The model stresses the importance of monetary institutions, which determine the underlying rules of the game. A purely discretionary environment contrasts with regimes, such as a gold standard or a paper-money constitution, in which monetary growth and inflation are determined via choices among alternative rules. The rule of law or equivalent commitments about future governmental behavior are important for inflation, just as they are for other areas that are influenced by possibly shifting public policies.

We are less comfortable about specifying fruitful approaches to framing positive theories of monetary institutions.²⁰ If we had retained the optimality criterion that we utilized for analyzing day-to-day monetary actions, and if we had assumed that the costs of implementing and enforcing monetary rules were small, then discretionary monetary policy would not be observed. Within the natural rate setting of our model, a positive theory would predict the selection of a rule (or its equivalent) – and the establishment of an accompanying enforcement apparatus – that would guarantee low and relatively stable rates of inflation.

Presumably, the substantial setup costs that are associated with erecting monetary or other institutions mean that changes in regime will be observed only infrequently. The relatively small experience with alternatives suggests – unlike for the case of regular operations within a given regime – the potential for substantial, persisting errors. Although we would be uncomfortable attempting to forecast a systematic direction of error in future institutional choices, we might be willing to label a particular past choice – such as the movement away from the remnants of the gold standard and fixed exchange rates – as a mistake.

²⁰ The distinction between choices of institutions and selections of policies within a given regime parallels Buchanan and Tullock's (1962) dichotomy between decisions at the constitutional and operating levels of government. Buchanan (1962) stresses the importance of the constitutional perspective in designing a satisfactory monetary policy.

The distinction between institutional choice and operating decisions within a given regime relates also to the economist's role as a policy adviser. In our model the economist has no useful day-to-day advice to offer to the monetary authority.²¹ If monetary institutions were set optimally, then the economist's counsel would also not enter at this level. The most likely general role for policy advice consists of identifying and designing improvements in present policy institutions. In the monetary area the major issue concerns arrangements that are preferable replacements for the present discretionary setup. We would like to know which mechanisms – such as commodity standards and legal restrictions on the behavior of paper money – would effectively (and cheaply) restrict the course of future money and prices.

References

- Barro, R. J. (1976): Rational Expectations and the Role of Monetary Policy, *J. Monetary Econ.* 2, January 1976: 1–32.
- Barro, R. J. (1977): Unanticipated Money Growth and Unemployment in the United States, *A.E.R.* 67, March 1977: 101–15.
- Barro, R. J. (1981): Unanticipated Money Growth and Economic Activity in the United States, in *Money, Expectations, and Business Cycles: Essays in Macroeconomics*, New York: Academic Press, 1981.
- Barro, R. J. (1983): Inflationary Finance under Discretion and Rules, *Canadian J. Econ.*, Vol. 16. February 1983, pp. 1–16.
- Barro, R. J./Gordon, D. B. (1981): A Positive Theory of Monetary Policy in a Natural-Rate Model, Working Paper no. 807, Cambridge, Mass.: Nat. Bur. Econ. Res., November 1981.
- Barro, R. J./Gordon, D. B. (1983): Rules, Discretion and Reputation in a Model of Monetary Policy, *J. Monetary Econ.* 12, July 1983: in press.
- Buchanan, J. M. (1962): The Criterion of Monetary Constitutions, in *In Search of a Monetary Constitution*, edited by Leland B. Yeager, Cambridge, Mass.: Harvard Univ. Press, 1962.
- Buchanan, J. M./Tullock, G. (1962): *The Calculus of Consent*, Ann Arbor: Univ. Michigan Press, 1962.

²¹ Perhaps this observation accounts for the Federal Reserve's attitude toward the unsolicited advice economists provide to it. The Federal Reserve appears interested mostly in "efficient" operation within a given policy regime-specifically, on what to do right now. Although many economists offer advice of this sort, there is little reason to believe that these suggestions would improve on the Fed's period-by-period optimization. More recently, much of economists' advice to the Fed has amounted to proposals for altering the underlying rules of the game. It is likely that the Federal Reserve is powerless to utilize these types of constitutional suggestions.

- Calvo, G. A. (1978): On the Time Consistency of Optimal Policy in a Monetary Economy, *Econometrica* 46, November 1978, pp. 1411–1428 (a).
- Calvo, G. A. (1978): Optimal Seigniorage from Money Creation: An Analysis in Terms of the Optimum Balance of Payments Deficit Problem, *J. Monetary Econ.* 4, August 1978, pp. 503–517 (b).
- Friedman, B. M. (1979): Optimal Expectation and the Extreme Information Assumptions of ‘Rational Expectations’ Macromodels, *J. Monetary Econ.* 5, January 1979, pp. 23–41.
- Friedman, J. W. (1971): A Non-cooperative Equilibrium for Supergames, *Rev. Econ. Studies* 38, January 1971, pp. 1–12.
- Gordon, D. B. (1980): Dynamic Equilibria with Purposive Policymaking, unpublished manuscript, Chicago: Univ. Chicago, Dept. Econ., November 1980.
- Hercowitz, Z. (1981): Money and the Dispersion of Relative Prices, *J.P.E.* 89, April 1981, pp. 328–356.
- Kreps, D. M./Wilson, R. B. (1980): On the Chain-Store Paradox and Predation: Reputation for Toughness, unpublished manuscript, Stanford, Calif.: Stanford Univ., July 1980.
- Kydland, F. E./Prescott, E. C. (1977): Rules Rather than Discretion: The Inconsistency of Optimal Plans, *J.P.E.* 85, June 1977, pp. 473–491.
- Lucas, R. E., Jr. (1972): Expectations and the Neutrality of Money, *J. Econ. Theory* 4, April 1972, pp. 103–124.
- Lucas, R. E., Jr. (1973): Some International Evidence on Output-Inflation Tradeoffs, *A.E.R.* 63, June 1973, pp. 326–334.
- Radner, R. (1979): Collusive Behavior in Noncooperative Epsilon-Equilibria of Oligopolies with Long but Finite Lines, unpublished manuscript, Murray Hill, N.J.: Bell Laboratories, August 1979.
- Sargent, T. J./Wallace, N. (1975): ‘Rational’ Expectations, the Optimal Monetary Instrument, and the Optimal Money Supply Rule, *J.P.E.* 83, April 1975, pp. 241–254.
- Taylor, J. B. (1975): Monetary Policy during a Transition to Rational Expectations. *J.P.E.* 83, October 1975, pp. 1009–1022.