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Abhandlungen

Standardisation in the Retail Banking Sector

Designing Functions for an Individualised Asset Allocation Advisory

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Abstract

This article is about individualising the process of giving advice to a retail customer in the field of asset allocation. With regard to this process, two main contributions are made by answering two questions. First, which objectives are relevant for a customer (beyond return and risk) and which functions are adequate to evaluate portfolios of investment alternatives with regard to these objectives? Based on empirical literature on customers' goals, the four objectives liquidity, variability, comprehensiveness, and manageability are identified as relevant. The background of each objective is discussed in order to formulate desirable properties of the objective functions. These properties are then used to axiomatically identify particular functions from fuzzy theory suitable for the given context.

The second question is: which selection function is adequate to select a particular portfolio out of a set of portfolios? To answer this question, again an axiomatic approach is chosen: Several properties are discussed and stated which shall reflect the customer's decision calculus. By requiring these properties, the selection function can be exactly specified.

The results can help financial services providers in two ways: On the one hand, they can provide their customers with a higher quality of their advisory services by taking into account more objectives than return and risk. On the other hand, as the derived functions are standardised, they can be used in software applications to support the advisory process which can then be offered at lower costs and thereby even to retail customers.

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Standardisierung im Retail Banking – Entwicklung von Funktionen für Bewertungsfragen im Rahmen individualisierter Anlageberatungsprozesse

Zusammenfassung

Der vorliegende Beitrag beschäftigt sich mit der Individualisierung von Anlageberatungsprozessen im Massenkundengeschäft bei Finanzdienstleistern. Hinsichtlich dieser Prozesse erweitert der Beitrag die Wissensbasis durch die Beantwortung zweier Fragen:

Die erste Frage lautet: Welche Ziele (neben Rendite und Risiko) sind für Kunden im Rahmen der Entscheidung für eine Kapitalanlage relevant und welche Zielfunktionen können Portfolios aus Anlagealternativen hinsichtlich dieser Ziele angemessen bewerten? Auf Basis bestehender empirischer Untersuchungen werden die Ziele Liquidierbarkeit, Veränderlichkeit, Verständlichkeit und Verwaltbarkeit hergeleitet. Im Anschluss werden die Hintergründe eines jedes dieser Ziele diskutiert, um erstrebenswerte Eigenschaften der jeweiligen Zielfunktionen zu formulieren. Diese Eigenschaften werden dann verwendet, um axiomatisch Funktionen aus dem Bereich der Fuzzy-Theorie abzuleiten, die im Kontext der Anlageberatung dazu geeignet sind, Portfolios aus Anlagealternativen hinsichtlich der Ziele zu bewerten.

Die zweite Frage stellt sich als: Welche Auswahlfunktion ist geeignet, um für einen bestimmten Kunden ein individualisiertes Portfolio aus einer Menge von Portfolios auszuwählen? Um diese Frage zu beantworten, wird auch hier auf eine axiomatische Vorgehensweise zurückgegriffen: Mehrere Eigenschaften einer Auswahlfunktion werden beschrieben, die das Entscheidungskalkül des Kunden abbilden sollen. Indem diese Eigenschaften gefordert werden, kann wiederum eine konkrete Auswahlfunktion hergeleitet werden.

Die Ergebnisse des Beitrags helfen Finanzdienstleistern in zweierlei Hinsicht: Einerseits können sie ihren Kunden eine bessere Qualität von Anlageberatungsprozessen bieten, indem sowohl mehr Ziele als lediglich Rendite und Risiko als auch kundenindividuelle Präferenzen bei der Auswahl eines Portfolios berücksichtigt werden können. Andererseits erlaubt die Standardisierung der entwickelten Funktionen, dass diese in Beratungsapplikationen verwendet werden, welche wiederum den Beratungsprozess unterstützen können. Hierdurch ist es möglich, diese individualisierte Beratungsdienstleistung zu geringeren Kosten und somit auch für das Massenkundengeschäft anzubieten.

