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Stagflation, Persistent Unemployment and the Permanence of Economic Shocks*

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Abstract

When changes occur, people do not know how long they will persist. Using a simple stochastic structure that incorporates temporary and permanent changes in an augmented IS-LM model, we show that rising prices and rising unemployment – stagflation – is likely to follow a large permanent reduction to productivity. All markets clear and all expectations are rational. People learn gradually the permanent values which the economy will reach following a permanent shock and gradually adjust anticipations. In our model, optimally perceived permanent values take the form of a Koyck lag of past observations.

I. Introduction

The principal choices that people make in a market economy – choices between present and future consumption, between labor and leisure, between real and monetary assets – depend on beliefs about the future. In forming their beliefs, individuals attempt to separate transitory and ephemeral changes from permanent and persistent changes. Even individuals who are fully informed about past and current values cannot be certain about future values. A basic inference problem that individuals face is to distinguish permanent values of variables like income, wages and prices from current values.[1]

This paper analyzes an economy in which decisions depend not just on the changes that occur but on their persistence. The economy is subject to real shocks to the labor, commodity and money markets and to nominal shocks to the money shock. Each shock has a permanent and transitory component. Indi-

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[1] Permanent income is related to *Friedman's* (1957) permanent income hypothesis. Beliefs about permanent wages play an important role in the determination of current labor supply decisions. See *Lucas/Rapping* (1969) and *Brunner/Meltzer* (1978).

viduals know the deterministic structure of the economic model and, also, know the stochastic structure. They use all available information to form optimal forecasts of the permanent values of relevant variables, but their information does not permit them to distinguish permanent and transitory changes as they occur.

All markets reach equilibrium and clear each period. The prices and quantities at which the markets clear depend, however, on the perceptions individuals in the aggregate hold about the persistence of the shocks that have occurred. Since people learn whether changes are permanent or transitory only by observing what has occurred, perceptions about permanent values change gradually, and differences between expected and actual permanent values can persist for a time.[2]

The distinction between permanent and transitory changes is particularly appealing for analysis of the labor market since it offers a potential means of reconciling sluggish adjustment of real wages and the persistence of 'unemployment' with market clearing and rational expectations. The usefulness of the distinction is not confined to the labor market. We show that a large permanent reduction in productivity causes, on impact, an increase in unemployment, which persists for a time, and an increase in prices. Real wages adjust toward their new equilibrium value as information about the permance of the shock accrues. During the transition real wages are 'sticky'. The permanent reduction of productivity generates many of the patterns described as stagflation – changes in prices, wages, output and real rates of return.[3]

The model of the economy, presented in section 2, is an augmented IS-LM model. There is an aggregate demand function, a money demand function, and a demand and supplies function for labor. The demand for labor is derived from a neo-classical production function; the supply of labor depends on both the current and the perceived permanent wage rate. The various shocks and their (known) stochastic structure are also presented in this section.

Section 3 explains the formation of beliefs or perceptions of permanent variables. First, we derive an optimal forecast of the current permanent value of any given shock. The forecasts are used to derive the structural beliefs[4] regarding permanent income, permanent wages and the permanent values of the other endogenous variables. Section 4 investigates the effects of various shocks on the equilibrium levels of employment, prices, output and the real wage rate and discusses the working of the model. Section 5 shows that large permanent decreases in productivity are followed by periods of persistent underemployment of

[2] *Friedman* (1979) obtains a similar result in different way.

[3] This seems to have been the case in 1973–1974. For alternative hypotheses that explain stagflation see *Friedman's* Nobel lecture (1977).

[4] This term is an adaptation of a concept due to *Turnovsky* (1977).

labor and by inflation. We show that inflation and unemployment can occur together – the pattern known as stagflation. A conclusion completes the paper.

II. The Model

The model developed in this section has many of the features of an IS-LM model that includes markets for commodities, money, bonds and labor.[5] Equilibrium prevails on all markets, so we eliminate one market, the bond market, from explicit consideration. Current and permanent values differ, and the differences affect the equilibrium position of each market.

Productivity, aggregate demand, labor supply, money demand and money supply are all subject to random shocks. Each shock has a permanent and a transitory component. The shocks to aggregate demand, labor supply and money demand are related through the budget constraint. For simplicity they are therefore entered in the various functions as the same shock, but each function has a different coefficient on the shock. Individuals in the economy have information on current and past values of all variables, but they are unable to observe the permanent and transitory components of each shock separately.[6] They use information on the current and past values of the variables to form optimal predictions of the components of each shock and to calculate permanent values of all variables. Expectations about the permanent values of the variables are rational in Muth's (1961) sense.

1. The Demand for Commodities

Demand for commodities is given by

$$(1) \qquad e_t = k + \alpha y_t^p + \beta\left[r_t - \left({}_t\, p_{t+1}^* - p_t\right)\right] + \varepsilon_t\,, \; 0 < \alpha < 1,\, \beta < 0,$$

where e_t, y_t^p and p_t are the logarithms of aggregate demand, real permanent income and the general price level respectively, r_t is the nominal rate of interest, ${}_t\, p_{t+1}^*$ is the logarithm of the price level expected to prevail by the public in period $t + 1$ as of period t, and ε_t is a random shock to aggregate demand. Eq. (1) states that aggregate demand depends on permanent income[7] and is inversely

[5] The model is a modified version of the model in *Cukierman* (1980). One important difference is that there is speculation in labor over time as in *Lucas/Rapping* (1969) and more recently in *Brunner/Meltzer* (1978).

[6] As we shall see, this information limitation disappears gradually as new information becomes available.

[7] This is in line with *Friedman's* (1957) permanent income consumption function.

related to the real rate of interest. The real rate is the nominal rate r_t minus the rate of inflation expected by the public, ${}_t p^*_{t+1} - p_t$.[8]

2. *The Supply of Commodities and the Labor Market*

Aggregate output, Y_t, is produced with a Cobb-Douglas production function. The aggregate capital stock is assumed fixed,[9] so aggregate output can be written as a function of labor input, L_t, and a random productivity factor, u_t,[10]

$$Y_t = e^{u_t} L_t^{\delta}, \quad 0 < \delta < 1 \tag{2}$$

The demand for labor is obtained from the first-order condition by equating the marginal product of labor to the real wage,

$$l_t^d = -\eta(w_t - u_t) + \eta \log \delta, \tag{3}$$

where $\eta \equiv 1/(1-\delta) > 0$, l_t^d is the logarithm of labor demanded, and w_t is the logarithm of the real wage rate in period t.

Labor supply is

$$l_t^s = \omega\left(w_t - w_t^p\right) + \eta \log \delta + \theta_2 \varepsilon_t, \tag{4}$$

where w_t^p is the logarithm of the real wage perceived as permanent in period t, and ω and θ_2 are positive constants. Workers compare the currently prevailing wage to the wage they currently perceive as permanent. Ceteris paribus, a decrease in w_t, or an increase in w_t^p decrease the current supply of labor.[11]

A rise in w_t relative to w_t^p induces workers to work now and substitute future for current leisure. When the current wage rate is below w_t^p, part of the labor force which looks for work abstains from accepting current employment. This group is counted as unemployed in the official statistics. When the actual and the permanent real wage rate are equal, unemployment is driven to zero.[12] This suggests that, within the context of the model, unemployment may be defined as

[8] The rate r_t is the yield to maturity on a one-period bond.

[9] We abstract from the long-run effects of investment on capital accumulation.

[10] Wherever a non-random variable appears both by itself and in log form, a capital letter is used for the variable and a lower case letter for the logarithm of the variable.

[11] For a derivation of a similar labor supply function from a microeconomic model see *Lucas/Rapping* (1969). For computational convenience we use $\eta \log \delta$ as the normalizing constant in labor supply.