Keywords: Portfolio Choice, Investment Decisions, Banks, Other Computer Software

JEL Classification: G11, G21, C88

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I. Introduction

"Private investors are turning away from financial advisers and choosing to control their own finances in the wake of the credit crunch."[1] The reason for such customers' mistrust might be found in the losses they suffered due to the credit crunch in recent years: Worldwide, assets of private households decreased in value by €3,360,000,000,000 from 2007 to 2009.[2] Regaining the customers' trust will require much effort from the banks. Amongst other activities, they will have to invest in improving the quality of their advisory services (which is often evaluated as poor[3]).[4] Improving the quality means to individualise the advisory services, i.e. to adapt them to a customer's individual properties, needs and knowledge. So far, individualised advisory services are cost-intensive due to a high level of human involvement in the process. That is why these services are currently mostly offered to customers belonging to the high net worth individual segments where the costs for manual individualisation are expected to be compensated by an adequate earning. To retail customers however, banks can offer such individualised advisory services at acceptable prices only if these services are supported via adequately designed information systems (IS), as this reduces manual interference and thus cost. Hence, the advisory process has to be standardised, as only then it can be supported via IS.

That is why this article follows an interdisciplinary approach: On the one hand, the quality of asset allocation advisory processes is improved; on the other hand and at the same time, these processes are standardised and can thus be supported via a software application. To improve quality, objectives which are relevant to customers – beyond risk and return – are determined and methods how to rate particular product categories (of financial services) with respect to these objectives are proposed. This helps to improve quality, as currently these objectives are usually not taken into account when giving IS-supported advice to retail customers. In ad-

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[1] *Ross* (2010).

[2] *Allianz Group* (2011).

[3] Cf. e.g., *Inderst/Ottaviani* (2010).

[4] Indeed, regaining customers' trust is not the only reason for banks to offer individualised advisory services; a higher quality of advisory services is more and more required by consumer protection laws (for instance in Europe, the AIFM Directive, or in the United States, the Credit CARD Act) and discussed at a regulatory level [for instance, the "Retail Financial Services Report" in Europe (*Commission of the European Communities* (2011)) or the "Consumer Financial Protection Bureau" in the United States (*Campbell* et al. (2011))].

dition, we axiomatically derive functions to rate combinations of product categories with respect to the determined objectives. This is important, as a solution for an individual customer's problem often consists not only of one, but of several product categories (i.e. a portfolio). The third contribution of this article to a higher quality of advisory services is to axiomatically derive a function to select one particular combination of product categories which fits best the customer's individual weights with respect to the objectives. This is particularly relevant, as – due to the additional objectives taken into account – no established selection function exists. All artefacts (namely the axiomatically derived functions and their input parameters) can then be implemented in a software application which supports individualised advisory processes. Such processes can then be offered at reasonable prices to customers belonging to the retail segment.

The paper is organised as follows: Section II describes the application scenario, namely a concept for an asset allocation advisory. Section III is split into three parts: First, empirical literature is analysed in order to identify objectives being relevant to customers in an asset allocation (III.1). Second, it is analysed how product categories (asset classes) can be rated with respect to the identified objectives (III.2). Third, we axiomatically derive functions to rate combinations of product categories regarding these objectives (III.3). In section IV, further steps of the asset allocation advisory are described, before section V designs a selection function for choosing a particular combination of product categories which fits the customer's needs best. To achieve this, first the literature concerning axiomatically derived functions for a multi-objective portfolio selection problem is analysed (V.1). Next, a selection function is determined from axiomatically defined requirements (V.2) and particular aspects of the derived function are discussed in (V.3) and (V.4). The last section sums up the results.

II. Application Scenario

In the following, we outline a concept for an individualised advisory process. Figure 1 depicts the steps defined the concept in order to find an individualised portfolio for a customer taking into account his or her individual preferences.[5]

[5] This paper focuses on retail customers, as this segment is currently not offered an advisory service of high quality, because costs are too high due to a high level

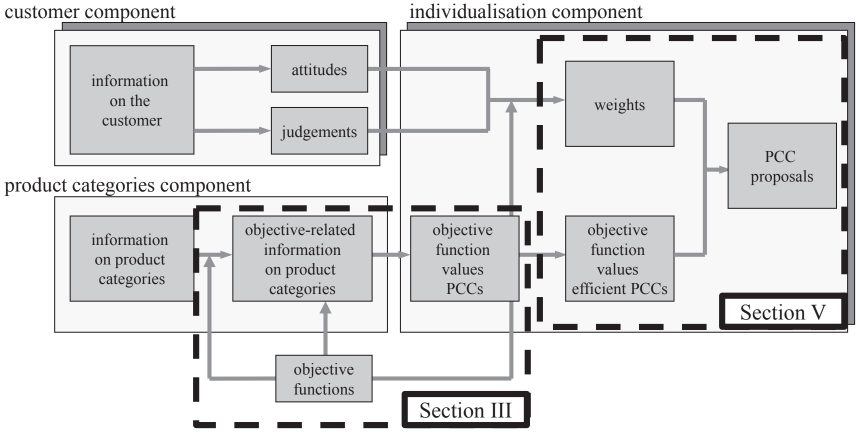

Figure 1: Concept of an Individualised Advisory Process[6]