[12] Since the focus of this paper is on cyclical unemployment, we do not discuss types of unemployment that arise for other reasons.

the difference between labor supply when $w_t = w_t^p$ and labor supply when the two wage rates differ. Hence the percentage rate of unemployment (n_t) is given by

$$n_t = \omega\left(w_t^p - w_t\right). \tag{5}$$

The formulation allows periods of over as well as unemployment and implies that the actual rate of unemployment can be on either side of zero.[13]

The term $\theta_2 \varepsilon_t$ in labor supply expresses the idea that Walras' Law applies to the random shocks affecting aggregate demand. A positive shock to aggregate demand must be balanced by one or more of the following changes: An increase in the supply of bonds, a decrease in the demand for money, or an increase in the supply of labor. A negative shock has the opposite responses in the bonds, money and labor markets. The term $\theta_2 \varepsilon_t$ states how much of the shock to aggregate demand takes the form of a change in supply of labor.[14]

3. *The Money Market*

The real demand for money is positively related to permanent income[15] and inversely related to the nominal rate of interest. The specific form of the demand function in nominal terms is given by

$$m_t^d = B + p_t + y_t^p + br_t + g\left(y_t - y_t^p\right) - \theta_1 \varepsilon_t, \qquad 1 > g > 0,\ b < 0,\ \theta_1 \geqq 0, \tag{6a}$$

where m^d is the natural logarithm of nominal money demand and B is a constant. The term $g\left(y_t - y_t^p\right)$ is the mirror image of the hypothesis that people relate their expenditure levels to permanent income even when permanent and actual income diverge. When permanent income is above actual income, the public reduces money balances to maintain spending. Conversely when actual income is above permanent income, the public increases money balances.[16] The term $-\theta_1 \varepsilon_t$ states that. Through Walras' Law, any shock to aggregate demand is

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[13] Unlike *Friedman* (1968), *Phelps* (1967) and *Lucas* (1973), unemployment is not caused by faulty perceptions about the price level but results from unavoidable errors in the perception of permanent real shocks and the existence of transitory real shocks.

[14] If $\theta_2 = 0$ individuals finance increased demand for goods only by borrowing and by running down their money balances.

[15] For a specification and estimation of a money demand function which depends on permanent income see *Friedman* (1959) and *Laidler* (1966).

[16] The model does not rule out the possibility that some of the synchronization of expenditures to permanent income is achieved by changing the excess demand for bonds. As a matter of fact the constraint $g < 1$ originates in the requirement that the excess de-

partly reflected as a shock to money demand in the opposite direction. The parameter θ_1 measures the portion of the shock to aggregate demand that individuals desire to finance by changing money balances.

The stock of money is given by

(6b)
$$m_t^s = m + \psi_t,$$

where m_t^s is the logarithm of the nominal shock, in period t; m is a constant, and ψ_t is a random shock to money.

4. Equilibrium[17]

Given the current realizations of the three shocks – ε_t, u_t, ψ_t – the permanent values of the wage rate (w_t^p), the level of permanent income (y_t^p) and the price level expected for next period (${}_t\, p_{t+1}^*$), we use the market clearing equations for the next commodities, labor and money markets and the production function to determine the current values of output (y_t), employment (l_t) the price level (p_t), the nominal rate of interest (r_t) and the real wage (w_t). Eqs. (7), (8) and (9) equate the quantity of commodities demanded (e_t) to y_t, and the quantities of money and labor demanded to the quantities supplied:

Commodities market

(7)
$$y_t = k + \alpha y_t^p + \beta\left[r_t - \left({}_t\, p_{t+1}^* - p_t\right)\right] + \varepsilon_t .$$

Money market

(8)
$$m + \psi_t = B + p_t + y_t^p + br_t + g\left(y_t - y_t^p\right) - \theta_1 \varepsilon_t .$$

mand for bonds is an increasing function of actual income and a decreasing function of permanent income. For more discussion of the bond market, see footnote 17.

[17] By using the budget constraint and the excess demands for labor, goods and money, we can derive the excess demand for bonds as a function of the variables and shocks in the model and use the resulting function to check the implications of the model. The model implies that, for sufficiently small values of θ_1 and θ_2, the excess demand for bonds is a decreasing function of ε_t; a positive shock to aggregate demand increases the demand for loans (excess supply of bonds) and the supply of labor and reduces the demand for money. For $g < 1$ and income velocity of money greater than 1, increases in $y_t - y_t^p$ increase the excess demand for bonds. Increases in the money stock and in the real rate of interest also increase the excess demand for bonds.

Labor market

(9) $$l_t = -\eta(w_t - u_t) + \eta\delta = \omega w_t - \omega w_t^p + \eta \log \delta + 0_2\, \varepsilon_t\,.$$

Eq. (10) restates the production function in logarithmic form

(10) $$y_t = u_t + \delta l_t\,.$$

5. *Permanent and Transitory Shocks – The Stochastic and Information Structures of the Ecoyomy*

Each of the three stochastic shocks ε_t, ψ_t, and u_t, has a transitory component and a permanent component,

(11) $$\varepsilon_t = \varepsilon_t^p + \varepsilon_t^q, \psi_t = \psi_t^p + \psi_t^q, u_t = u_t^p + u_t^q$$

An intuitive interpretation of the formal definition in (11) is that any shock which remains at its current value until something else happens is a permanent shock, whereas any shock which affects the system for only one period is a transitory shock.[18] Let $x_t \equiv x_t^p + x_t^q$, where $x_t^i = \varepsilon_t^i, \psi_t^i, u_t^i$ and $I = q, p$. We assume that

(12) $$\Delta x_t^p \sim N\left(0, \sigma_{xp}^2\right), x_t^q \sim N\left(0, \sigma_{xq}^2\right),$$

where $\Delta x_t^p \equiv x_t^p + x_{t-1}^q$. The permanent component, x_t^p, of each shock is expected to remain at its previous value unless the change in this component deviates from its expected value. The expected value of a change in x^p is zero; x_t^p is a random walk. The transitory component x_t^q is expected to vanish to zero unless another transitory shock hits the system in the next period. $\Delta\varepsilon_t^p, \Delta\psi_t^p, \Delta u_t^p, \varepsilon_t^q, \psi_t^q$ and u_t^q are mutually and serially uncorrelated.

In each period, *t*, individuals in the economy have full information about the current and past values of the endogenous variables like y_t, r_t, p_t, and w_t and the current money stock m_t. They also have beliefs about the current values of permanent income, the permanent real wage rate and the future expected price

[18] In extreme interpretation of a 'permanent shock' cannot coexist with the notion that such a shock is subject to some non-degenerate probability distribution. If the shock is 'permanent' in the strictest sense it is expected to remain at its current level with probability 1 forever. Such a notion of permanence does not seem useful for the analysis of the type of random shocks that affect economic systems. Therefore a less extreme definition of permanence is used in the text. The meaning FI 'permanent' is not independent of the stochastic structure.

level.[19] For given beliefs about ${}_t p^*_{t+1}, y^q_t, w^q_t$ and the current values of y_t, r_t, p_t and m, individuals in the economy can solve for the current values of the shocks ε_t, ψ_t and u_t.

Individuals observe only the sum of the permanent and transitory components of each shock and cannot separate the two reliably. Moreover this lack of information is not entirely dispelled by the passage of time. Even when they are in period *t*, individuals do not know with certainty how much of the shock $x_{t-j}, j \geqq 1$, that they observed in the past is due to transitory changes and how much is due to permanent changes. And there is nothing in the aggregate statistics computed by the appropriate agencies which removes this lack of information with the passage of time. The fundamental inference problem confronting individuals arises from their inablility to separate the two components of each shock.

The information structure of the model differs from the structure encountered in models which are based on a confusion between aggregate and relative price changes. See *Lucas* (1973), *Barro* (1976), *Cukierman/Wachtel* (1979) and *Cukierman* (1979). In these models, the publication of a general price index dispels confusion immediately. In our model, new information is useful but does not eliminate the confusion. There is no aggregate statistical data which informs people in the economy about the extent to which a shock to past productivity, to aggregate demand or to the money supply is permanent or transitory.[20]

III. The Formation of Beliefs about Permanent Variables

1. Optimal Forecasts of Permanent Values of the Exogenous Shocks

This section explains how individuals form optimal forecasts of the various shocks and modify their beliefs or perceptions about permanent income. permanent wages and the permanent values of the other endogenous variables. We focus on the optimal prediction of x^p_t given the information on $x_t, x_{t-1}, x_{t-2} \ldots$ which is available in period *t*. The information set of period *t* includes the current value and all past values of *x*, and is denoted I_t. The problem of forming an optimal forecast about x^p_t given I_t can now be formulated as follows: Given

[19] Differential information and beliefs are ignored.

[20] This is the fundamental thesis of *Brunner/Meltzer* (1978). If the shock is permanent it becomes relevant for the prediction of the future course of the economy. If it is transitory, it is irrelevant as far as the prediction of the future is concerned. The oil shocks of the seventies, changes in rainfall, and the famous disappearance of the Peruvian anchovies are examples of real shocks that cannot be accurately labelled as permanent and transitory when they occur.

$$x_t = \Delta x_t^p + \Delta x_{t-1}^p + \ldots + \Delta x_{t-n}^p + \ldots + x_t^q,$$
$$x_{t-1} = \Delta x_{t-1}^p + \ldots + \Delta x_{t-n}^p + \ldots + x_{t-1}^q,$$

(13) $$x_{t-n} = . \qquad \Delta x_{t-n}^p + \ldots + x_{t-n}^q$$
$$\vdots$$

form an optimal forecast of

(14) $$x_t^p = \Delta x_t^p + \Delta x_{t-1}^p + \ldots + = \Delta x_{t-n}^p + \ldots$$

Since x_t^p has at least one stochastic component in common with each of x_t, x_{t-1} , x_{t-n}, ..., it is correlated with each of these *x*'s, Hence the point estimate of the permanent component made in period *t* for that period has a lower variance if the information about all past realizations of *x* is used in the computation of the forecast. More formally, 'the minimum variance point estimate of x_t^p given I_t, is equal to the conditional expectation of x_t^p, where all the observations on *x* up to and including period *t* enter into the conditioning set, I_t. But conditional expectations of variables from multinormal distributions are linear functions of the conditioning variables.[21] *Muth* (1960) has shown that the best (minimum variance) linear estimator of x_t^p given the information set I_t, is[22]

(15a) $$Ex_t^p = \lambda \sum_{i=o}^{\infty} (1-\lambda)^i x_{t-i},$$

where

(15b) $$\lambda = \sqrt{a + \frac{a^2}{4}} - \frac{a}{2}. \qquad a \equiv \frac{\sigma_{xp}^2}{\sigma_{xq}^2}.$$

It follows that the prediction in (15a) is the best estimator of the permanent component of period *t* given information on *x* up to and including that period and is therefore the rational expectation of the level of the permanent component in period *t*. Four features of Ex_t^p should be noted.

[21] This is a direct consequence of theorem 3.10 in *Graybill* (1961, p.63).

[22] See *Muth* (1960, pp. 302–304). For the particular stochastic process used here the predictor in (15) is also the minimum variance estimator of the *actual* value of *x* for period *t*+1 given the information set I_t. This can be seen by noting that by Wold's decomposition theorem Δx_{t+1} can be expressed as the first order moving average process $\Delta x_{t+1} = \gamma_{t+1} - (1-\lambda)\gamma_t$, where γ_t is white noise. Expressing γ_{t+1} as an infinite lag on values of Δx up to and including period *t* (the set I_t) by using the lag operator and rearranging, it can be seen that the best prediction of x_{t+1} given I_t, is also given by (15). Note that this feature is specific to the particular stochastic process used here.

First, the optimal prediction of Ex_t^p takes the form of a distributed lag of past values. All past values of the observed shocks, x_{t-i}, i=1, 2, …, are used to forecast the permanent component. The reason that the rational expectation has the form of a distributed lag can be found in the structure of information. From (14) we see that x_t^p is influenced by each of the terms Δx_{t-i}^p, i=0, 1, 2, …. All of these terms are unknown in period t, and every past value of x contains information about the permanent component, so it is rational to use this information to predict x_t^p.[23] Stated less formally, our result emphasizes the fact that, even several periods after the realization of a permanent shock that changes x_t, people in the economy cannot be certain about how much of the change is permanent and how much is transient. They therefore find it useful to use current and past values of the x's to decide on the permanent value of x.

Second, each past value of x enters the prediction formula with a positive weight. The higher the observed frequency of large x's in the past, the more evidence there is that x is permanently high, so the expectation regarding the permanent component is higher too.

Third, the weights of the optimal forecast in (15a) assume the form of a Koyck distributed lag and sum to unity.[24]

Fourth, the sum, $S_n \equiv \lambda \sum_{t=0}^{n-1} (1-\lambda)^i$, of the first n coefficient is an increasing function of the ratio, a, of the variance of permanent to the variance of temporary shocks.[25] Since the weights sum to unity, the larger is a, the faster people learn about a permanent change when such a change occurs. As $a \to 0$, $\lambda \to 0$ and the weights given to all past history are nearly equal. In this case, the transitory shocks cancel; the permanent component is virtually constant over time, and people estimate x_t^p by giving all past information equal weight. It is as if

[23] *Muth* (1961, p. 320), *Frenkel* (1975) and *Mussa* (1975) analyze particular cases in which a distributed lag is the optimal predictor of a variable. In each of these cases, the result depends on some restrictive assumptions about the deterministic structure of the economy. As can be seen above, the stochastic structure presented here can coexist with a wide variety of structural models. As is well known, a large number of empirical studies have found distributed lags useful.

[24] Inspection of (15b) shows that for any $a \geqq 0, 0 \leqq \lambda < 1$. Hence whatever the ratio between σ_{xp}^2 and σ_{xq}^2 the expression in (15a) is a Koyck distributed lag.

[25] This can be seen by noting that $S_n = 1-(1-\lambda)^n$ “. Since $\lambda < 1$ this implies that S_n increases with λ. Differentiating (15b) with respect to a we obtain

$$\frac{\partial \lambda}{\partial a} = \frac{1}{2}\left(\left[\left(\frac{1+a+a^2/4}{a+a^2/4}\right)\right]^{\frac{1}{2}} - 1\right),$$

which must be positive. It follows that

$$\frac{\partial Sn}{\partial a} = \frac{\partial Sn}{\partial \lambda}\frac{\partial \lambda}{\partial a} > 0\cdot$$

they estimate a constant mean using all the observations they have. At the other extreme, $a \to \infty$, and $\lambda \to 0$. Almost all the variation in x is now caused by variations in the permanent component, so virtually all the weight is concentrated on the most recent past period, and no weight is given to earlier observations of x.[26]

2. *Solutions for the Permanent Values*

By using (15a) for each of the observations on the various shocks in (11), we can find the public's belief or perception about the permanent component of each shock in period t. Let $E\left[\varepsilon_t^p \mid I_t\right]$, $E\left[\psi_t^p \mid I_t\right]$ and $E\left[u_t^p \mid I_t\right]$ represent, respectively, the beliefs about the permanent values of the shocks to aggregate demand, money supply and productivity in period t.[27] By assumption, the structure of the economic model is known to everyone. Knowledge of the structure is used with the beliefs about the shocks to form (rational) beliefs about the permanent values of the endogenous variables in period t.