The knowledge the financial services provider (FSP) holds on the customer is stored in the customer component. It uses information on the customer[7] (e.g., job, financial status) for deriving the customer's attitudes[8] and the FSP's judgement on the customer[9] for advisory-relevant aspects.

The product categories component holds the relevant knowledge on the product categories considered in an advisory. A product category (PC) contains similar products and is comparable to an asset class, as e.g. life insurance or pension funds.[10] The product categories component transforms information on product categories (for instance historical returns or experts' estimations) into objective-related information on product categories.

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of manual interference. This focus on retail customers does not mean any loss of generalisation: The results can be applied to other customer segments as well. In addition, as the article at hand standardises the advisory process, also the complexity of this process is reduced. As a consequence, most financial services providers (FSPs) are able to make use of the results.

[6] Following *Buhl* et al. (2004), p. 431.

[7] In this article, the term customer is consistently used to describe the person belonging to the retail segment who receives advice from the FSP.

[8] For instance, the customer's risk aversion. For the quantification of attitudes cf. *Thurstone* (1931).

[9] For instance, the customer's risk carrying capacity.

[10] For further examples, please refer to Table 3.

Thereby, each PC is assigned a numerical value for its contribution to each objective considered in the advisory. The individualisation component uses the output of the other two components. On the customer side, judgements and attitudes are transformed into weights reflecting the importance of the different objectives to a particular customer. On the product side, different product categories are combined to so called product categories combinations (PCCs), equal to a portfolio (e.g., 25 % mutual funds, 50 % pension funds, and 25 % cash funds). Deriving objective functions which are used to assign objective function values (to each) PCCs for all objectives considered is one of the two main purposes of this article (section III). This step leaves a number of – with regard to the objective function values – dominated PCCs which are excluded from the solution set. Finally, the individualisation component selects and prioritises PCCs proposals out of the set of efficient PCCs based on customer preferences, using a selection function which comprises both weights and objective function values (of the) efficient PCCs. Designing this selection function is the second main purpose of this article (section V).

The concept was applied in different advisory scenarios which vary regarding the objectives that have to be taken into account: For each objective function considered, the objective-related information about PCs (stored in the product categories component) as well as the objective function values (of a) PCC and the weights (both stored in the individualisation component) have to be computed. For illustrating purposes, we use an asset allocation advisory scenario as an example for the individualised advisory process. In addition, we provide a numeric example to demonstrate the process step by step.

III. Objective Functions for an Individualised Asset Allocation Advisory

This section mainly addresses three areas within the application scenario outlined in the previous section: First, based on empirical studies, those objectives are identified which are relevant to a real-world customer in an asset allocation advisory. Second, it is illustrated how different PCs can be evaluated concerning the identified objectives. The major part however addresses a third aspect: Very often, an individualised solution does not only consist of one, but of several PCs. As described above, such a solution is named PCC. The question to be answered is therefore: How should one evaluate such a combination of product categories with respect to the particular objectives? Based on an axiomatic approach, one mathematical function is derived for each objective identified earlier. The results of these objective functions can be used to evaluate PCCs regarding the objectives.

1. Customers' Objectives

The objective functions which are applied in the asset allocation advisory are derived from analysing empirical literature on customers' objectives.[11] The authors consider the survey "Debit and Credit 6" as most suitable, as it is based on questioning more than 10,000 respondents[12] and was conducted for the sixth time in 2005. Table 1 denotes statements concerning a private investment and the percentage (%) of respondents who claimed the corresponding criteria to be "very important" or "fairly important".[13]

The concept outlined in section II shall be the base for a decision support system to give advice to a customer. The number of criteria (31) for an ideal investment listed in Table 1 is by far too high; most customers are overstrained by facing so many facts.[14] Therefore, the catalogue of criteria is reduced by means of the following two steps:

a) combining/aggregating several criteria

b) examining whether a particular criterion is relevant for the given situation (here: providing advisory for an asset allocation).