Formally, the variables y_t^p, l_t^p, p_t^p, w_t^p and r_t^p, which represent the beliefs of the public in period t about permanent income, permanent labor input, permanent prices, permanent real wages and the permanent value of the nominal interest rate, are solved from the system of eqs. (7)–(10) after substituting the beliefs about the permanent values of the various shocks, $E\varepsilon_t^p$, $E\psi_t^p$, Eu_t^p. Eqs. (7')–(10') show the adjustments.

(7') $$y_t^p = k + \alpha y_t^p + \beta r_t^p + E\varepsilon_t^p ,$$

(8') $$m + E\psi_t^p = B + p_t^p + y_t^p + br_t^p - \theta_1 E\varepsilon_t^p ,$$

(9') $$l_t^p = -\eta\left(w_t^p - Eu_t^p\right) + \eta \log \delta = \eta \log \delta + \theta_2 E\varepsilon_t^p ,$$

(10') $$y_t^p = Eu_t^p + \delta l_t^p .$$

Only perceptions of the permanent values remain; p_t^p has been substituted for ${}_t p_{t+1}^*$ to reflect the fact that, given the information available in period t, the best forecast of the price level of period $t + 1$ is the permanent level of prices that the public perceives in period t.[28]

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[26] See also *Muth* (1960) end of section 3.

[27] For notational convenience the conditioning information set, I_t, will be deleted in future references to the beliefs.

[28] This is a consequence of the essentially stationary specification of the money supply process. The model can be easily extended to incorporate a (non-stochastic) positive rate of growth of the money supply.

Table 1

Solutions for Permanent Values[a]

$$w_t^p = -\frac{\theta_2}{\eta} E\varepsilon_t^p + Eu_t^p$$

$$l_t^p = \theta_2 E\varepsilon_t^p + \eta \log \delta$$

$$y_t^p = \delta\theta_2 E\varepsilon_t^p + Eu_t^p + \delta\eta \log \delta$$

$$p_t^p = \left[\theta_1 + \frac{b}{\beta} - \delta\theta_2 \left[1 + \frac{b}{\beta}(1-\alpha)\right]\right] E\varepsilon_t^p - \left[1 + \frac{b}{\beta}(1-\alpha)\right] Eu_t^p + E\psi_t^p + K_p^p$$

$$r_t^p = \frac{1}{\beta}[(1-\alpha)\delta\theta_2 - 1] E\varepsilon_t^p + \frac{1-\alpha}{\beta} Eu_t^p + K_r^p$$

[a] K_p^p and K_r^p are combinations of parameters and have no importance for the discussion.

Table 1 shows the solutions for the permanent values in terms; of the perceived permanent shocks.[29] All permanent shocks shift the permanent values of the endogenous variables in the same direction as the initial shocks shown in eqs. (7) to (10). We discuss the effects of the three shocks in turn.

Permanent shocks to the money stock, $E\psi_t^p$, affect only the permanent price level. This is a reflection of the monetary neutrality of the model. Once the change in money stock is perceived, it is reflected in p_t^p and therefore in today's forecast of next period's price, ${}_t p_{t+1}^*$; all real variables are unaffected.

Permanent real shocks to spending, $E\varepsilon_t^p$, include permanent changes in tastes for current relative to future consumption and permanent changes in the real value of government spending. Such changes have permanent real effects. An increase in $E\varepsilon_t^p$, permanently increases employment, real permanent output, and changes the (real and nominal) permanent interest rate, but lowers the permanent real wage, w_t^p.[30] The effect on the permanent price level is ambiguous.

The responses of permanent employment, output and real wages to $E\varepsilon_t^p$ are entirely the result of the effect on permanent labor supply, given by θ_2. Workers increase the permanent supply of labor as part of their adjustment to a positive

[29] Hereafter, we use the term 'permanent value' to refer to the perceptions of the permanent values of the shocks and of the endogenous variables. The perceived permanent values differ from the true permanent values except on a set of measure zero. Behavior is always in terms of the perceived values, and the two sets – perceived and true – will differ almost always.

[30] The permanent values of the real and nominal interest rate coincide because the perceived permanent rate of inflation is zero. This is a consequence of our assumption of a stationary money stock.

shock to permanent aggregate demand. An increase in l_t^p lowers permanent real wages and raises output. If θ_2, is relatively small, the ambiguity about the response of p_t^p is removed. In this case, the permanent price level rises once the shock is perceived.

Permanent changes in productivity also induce non-neutral responses. Increased permanent productivity increases permanent output and the permanent real wage. Increased permanent output lowers the permanent price level and the real rate of interest. Permanent employment remains unchanged because we have assumed a vertical long-run supply curve of labor with respect to w_t^p. If we permit the elasticity of labor supply with respect to w_t to exceed the elasticity with respect to w_t^p, in eq. (4), the long-run supply of labor is positively sloped. Permanent increases in productivity increase l_t^p. The direction of all other permanent responses remains the same as in table 1.

The oil shocks of 1974 and 1979 can be treated as permanent reductions in productivity. Once the public recognizes the permanent nature of the shocks, permanent output, permanent income and permanent real wages fall. Prices and real interest rates are permanently higher, but the permanent price level does not continue to rise unless one of the permanent shocks changes from period to period.

Adjustments of beliefs about the permanent values of the endogenous variables in table 1 take the form of distributed lags of past values of the shocks to spending. productivity and money. This follows from the fact that the optimal forecasts of the permanent components of the variables depend only on distributed lags of past values of the observed values of the various shocks. A large body of empirical work on employment, output, prices, wages and interest rate is consistent with this finding, but provides no explanation for either sluggish adjustment or the persistence of response to change. Many of these empirical findings have been dismissed as evidence of irrational neglect of available information. or explained by high costs of learning, as in *Friedman* (1979), or the accelerator, in *Lucas* (1975).

Our model suggests that gradual adjustment is not irrational and should be found in empirical studies whenever the variance of the permanent shock relative to the variance of the transitory shock is not infinite. Gradual adjustment is a result of the fundamental inference problem that people face: the problem of separating permanent and transitory changes when those changes are not observed separately.

IV. The Working of the Model

The perceived permanent values summarize all the available information about the present and future, but they do not fully describe the present. New shocks can occur each period and perceptions about the permanence of shocks change. The actual values of the endogenous variables depend both on the perceptions and on the shocks. In this section, we solve for the actual values of the endogenous variables and analyze the effects of the shocks on the principal variables of the model.

1. Solutions for Actual Values

By substituting the permanent values from table 1 into eqs. (7)–(10) and solving for the equilibrium values of the endogenous variables, we can obtain y_t, l_t, w_t, p_t and r_t as functions of the actual shocks, $x_t = \varepsilon_t, u_t, \psi_t$, and the perceived permanent values of the same shocks, Ex_t^p. The solutions are shown in table 2.[31]

2. Responses to Productivity Changes

If all shocks are permanent and fully perceived, $u_t = Eu_t^p$. Fully perceived permanent shocks to productivity change output and real wages in direct proportion to the size of the shock. Employment, l_t, is unaffected.

A transitory shock is at the opposite extreme. We can analyze a transitory shock by examining the effect of a change in u_t holding $Eu_t^p = u_t^p$. The increase in u_t raises real wages, employment. and output and lowers the price level. Since a transitory shock is a one-time change, people expect the actual values to return to the permanent values.

Inability to separate permanent and transitory changes makes the adjustment to any shock a mixture of the responses to permanent and transitory Table 2 changes. At first, the change in u_t (ε_t or ψ_t) is not correctly perceived to be permanent or transitory. As the permanent or transitory nature of the shock becomes clearer, the responses of the endogenous variables change.

[31] We obtain the equations in table 1 by taking expectations of the values in table 2, conditional on I_t. This demonstrates that the endogenous variables have the stochastic structure for which the permanent values are defined, as required for a rational expectations equilibrium. To avoid a lengthy and not very illuminating expression, we have not substituted for the permanent values in the r_t equation.

Table 2

Solutions for Permanent Values[a]

$$w_t = \frac{\eta}{\eta+\omega}\left(u_t + \frac{\omega}{\eta}Eu_t^p\right) - \frac{\theta_2}{\eta+\omega}\left(\varepsilon_t + \frac{\omega}{\eta}E\varepsilon_t^p\right)$$

$$l_t = \frac{\eta\omega}{\eta+\omega}\left(u_t - Eu_t^p\right) + \frac{\theta_2}{\eta+\omega}\left(\eta\varepsilon_t + \omega E\varepsilon_t^p\right) + \eta\log\delta$$

$$y_t = \frac{1}{\eta+\omega}\left[\left[\eta+\omega(1+\delta\eta)\right]u_t - \delta\eta\omega Eu_t^p\right] + \frac{\delta\theta_2}{\eta+\omega}\left(\eta\varepsilon_t + \omega E\varepsilon_t^p\right) + \delta\eta\log\delta$$

$$p_t = \frac{1}{\beta(1-b)}\begin{Bmatrix} \beta\psi_t + \left[b+\theta_1\beta - \frac{(b+\beta g)\delta\theta_2\eta}{\eta+\omega}\right]\varepsilon_t - (b+\beta g)\frac{\left[\eta+\omega(1+\delta\eta)\right]}{\eta+\omega}u_t \\ +(b+\beta g)\frac{\delta\eta\omega}{\eta+\omega}w_t^p + \left[\alpha b - \beta(1-g)\right]y_t^p - b\beta p_t^p \end{Bmatrix} + K_p$$

$$= \frac{1}{\beta(1-b)}\begin{Bmatrix} \beta\left(\psi_t - bE\psi_t^p\right) - \frac{\left[\eta+\omega(1+\delta\eta)\right](b+\beta g)}{\eta+\omega}u_t \\ +\left[\theta_1\beta + b - \frac{\theta_2\delta\eta(b+\beta g)}{\eta+\omega}\right]\varepsilon_t \\ +\left[\frac{(b+\beta g)}{\eta+\omega}\delta\eta\omega - \beta(1-g) + b\left[\beta+\alpha+b(1-\alpha)\right]\right]Eu_t^p \\ -\left\{\left[\frac{(b+\beta g)\omega}{\eta+\omega} - \alpha b + \beta(1-g)\right]\delta\theta_2 + b\varphi_1\right\}E\varepsilon_t^p \end{Bmatrix} + K_p$$

$$r_t = \frac{1}{\beta(1-b)}\begin{Bmatrix} -\beta\psi_t + \frac{1+\beta g}{\eta+\omega}(\eta+\omega(1+\delta\eta))u_t + \left[\frac{1+\beta g}{\eta+\omega}\delta\theta_2\eta - (1-\theta_1\beta)\right]\varepsilon_t \\ -\frac{(1+\beta g)}{\eta+\omega}\delta\eta\omega w_t^p - \left[\alpha - \beta(1-g)\right]y_t^p + \beta p_t^p \end{Bmatrix} + K_r$$

$$\varphi_1 \equiv \theta_1\beta + b - \delta\theta_2\left[\beta + b(1-\alpha)\right]$$

[a] K_p^p and K_r^p are combinations of parameters and have no importance for the discussion.

Suppose there is a permanent change in u_t. Initially Eu_t^p changes very little. If the shock is positive, real wages, employment, and output rise, and the price level falls. As the perception of permanence increases, Eu_t^p starts to adjust. Adjustment of Eu_t^p reinforces the effect of u_t, on w_t and, wholly or partly, offsets the effects of u_t on employment and output.

The distinction between permanent and actual changes helps to explain why real wages appear to be 'sticky'. The initial response of the real wage to a permanent productivity shock is a fraction, $\eta / (\eta + \omega)$, of the response to a fully perceived permanent shock. Once the permanence of the shock is recognized, the effect of u_t on the real wage is reinforced by the response of of w_t to Eu_t^p. Eq. (15) implies, however, that Eu_t^p adjusts gradually, so the full adjustment of the real wage occurs gradually.

The 'stickiness' of real wages means that the short-run elasticity of the real wage with respect to productivity is smaller than the long-run elasticity. The short-run elasticity of employment is larger than the long-run elasticity, however. Employment rises in response to a positive shock to productivity (and falls in response to a negative shock). The response to the actual shock increases the demand for labor. Growing recognition of the permanence of the shock, and the resulting gradual adjustment of real wages, reduces the demand for labor and lowers employment.

The response of employment to a permanent productivity shock is reflected in output. Output initially overshoots, then gradually adjusts as the permanence of the shock is perceived and the increased or reduced productivity becomes fully reflected in the prevailing real wage. Permanent reductions in productivity permanently lower output. The unchanged labor force has lower productivity and produces less.

During adjustment of perceived permanent changes to actual changes in productivity, the price level may rise or fall. The speed with which perception of the permanence of the shock grows and the relative effects of w^p, y^p and p determine the direction of adjustment. The ambiguity in the response of p to Eu_t^p arises because the positive effects of Eu_t^p on w_t^p and y_t^p (for β sufficiently small) combine with a negative effect of Eu_t^p on p_t^p.

The long-run adjustment. after the permanence of the productivity shock is fully perceived, is not in doubt. Permanent increases in productivity lower, and permanent reductions raise, the excess demand for output. The long-run elasticity of p_t, with respect to u_t Eu_t, is $-[1 + (\mathrm{b} / \beta)(1 - \alpha)]$ which is unambiguously negative.

3. *Responses to Monetary Shocks*

Our model is neo-classical, so money does not affect output, employment or wages. The only effects of actual and perceived monetary shocks are on the price level and the hate of interest. A change in ψ_t, changes the price level by changing the excess supply of money and market interest rates. The change in interest rates changes spending. A change in $E\psi_t^p$ also affects spending by changing the perceived price level and the excess demand or supply on the commodity market.

The equation for p_t in table 2 shows that when all monetary shocks are permanent ($\psi_t = E\psi_t^p = \psi_t^p$), the price level changes equiproportionally. Transitory shocks change the price level less than proportionally because the transitory shock temporarily changes interest rates and the demand for money in an offsetting direction. If a permanent change in money is less than fully perceived, the price level undershoots the stationary equilibrium value. The price level remains below the stationary equilibrium value following monetary expansion and above it following monetary contraction until the permanence of the shock is correctly perceived.

Substituting for p_t^p in the equation for r_t in table 2 and expressing r_t in terms of $\psi_t - E\psi_t^p$ shows that an unforeseen permanent increase in money lowers the nominal rate of interest.[32] Market interest rates fall in response to increases in money and rise as the permanence of the change in money is perceived. The positive effect of perceived changes in money on interest rates is a result of their effect on perceived or anticipated prices. Once people expect the price level to rise, they reduce real balances. This causes a decrease in the current real rate of interest and therefore increases aggregate spending and the price level. The rise in the nominal rate re-establishes equilibrium in the money market.

4. *Responses to Spending Shocks*

Changes in actual and perceived aggregate demand also affect the equilibrium positions of the output, labor, money and bond markets. Table 2 shows that the effects on output, employment and wages depend on the response of labor supply. If $\theta_2 = 0$, the supply of labor is independent of the shock to aggregate demand, and the real wage and output are unaffected by – perceived and non-perceived – shocks to spending. The entire effect of the spending shocks is borne by the price level and the rate of interest. Prices and market rates of interest rise in response to positive shocks and fall in response to negative shocks.