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[11] One might also choose a normative approach, deriving the objective functions from criteria which "should" be taken into account for instance from a rational point of view. However, such a normative approach would have to rely on arguments only, whereas the chosen approach starts from an empirical base and includes rational points of view wherever possible.

[12] *TNS Infratest Finanzforschung* (2005), p. 98.

[13] As the empirical investigation was conducted before the credit crunch, the percentages concerning the risk-related statements might be even higher today. However, for evaluating the relevance of a statement, the percentage of respondents who considered it very or fairly important is deliberately not taken into account, as the importance is different for each customer: An individualised advisory should take into account how important these aspects are for the individual customer who is advised and not use an average value – as the ones listed in Table 1 – for all customers. Consider the example of 28 % of the respondents claiming that "An Investment where I can get directly involved myself" is (very) important. This does not necessarily mean, that this criterion is (very) important to *all* customers. Indeed, for one particular customer, this criterion might not be important at all, because he does not want to deal with any financial affairs; this possibility (not necessity!) is represented in the advisory process by choosing the weight of the corresponding objective accordingly (cf. section IV.1).

[14] Cf. e.g., *Duncan* (1980) or *Miller* (1956).

Table 1

Criteria for the Ideal Investment[15]

Statement	%	Statement	%
Risk-free Investment	91	An Investment with permanent state subsidisation	58
An Investment that is profitable in the long term	91	An Investment with no speculation time limits to worry about	58
An Investment that is not devalued by inflation, currency changeover, international financial crises, etc.	91	An I where the risk is spread	55
An Investment which constantly increases in value	90	An Investment with which friends/acquaintances/colleagues have been successful	55
An Investment for which I get back at least what I paid in	90	An Investment that is recommended by independent consumer protection organisations	51
An Investment which does not cost me extra charges or fees	89	An Investment that I can, if required, convert into another form of investment	49
An Investment I know everything about	88	An Investment handled by my employer	49
An Investment which requires little attention	87	An Investment where losses are tax-deductible	47
An Investment with a guaranteed minimum return	87	An Investment recommended by the financial press	43
An Investment also for small amounts	86	An Investment recommended by my tax/ financial advisor	42
Constant, unchanging return (e.g. fixed interest rate)	84	An Investment which gives me the possibility to make a fast profit	38
An Investment I can deal with myself, without having to rely on experts	83	An Investment I can control comfortably by telephone or computer	30
A tried and tested Investment Product	82	An Investment where I can get directly involved myself	28
An Investment to which I have fast access	76	Investment in areas that interest me personally	23
An Investment recommended by my bank/savings bank	68	An Investment where the money is invested in ecological projects	21
An Investment offering tax savings/tax concessions	61		

15 *TNS Infratest Finanzforschung* (2005), p. 98 ff.

Table 2

Objectives for an Individualised Asset Allocation Advisory Derived from Empirical Criteria[16]

Empirical Criteria	Objective
"An investment that is profitable in the long term", "An investment which constantly increases in value", "An investment for which I get back at least what I paid in"	Return
"Risk-free investment", "An investment that is not devalued by inflation, currency changeover, international financial crises, etc.", "Constant, unchanging return (e.g. fixed interest rate)", "An investment where the risk is spread", "An investment with a guaranteed minimum return"	Risk
"An investment to which I have fast access", "An investment that I can, if required, convert into another form of investment"	Liquidity
"An investment where I can get directly involved myself"	Variability
"An investment I know everything about", "An investment that I can deal with myself, without having to rely on experts", "A tried and tested investment product"	Comprehensiveness
"An investment which requires little attention", "An investment I can control comfortably by telephone or computer"	Manageability

The results of step (a) can be seen in Table 2 which contains the proposed assignment of the empirical criteria to particular objectives.[17]

[16] Cf. *Buhl* et al. (2008).