If we may treat government spending as one type of spending shock. we see that, in the absence of an effect on labor supply, government spending has no effect on output and employment. The initial effect of positive ε_t, increases aggregate spending and reduces the demand for money. The excess demand for output raises the price level and the market rate of interest. Gradually the higher level of spending is perceived to be permanent. As this occurs the price level ris-

[32] Such a phenomenon has been observed by *Cagan* (1972) for the U.S. and by *Harberger* (1963) for Chile. *Frenkel* (1975) and *Mussa* (1975) provide an explanation of this phenomenon that is based on an expectation formation process which combines adaptive and regressive elements. However the class for which their process is rather restricted. See *Mussa* (1975).

es further to reach a new permanently higher level. The market rate of interest remains at a higher level after the permanence of the shock is fully perceived.[33]

In the more general case of $\theta_2 > 0$, positive shocks to aggregate demand increase employment and output and lower real wages. Perception that the positive shocks are permanent reinforces the initial response, further increasing output and employment and lowering the real wage. Both the perceived wage (table 1) and the actual wage (table 2) fall in response to the rise in $E\varepsilon_t^p$, but the former falls more than the latter.

Analysis of the permanent shock to spending implies that such changes cannot affect employment unless people are willing to work more, at lower wages, to buy the additional output they produce. The analysis implies, however, that positive transitory shocks to spending also increase output by shifting the labor supply function and lowering the real wage.

V. Stagflation

The sluggish adjustment of wages. prices, output and employment that results from the confusion between permanent and transitory shocks can produce rising unemployment and rising prices in response to a permanent decrease in productivity. This pattern, known as stagflation, is at times, taken as evidence that prices are set without regard to market conditions. We show that this is incorrect. 'Stagflation' can occur even in a neo-classical framework when there is uncertainty about the permanence of shocks.

This section analyzes unemployment and inflation separately. Then we combine the responses and discuss the 'stagflation' that followed the 1973–1974 oil shock.

1. Unemployment

Eq. (5) makes unemployment dependent on the difference $w_t - w_t^p$. By substituting from tables (1) and (2) and leading unemployment *j* periods, we obtain

(16a) $$n_{t+j} = \frac{\omega\eta}{\eta+\omega}\left(Eu_{t+j}^p - u_{t+j}\right) + \frac{\omega\theta_2}{\eta+\omega}\left(\varepsilon_{t+j} - E\varepsilon_{t+j}^p\right),$$

[33] The effects of ε_t and $E\varepsilon_t^p$ on the price level, given $\theta_2 = 0$, are

$$\frac{\partial p_t}{\partial \varepsilon_t} = \frac{b+\theta_1\beta}{\beta(1-b)} > 0 \text{ and } \frac{\partial p_t}{\partial E\varepsilon_t^p} = \frac{-b(\beta\theta_1+b)}{\beta(1-b)} > 0$$

for small values of β. The rise in the market rate reflects the rise in the real rate. Substituting $p_t^p - p_t$ from tables I and 2, we can obtain the real rate $\upsilon_t = r_t - \left(p_t^p - p_t\right)$. For $\theta_2 = 0$, $\partial\upsilon_t/\partial\varepsilon_t = -1/\beta > 0$ and $\delta\upsilon_t / \partial E\varepsilon_t^p = 0$.

or

$$n_{t+j} = A_1\left(Eu^p_{t+j} - u_{t+j}\right) + A_2\left(\varepsilon_{t+j} - E\varepsilon^p_{t+j}\right). \tag{16b}$$

Unemployment is an increasing function of the difference between the permanent level of productivity and its current value and a decreasing function of the difference between the perceived and actual levels of aggregate demand.

When the actual values of productivity and aggregate demand are equal respectively to their perceived permanent components, the unemployment rate is zero. When people believe that the permanent level of productivity is higher than the actual level, they refuse offers of employment at wages below the wage they believe should prevail, so unemployment is positive. Conversely, people supply labor in excess of the amount demanded at the current wage when they believe that the current wage and current productivity are above permanent levels. Similarly, when people believe that current aggregate demand is temporarily high, they must also believe that labor supply is temporarily high. They estimate that the current permanent wage is above the actual wage. Measured unemployment increases.

The unemployment rate is not stationary. Each observation on unemployment, output, prices, wages and interest rates leads to revision of beliefs about permanent and transitory components, and each revision of beliefs changes perceived and actual values. But each period also brings additional shocks. The fundamental inference problem remains.

To illustrate the inference problem, and its influence on unemployment, we start from a position in which the only reason for a difference between actual and perceived permanent productivity and aggregate demand is that the actual shocks include a transitory component.[34] Formally,

$$E_q u_{t-1} = u^p_{t-1} = Eu^p_{t-1}\,,\; E_q \varepsilon_{t-1} = \varepsilon^p_{t-1} = E\varepsilon^p_{t-1}, \tag{17}$$

The only reason that (positive or negative) unemployment occurs is that productivity and aggregate demand are subject to transitory shocks. Unemployment is a white noise process with mean zero.

Suppose now that in period t a large permanent shock to productivity reduces productivity by a large constant, Du^p. People observe the effects of the shock, but they do not know whether the observed changes are permanent or transitory adjustments. For convenience, we assume that, without the shock, permanent productivity and permanent aggregate demand would have been constant at u^p

[34] The index q on the expectation operator indicates that the expectation is over the distribution of the transitory component of the variable.

and ε^p respectively.[35] The rate of unemployment before and after the shock is given by (18) and (19) respectively.[36]

$$n_{t-j} = -A_1 [1 -], \tag{18}$$

$$n_{t+j} = -A_1 \left[1 - \lambda_u \sum_{i=o}^{j} (1-\lambda_u)^i \right] Du^p$$

$$+ A_1 \left[\lambda_u \sum_{i=1}^{j} (1-\lambda_u)^i \, u^q_{t+j-i} - (1-\lambda_u) u^q_{t+j}\right]$$

$$- A_1 \left[\lambda_\varepsilon \sum_{i=1}^{j} (1-\lambda_\varepsilon) \varepsilon^q_{t+j-i} - (1-\lambda_\varepsilon) \varepsilon^q_{t+j}\right], \quad j \geqq 0, \tag{19}$$

where

$$\lambda_x \equiv \left[\frac{\sigma^2_{xp}}{\sigma^2_{xq}} + \frac{1}{4}\left(\frac{\sigma^2_{xp}}{\sigma^2_{xq}}\right)^2\right]^{\frac{1}{2}} - \frac{1}{2}\frac{\sigma^2_{xp}}{\sigma^2_{xq}}, \; x = u, \varepsilon.$$

By taking expected values (over u^q_t and ε^q_t)of eqs. (18) and (19) before and after the permanent reduction in productivity, Du^p, we can compare the expected values of the rate of unemployment over the distribution of transitory shocks,

$$E_q n_{t-1} = 0. \tag{18'}$$

$$E_q n_{t+j} = -A_1 \left[1 - \lambda_u \sum_{i=o}^{j} (1-\lambda_u)^i \right] Du^p, \; j \geqq 0. \tag{19'}$$

Eq. (18') shows that the expected value of the rate of unemployment before the change in permanent productivity is zero: unemployment deviates from its

[35] In fact additional permanent shocks will continue to affect the economy according to the stochastic processes in (11) and (12). However if the permanent shock to productivity that occurs in period t large in comparison to σ^2_{up} and to $A^2_2 \sigma^2_{up}$ it will dominate the economy for a while. See also the discussion at the end of the subsection on unemployment and business cycles which follows.