[17] Step (b) can also be seen from Table 2, as it does not include all criteria which are listed in Table 1. The objectives not listed are not considered for the following reasoning: The criteria "An Investment which does not cost me extra charges or fees", "An Investment offering tax savings/tax concessions", "An Investment where losses are tax-deductible" and "An Investment with permanent state subsidisation" have influence on the cash-flows associated with a PC and are therefore integrated in the objective return. The statement "An Investment also for small amounts" is covered via assumption A4 below, which explicitly does not require a minimum share of a PC. "An Investment recommended by my tax/financial advisor", "An Investment recommended by my bank/savings bank", "An Investment recommended by the financial press", "An Investment that is recommended by independent consumer protection organisations", and "An Investment with which friends/acquaintances/colleagues have been successful" are requirements which concern external/ other sources of advisory – therefore they need not be depicted in a decision support system which itself aims at providing advisory. The criteria "An Investment where there are no speculation time limits to worry about" and "An Investment which gives me the possibility to make a fast profit" express the goal to realize 'quick wins' in the short run. Therefore, they are not important for the long term application scenario 'private asset allocation'. "Investment in areas

Based on these considerations the first assumption shall be formulated:

A1: There are $n = 6$ (conflicting) objective functions that are to be maximised or minimised.

2. *Objective-Related Information*

So far, six objectives were determined which shall be taken into account in the individualised financial services advisory for asset allocation. Before assessing the objective function values of a PCC, the evaluation of PCs with respect to the considered objectives has to be addressed. Hence, two further assumptions concerning the available PCs and their objective-related pieces of information are stated:

A2: Π_P is a set of r product categories PC_l, $l \in \{1, \ldots, r\}$ which represent available asset classes:

$$\Pi_P = \{PC_1, \ldots, PC_r\}$$

A3: Each of the r product categories PC_l is defined by a vector containing $n = 6$ objective-related pieces of information about PCs p_{zl}, $z \in \{1, \ldots, 6\}$ and $l \in \{1, \ldots, r\}$:

$PC_l = (p_{1l}, p_{2l}, \ldots, p_{6l})$ with $p_{zl} > 0$ for $z \in \{1, 2\}$, $p_{zl} \in (0; 1]$ for $z \in \{3, \ldots, 6\}$ and covariance matrix $p_{2l} = \begin{pmatrix} p_{211} & \cdots & p_{21r} \\ \vdots & \ddots & \vdots \\ p_{2r1} & \cdots & p_{2rr} \end{pmatrix}$, where p_{2lk} represents the covariance of PC_l and PC_k.

An important issue is the assessment of the objective-related pieces of information $p_{1l}, p_{2l}, \ldots, p_{6l}$. Concerning the objectives return and risk, there is a plethora of methods to measure these objectives. For instance, they can be measured based on historical data by means of expected value and variance, which can also be applied for the given context. In contrast, such measures do hardly exist for the other four objectives identified in Table 2. Concerning the liquidity of stocks, there are measures like the bid-ask-spread.[18] Such measures, however, rely on the existence of market places which are, for example, only available for some of the PCs.

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that interest me personally", "An Investment that is handled by my employer" and "An Investment where the money is invested in ecological projects" can be used as criteria to sort out in advance those PC which do not meet these requirements.

[18] Cf. e.g., *Amihud* (2002).

As a consequence, for those PCs, where the necessary data for determining the bid-ask-spread are available, the bid-ask-spread is a valid starting point. Nevertheless, further approaches are needed to provide information related to the four objectives identified in Table 2, because they are of utmost importance to the customer and should therefore be integrated in an individualised asset allocation advisory. Information related to these objectives can for instance be derived from experts' estimations as depicted in Table 3.

The values are based on a survey of 209 financial advisors who were asked how good they judge a PC concerning particular criteria on a scale from 1 (lowest) to 5 (highest). Admittedly, the criteria used in Table 3 are neither congruent with the criteria used in *TNS Infratest Finanzforschung* (2005)[19] nor with the objectives derived above. But the criteria can be mapped to the objectives; for instance "familiarity of the investor" is an indicator for "comprehensiveness". Using such data, it is possible to derive information related to the particular objectives, i.e. the objective-related information p_3 to p_6 and they can be therefore assumed as given.[20]

In Table 4, an example is given that will be used throughout the article to illustrate the advisory process.

Given are three PCs with objective-related pieces of information and covariances as given in Table 4. The values were chosen for illustrational purposes.

So far, eacry h product categohas been evaluated separately. However, since it is highly likely that the customer's needs are not satisfied by only one product category, functions are needed to evaluate combinations of product categories. These are the objective functions in Figure 1 to be designed in the next section.

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[19] *TNS Infratest Finanzforschung* (2005), p. 98 ff.

[20] The proposed approach is by no means restricted to a particular set of PCs for the given application scenario; this is due to the fact that all categories of products can be evaluated concerning their information related to the objectives considered appropriate for the particular advisory.

Table 3

Experts' Estimation on Product Categories with Respect to Particular Criteria[21] (Own Translation)

Product Category	Conservation of nominal monetary value	Conservation of real monetary value	Good return of pay-out	Long-term increase in value	Short-term financial gain	Fast liquidity	Tax savings / Tax concessions	Small denomination	Minimum activity required from investor	Familiarity of the investor	Continuous information flow to the investor	Involvement of / decisions by the investor
Stock funds	3.60	3.74	2.62	3.96	3.40	4.67	2.50	4.75	3.68	3.15	3.31	1.69
Building society savings agreement	3.78	2.99	1.87	2.61	1.23	1.53	3.70	2.77	3.50	3.46	3.21	1.88
Berlin loan	3.77	3.19	3.12	2.92	1.42	1.23	4.69	2.39	3.30	2.54	2.86	1.61
Noble metals	2.01	2.25	1.11	2.79	1.92	3.72	1.24	3.61	3.10	3.34	3.03	1.89
Fixed interest securities	4.33	4.02	4.16	3.57	2.51	4.40	2.37	4.13	3.71	3.95	3.45	1.90
Life insurance	3.72	3.39	2.64	3.87	1.20	1.50	4.06	2.64	3.57	3.25	2.61	1.54
Open property funds	3.56	3.69	3.14	4.12	1.57	3.75	2.92	4.24	3.75	2.90	3.12	1.61
Company shares	2.92	3.04	2.99	3.02	1.99	1.70	3.09	1.80	2.29	2.16	2.58	3.38
Options bonds	3.60	3.29	2.48	3.27	3.52	4.33	2.40	3.46	2.66	2.34	2.47	1.80
Warrants	2.88	3.04	1.45	3.02	4.50	4.59	1.78	3.96	2.31	2.44	2.75	1.78
Bond funds	4.05	3.88	4.14	3.93	2.35	4.67	2.12	4.55	3.84	3.58	3.39	1.52
Savings	3.97	3.07	2.35	2.53	1.50	4.34	1.50	4.58	4.24	4.53	4.25	2.35
Standard stocks	3.64	3.80	2.76	3.94	4.26	4.67	2.48	4.20	2.49	3.34	3.64	3.59
Growth stocks	3.52	3.77	2.49	4.20	3.73	4.33	2.37	3.74	2.54	2.84	3.01	2.82
Currency bond	3.24	3.09	3.94	3.33	3.02	3.92	2.05	2.42	2.40	2.44	2.48	1.66
Convertible bond	3.56	3.25	2.74	3.53	3.12	4.20	2.14	3.37	2.44	2.40	2.50	1.84

[21] *Ruda* (1988), p. 98 ff.

Table 4

Example 1 – Product Categories and Their Objective-Related Pieces of Information

PC	return	risk	liquidity	variability	comprehen-siveness	manageability	Cov	1	2	3
PC_1	0.10	0.19	0.40	0.15	0.40	0.30	1	0.0361	–0.013	–0.0114
PC_2	0.01	0.01	0.90	0.90	0.90	0.09	2	–0.013	0.0001	–0.0004
PC_3	0.03	0.10	0.05	0.05	0.30	0.05	3	–0.0114	–0.0004	0.0100

3. Objective Functions[22]

The step of combining PC to PCCs is described and formally noted in assumption (A4):

A4: The PC_l can be combined to $PCCs$. Each of the $PCCs$ is defined by a vector $PCC(x_1, \ldots, x_r) = (x_1, \ldots, x_r, f_1, \ldots, f_6)$,

with $\sum_{l=1}^{r} x_l = 1$ and $x_l \geq 0$, $\forall l \in \{1, \ldots, r\}$.

x_l, $l \in \{1, \ldots, r\}$: share of the PC_l in a PCC.

f_z, $z \in \{1, \ldots, 6\}$: $n = 6$ *objective function values of a PCC.*

In this section, we address the question of which functions are adequate to determine f_3, f_4, f_5, and f_6, i.e. four of the $n = 6$ objective function values of a PCC.