[36] Eq. (19) can be derived as follows. Rewrite (15a) as

$$Ex^p_{t+j} = \lambda \sum_{i=0}^{j} (1-\lambda)^i \, x_{t+j \quad i} + (1-\lambda)^{j+1} Ex^p_{t-1}$$

for $x = u, \varepsilon$, substitute the result in (16b) and use the assumptions that $E_q x_{t-1} = Ex^p_{t-1}$ for $x = u, \varepsilon$, that ε^p_{t+j} is constant for a sufficient length of time and that $u^p_{t+s} = u^p$ for $s < 0$ and $u^p_{t+s} = u^p + Du^p$ for $s \geqq 0$.

expected value only because of the occurrence of transitory shocks as seen in (18). After the reduction in permanent productivity. the expected value of unemployment becomes positive since $1-\lambda_u \sum_{i=o}^{j} \left(1-\lambda_u\right)^i > 0$ for all i and $Du^p < 0$. The expected value of unemployment is largest immediately after the decrease in permanent productivity then falls monotonically to zero as information about the permanence of the shock accrues. The length of time during which unemployment persists depends on the relative variances of permanent and transitory shocks. The larger the variance of permanent productivity shocks, σ_{up}^2, relative to the variance of transitory productivity shocks, σ_{uq}^2, the shorter and the less severe is the period of unemployment following a shock of given magnitude.

Inspection of (19') also suggests that unemployment persists on the same side of zero for several periods. Persistent unemployment is a consequence of the more gradual adjustment of permanent productivity and permanent wages than of actual productivity and the actual wage rate. Initially, people interpret most of the decrease in the equilibrium real wage as transitory. If the shock persists, beliefs adjust gradually: the difference between perceived permanent wages and actual wages decreases, and the expected value of the rate of unemployment decreases. Formally, as j increases from 0 the coefficient of Du^p decreases in absolute value towards 0.

2. *Persistent Unemployment and Business Cycles*

Eq. (16) implies that the rate of unemployment in period $t+j$ is a linear combination of the forecast errors of the productivity and aggregate demand shocks. Given the information set I_{t+j} which includes the actual values of those two shocks up to and including period $t+j$, those forecast errors are serially uncorrelated. Therefore the rate of unemployment conditioned on I_{t+j} is also serially uncorrelated.[37] However, if a sample of unemployment rates is drawn for a period following a relatively large permanent shock, serial correlation will be found in the sample. This serial correlation is a property of the sample and provides no information about the underlying population or about a larger sample drawn from the same population, or about a sample of equal size drawn for a different time period.

Evidence of ex post serial correlation in a particular sample is not evidence of inefficient use of information. Rational agents, looking back on the period, find support for the hypothesis that a large permanent shock occurred but was misperceived at the time.

[37] We are indebted to Michael Parkin, Patrick Minford and Walter Wasserfallen for insisting on this point.

At the time that the serial correlation is generated, there is no way in which people can use this evidence to improve their forecast. The reason is that the covariance between the forecast errors in the population is zero. In retrospect, however, the same people realize that a large negative shock to permanent productivity has occurred causing perceptions about productivity to be higher than actual productivity for a while and inducing a finite period of serial correlation in the rate of unemployment. By the time people realize that a large permanent shock has occurred, this information can no longer be used to improve the decisions they made in past periods. It is possible to show formally that the covariance between adjacent forecast errors conditioned on the knowledge (to the economist but not to the public) that a large permanent shock occurred in period t will be non-zero and large fog several periods after the shock.[38] Footnote 38 shows that if $\left(\Delta x_t^p\right)^2$ is large relative to σ_p^2 the conditional expected value is positive and large for several periods following a large permanent shock. Standard tests for serial correlation may then detect serially correlated forecast errors and serially correlated unemployment. If $\left(\Delta x_t^p\right)^2 < \sigma_p^2$, there is negative serial correlation. In this case serial correlation is bound from above by σ_p^2, so it is less likely to be detected in sample data.

Looking backward, economists and statisticians can identify some periods in which there is serially correlated unemployment or, more generally, there are serially correlated deviations of real variables from their trend. The amplitude and duration of unemployment. in such periods, depends on the relative variance of permanent and transitory components. The larger the relative variance

[38] Let e be the forecast error. Then

$$E\left[e_{t+j+1}, e_{t+j} \mid \Delta x_t^p\right]$$

$$= E\begin{bmatrix} x_{t+j+1}^q - \lambda\left(x_{t+j}^q + (1-\lambda)x_{t+j-1}^q + \ldots\right) \\ + \Delta x_{t+j+1}^p + (1-\lambda)\Delta x_{t+j}^p + \ldots + (1-\lambda)^{j+1}\Delta x_t^p + \ldots \end{bmatrix}$$

$$x\begin{bmatrix} x_{t+j}^q - \lambda\left(x_{t+j}^q + (1-\lambda)x_{t+j-2}^q + \ldots\right) \\ + \Delta x_{t+j}^p + (1-\lambda)\Delta x_{t+j-1}^p + \ldots + (1-\lambda)^{j}\Delta x_t^p + \ldots \end{bmatrix}$$

$$= \frac{-\lambda\sigma_q^2}{2-\lambda} + (1-\lambda)\sigma_q^2\left[1 + (1-\lambda)^2 + \ldots + (1-\lambda)^{2(j-1)} + (1-\lambda)^{2j} + \ldots\right]$$

$$+ \left(\Delta x_i^p\right)^2 (1-\lambda)^{2j+1}$$

$$= \frac{-\lambda\sigma_q^2}{2-\lambda} + \sigma_q^2\frac{1-\lambda}{\lambda(2-\lambda)} + (1-\lambda)^{2j+1}\left(\left(\Delta x_t^p\right)^2 - \sigma_p^2\right)$$

$$= \frac{1}{\lambda(2-\lambda)}\left[(1-\lambda)\sigma_p^2 - \lambda^2\sigma_q^2\right] + (1-\lambda)^{2j+1}\left(\left(\Delta x_t^p\right)^2 - \sigma_p^2\right)$$

$$= (1-\lambda)^{2j+1}\left(\left(\Delta x_t^p\right)^2 - \sigma_p^2\right) > 0 \;\; as \;\; \left(\Delta x_t^p\right)^2 > \sigma_p^2.$$

of the permanent component, the faster is the speed of adjustment to the shock. For the same realization of a given large permanent shock a relatively large variance of permanent shocks lowers the amplitude and decreases the duration of unemployment. Conversely a relatively small variance of permanent shocks implies that a given permanent shock lengthens the duration.

Throughout the discussion of unemployment, we have made the convenient assumption that the only permanent change that occurs is the permanent productivity shock, Du^p. This permitted us to trace the effects of a permanent change in isolation.

We can now remove the assumption of a single permanent shock and discuss the qualitative effects of a large permanent productivity change in a world in which u_t^p and ε_t^p change continuously over time [in line with the assumptions in (12)] but usually by relatively small increments. If we now superimpose on this economy an unusually large negative permanent productivity shock[39] in period t, it will dominate the scene for some time and all our results above will follow. In particular the lower the variance of permanent productivity shocks, the longer it takes people to learn about a permanent change of a given magnitude and the longer the period of unemployment following the shock.

3. *Expected Inflation*

All of the shocks in the model affect the level of variables. Unless there are repeated shocks in the same direction, measured rates of change decline toward zero. There is no permanent inflation.[40]

Individuals form expectations about the price level they expect to prevail in period t+1 on the basis of information available in period t. The perceived or anticipated price level is ${}_t p_t^* = p_t^{\mathrm{P}}$. The anticipated rate of inflation, after substituting the equations in tables 1 and 2, and cancelling terms, is

$$(20) \qquad {}_t p_{t+1}^* - p_t = -\frac{1}{1-b}\left(\psi_t -, E\psi_t^p\right) + \left(\frac{b}{\beta} + g\right)\frac{\eta + \omega(1+\delta\eta)}{(1-b)(\eta+\omega)}\left(u_t - Eu_t^p\right)$$
$$-\frac{(\eta+\omega)(\theta_1 + b/\beta) - \theta_2\delta\eta(g + b/\beta)}{(1-b)(\eta+\omega)}\left(\varepsilon_t - E\varepsilon_t^p\right).$$

[39] By the assumption (12) the probability that such a shock will occur is small but not zero. Hence large shocks will not occur very often but once they do, they will trigger the various effects described in the text.