We mentioned earlier, that literature on portfolio theory provides a plethora of methods to measure return and risk of PCs. In many cases, this literature also uses these measures as input factors to calculate aggregated values on the portfolio level (PCCs). The first and most famous approach was the one by *Markowitz* (1952). To the best of the authors' knowledge, such methods lack for the other four objectives identified in Table 2, which are also relevant for individualised financial services ad-

[22] This section is based on but fundamentally extends and reformulates the argumentation of *Buhl* et al. (2005).

visory.[23] Hence, *one* suitable function for each of the objectives liquidity, variability, comprehensiveness and manageability shall be identified.

According to the assumptions, there is a set Π_P of *PCs*, which are evaluated with respect to the six objectives determined above. Each of these *PCs* has a share x_l in a particular *PCC*. The main question is: How and to which extent should the information of the different *PCs* related to a particular objective impact the function value of the *PCC* with respect to this objective?

To answer these questions and select one particular function out of many possible alternatives, a two-step approach is used to narrow down the number of suitable functions.

First, the objective-related pieces of information cannot be considered exact values, because – as described earlier – this kind of information is mostly based on experts' estimations and therefore more or less vague. Due to this vagueness, operators from fuzzy theory are used as functions to calculate aggregated objective function values on the level of PCCs. Fuzzy theory explicitly addresses vague decision-making and uses such operators to derive new membership functions via a conjunction[24] of two existing membership functions. A2 is in accordance with fuzzy theory, when it defines [0; 1] as the value ranges of the objective-related pieces of information p_3, p_4, p_5, p_6. For reasons of consistency, the value range of the results of the objective functions is assumed to be [0; 1], too.

Given this restriction of taking the operators from fuzzy theory, the next step is to state desirable properties in terms of requirements derived from the application context, in our case an asset allocation advisory. This axiomatic method consists of two steps: First, desirable properties (which represent requirements) of the function are formulated. Then a function is derived from these properties, so that it meets the corresponding requirements.[25]

The authors decided to use this relatively formal way (also in section IV) to determine adequate objective functions for several reasons: First, making the requirements explicit allows for a transparent discussion,

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[23] Concerning liquidity, functions exists to measure the values of a PCC for this objective. The potentials and shortcomings of these approaches are discussed in section III.3.a).

[24] For instance: union, intersection, complement, cf. *Zimmermann* (2001).

[25] Axiomatic approaches are, for instance, also used in used in *Tsoukiàs* (1994), *Greco* et al. (2004), *Buhl* et al. (2007) or *Alonso-Meijide* et al. (2007).

whether the results of the objective functions are indeed desirable from a rational point of view. In addition, the effects of dropping / adding particular requirements can be analysed and thus the objective functions can be developed further. The transparency with respect to the requirements and their effects makes it also easier for the FSP to explain its evaluation or recommendation to its customers.

In the following sections, one particular objective function for each objective is selected out of several alternatives provided by fuzzy theory.

a) Liquidity

Besides maximising return and minimising risk, literature on asset allocation often refers to the objective "liquidity": "A typical risk for an investor who holds illiquid assets is that it might be impossible to sell the assets rapidly, making possible for an investor to run out of cash. A second form of liquidity risk is that selling an illiquid asset might cause the transaction price to drop."[26] Indeed, literature provides measures to determine the liquidity of a combination of stocks[27]; however, these approaches are not fully applicable to our context, due to the reasons already mentioned earlier: many of the PCs are not traded at transparent market places like – for instance – a stock exchange. As a consequence, the bid-ask-spread – which is input to the existing objective functions – is not determinable for them. However, the aggregation functions proposed in literature remain potential candidates for the objective functions, as they can also aggregate liquidity-related pieces of information that were determined in a different way than the bid-ask-spread. Nevertheless, we follow the axiomatic approach outlined earlier in order to provide transparency on the properties that come along with the objective function chosen in the end.

Hence, a function f_3 shall be determined[28] which derives a function value for the objective liquidity based on the objective-related information of each PC being part of the PCC under consideration. Simultaneously, the function shall meet the requirements as defined in what follows. First, aggregating the objective-related information of two PCs is considered; afterwards, the results are extended to more than two PCs.