[40] The qualitative results are unaffected if we allow money stock to grow secularly.

Eq. (20) shows that when the actual shocks are equal to their perceived permanent values, the expected rate of inflation is zero. Elsewhere, the expected rate of inflation is directly or inversely related (depending on the shock under consideration) to the difference between the current realization of the shock and the perceived permanent value.

An increase in the money stock that is perceived as transitory induces a transitory increase in the price level. People believe that the price level will return to the permanent level, so $\psi_t - E\psi_t^p > 0$ implies a negative expected rate of price change. A temporary, positive shock to productivity has the opposite effect. The shock lowers the price level but the lower price level is not expected to persist. The expected rate of price change is positive.

In general, any shock that causes x to deviate from $Ex_t^p\left(x_t = \psi_t, \varepsilon_t, u_t\right)$ affects the expected rate of inflation in a direction opposite to the effect of x_t, on the current price level.[41] This general rule leads to two propositions.

First, a maintained increase in the permanent value of a shock that starts in period t causes an initial change in the anticipated rate of price change (inflation) opposite to the initial effect on the price level. Expected inflation reverses direction gradually. The reasoning is implicit in the gradual adjustment of permanent perceptions implied by eq. (15) and the fact that in (20) actual and permanent values have opposite effects on inflation.

Second, the expected rate of inflation and the duration of expected inflation following a permanent shock depend on the relative variances of the permanent and transitory compones of the shock. This follows from the demonstration, in footnote 25 above, that the sum, S_n, of the first n coefficients of the distributed lag increases as the relative variance increases.

4. *Stagflation*

A permanent negative shock to productivity affects both unemployment and the rate of price change. This section brings some of our findings together to provide an explanation of the occurrence of stagnation – rising prices and unemployment.

Suppose that a permanent, negative shock to productivity occurs in period t. Initially. the shock is not perceived as permanent. We know from the discussions of prices, unemployment and inflation that a shock of this kind initially causes the unemployment rate to increase and the price level to rise. The measured rate of price change is positive. Since the reduction in u_t, is treated as tem-

[41] Where the total response is ambiguous, as is the case for ε_t with $\theta_2 \neq 0$, the proposition applies to the positive and negative components of the response.

porary, the public expects future prices to be lower and future employment to be higher.

Because people do not correctly perceive the permanence of the productivity shock, they consume at levels more nearly consistent with past than future values of y^p. Real wages fall on impact but not enough to prevent unemployment. The unemployed seek work at a real wage, w^p that is not fully adjusted to the permanently lower level of productivity in the economy. Workers reject offers of employment at the current wage. Since everyone believes that the economy can sustain a real wage higher than the current wage, the workers' response is rational.

Output declines first because of the permanent reduction in productivity and second because labor withdraws some of its services. Since aggregate demand declines initially just a bit, there is excess demand in the commodity market and upward pressure on the price level. Thus a large permanent negative productivity shock increases on impact both the rate of price change and unemployment.

Persistence of the negative productivity shocks erodes the belief that the shock is transitory. Eu_t^p declines, and the initial responses of employment, output and measured inflation are reversed. Real wages decline further, and perceived permanent real wages fall. People accept employment at lower real wages. Unemployment and the measured rate of inflation fall.

The speed of adjustment to a permanent productivity shock, and the persistence of stagflation, depend on the relative variance of permanent and transitory shocks. Infrequent permanent shocks lengthen the period of stagflation following a particular shock Because people are unaccustomed to permanent shocks, they take longer to believe that the productivity shock is permanent after it occurs. As the frequency of the shocks increases, the variance of the permanent component rises. and the speed of adjustment increases.[42]

A permanent negative productivity shock appears capable of generating the pattern of responses popularly called stagflation. Stagflation results from the inability to accurately separate permanent and transitory components of a productivity shock in a world of uncertainty. Rational responses in a world of uncertainty, with prices and quantities at market clearing values, appears to be consistent with the pattern described by stagflation.

[42] The sequence of events following a permanent productivity shock resembles the response to the 1973–1974 oil shocks and the resulting stagflation. The actual sequence was of course a mixture of the shock and the responses to fiscal and monetary policies.

VI. Conclusion

We have used a standard equilibrium model in which all expectations are rational and all markets clear instantaneously to generate sequences of persistent unemployment. The distinctive feature of the theory is that the permanent and transitory components of the shocks that affect the economy are not known in advance and are not revealed for some time after they occur. People must solve an inference problem to distinguish permanent and transitory components of the data they observe. Long after shocks occur, rational individuals, who use all available information, can remain uncertain about the persistence of shocks.

Individuals' decisions depend on their belief about the permanent values, so they forecast these values. Since the permanent components of past shocks have some elements in common with the current permanent component, optimal forecasts of permanent values use observed values of past and current shocks. For the specific stochastic structure we use, the rational expectation of the current value of the permanent component has the form of a Koyck lag on past observed values of the shock.

Although the adjustment paths in our models depend on a relatively simple stochastic structure, the qualitative implications for stagflation are more general. Rising prices and rising unemployment–stagnation–is consistent with the model under conditions that appear to have been realized in the recent past. Most of these implications, and others, are developed fully in the text, so we summarize our findings briefly.

A permanent reduction in productivity increases unemployment on impact. Even though there is no accelerator in the model, unemployment persists for a time if the shock is large. The reason is that perceptions of the permanence of the lower level of productivity form slowly. Real wages decline on impact but do not fully respond to the shock until the permanence of the shock is recognized. During the transition, the labor market clears at the prevailing real wage, but the level of employment remains below the level at which anticipated and actual wages are equal.

Unemployment results from the gradual adjustment of beliefs. When workers believe that current wages are less than the wages they perceive as permanent, they remain unemployed. The converse is also true. When real wages exceed their perceived permanent value, workers supply more labor and unemployment is negative.

Persistent unemployment does not depend on an accelerator as in *Lucas* (1975). Persistence increases (adjustment slows) as the variance of permanent shocks decreases relative to the variance of transitory shocks. Slow adjustment implies that at times unemployment is serially correlated. Evidence of serial correlation in a particular sample can be observed by looking backward. At the time that the unemployment occurs, unemployment rates appear to be serially uncorrelated.

Theories of persistent unemployment that rely on some kind of cost-of-adjustment hypothesis do not generate simultaneous increases in prices, reductions in employment and in real wages. In our model, a reduction in permanent productivity raises the price level at the same time that unemployment increases and subsequently lowers the real wage. On impact, the measured rate of inflation rises. As perception of the permanence of the shock grows, the measured rate of inflation declines and real wages converge to their lower permanent value.

Our model implies that expectations form and decay gradually. Much of our quantitative knowledge about the economy comes from studies that use distributed lags to obtain expected values as a weighted average of past values. *Poole* (1976) argues that such estimates produce lag structures that are too long to be consistent with rational expectations. We find his criticism too sweeping. If the fundamental inference problem in a world of uncertainty is the problem of judging the persistence of shocks. distributed lags are not just an acceptable procedure, they are an optimal means of forecasting.[43] The particular form taken by the optimal predictor here is a consequence of the particular stochastic form assumed for the permanent and transitory components of the shocks. For other assumptions the optimal predictor will not necessarily be of the Koyck type. However it will almost always be a distributed lag because it is optimal to use all past information when the inference problem involves the separation of permanent from transitory changes.

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[43] *Friedman* (1979) also makes this point in the context of a single equation model.

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