[26] *Kempf/Uhrig-Homburg* (2000), p. 27.

[27] Cf. e.g., *Amihud* (2002), p. 37.

[28] We start with f_3, as we assume f_1 and f_2 to be the functions concerning the objectives return and risk.

If two liquidity-related pieces of information differ, it is natural that the "negative" impact of the lower is compensated by the "positive" impact of the higher value (et vice versa). Hence, the result of the objective function shall represent a trade-off between the two values: The aggregated function value for liquidity should be in between the objective-related pieces of information of the two PCCs, i.e., f_3 shall possess the Mean Value Property.[29]

(RA1) Mean Value Property: It holds for all $p_1, p_2 \in [0;1]$:
$\min(p_1, p_2) \leq f_3(p_1, p_2) \leq \max(p_1, p_2)$.

The higher the objective-related pieces of information, the higher the resulting objective function value of the PCC shall be (et vice versa). This requirement corresponds to the human expectation that combining PCs with high liquidity results in a high liquidity of the PCCs (et vice versa). Therefore, monotony is required:

(RA2) Monotony: f_3 increases monotonically in both directions, i.e. it holds for each $u, p_1, v, p_2 \in [0; 1]$:
If $u \leq p_1$ and $v \leq p_2$, then $f_3(u, v) \leq f_3(p_1, p_2)$.

The liquidity of a PCC should not depend on the fact which objective-related information goes first into the computation of the objective function value. Consequently, the order of computing the liquidity-related information of the PCs shall have no impact on the result of the aggregated objective function value:

(RA3) Commutativity: f_3 is commutative, i.e. it holds for each $p_1, p_2 \in [0; 1]$:
$f_3(p_1, p_2) = f_3(p_2, p_1)$.

Small changes of the objective-related pieces of information to be aggregated shall also result in small changes of the objective function value of the PCC. Therefore continuity is required:

(RA4) Continuity: f_3 is continuous in both variables.

[29] One might also argue that – in analogy with the objectives return and risk – the liquidity function values of two PCs correlate with each other and that the corresponding covariances should be taken into account. However, (as with return and risk) this would require an additional objective function (e.g. liquidity variance). For reasons of simplicity and consistency with existing literature, we omitted this alternative, consider it however an interesting topic for further research.

If the objective-related information is equal for both PCs, the result of the objective function for the PCC is equal to the objective-related information. The authors consider it as a natural behaviour that aggregating two PCs with equal liquidity-related information results in the same liquidity-related function value for the PCC. Therefore, f_3 shall be idempotent:

(RA5) Idempotence: It holds for each $p \in [0; 1]$: $f_3(p, p) = p$.

In general, it depends on the particular decision situation whether the resulting function value for liquidity of the PCC should be determined by both PCs (AND-conjunction) or only by one PC (OR-conjunction). Consequently, different operators might be suitable. However, selecting an operator for each decision situation anew is not applicable in practice.[30] Therefore, several parametrisable operators exists, which use an input parameter (termed γ in the following) to adapt the behaviour of the operator to the particular decision situation. Empirical results illustrate that operators with a parameter represent human evaluations of alternatives much better than operators without a parameter.[31]

(RA6) Adaptivity: The objective function shall be parametrisable.

At this point, it shall be analysed which functions meet the requirements so far. This step leads to operators belonging to the class of Averaging Operators[32]. The results of Averaging Operators lie between the minimum and the maximum operator (RA1). Table 5 lists common Averaging Operators which meet the requirements (RA1) to (RA6).

So far, no single operator can be derived, as all three operators listed in Table 5 meet the requirements stated above. However, the requirements above only refer to aggregating two liquidity-related pieces of information which are *equally weighted*. In contrast, our problem requires the aggregation of more than two liquidity-related pieces of information. Moreover, concerning liquidity, these pieces of information differ in their relevance for the aggregated value as will be discussed next.

The influence a PC's liquidity has on the liquidity of a PCC depends proportionally on the share the PC has in the PCC. To put it another way: only that share a particular PC has in a PCC is as liquid as the PC. Gen-

[30] Cf. *Werners* (1984), p. 297.

[31] Cf. e.g., *Zimmermann/Zysno* (1980) or *Zimmermann/Zysno* (1983).

[32] Cf. *Zimmermann* (2001), p. 36.

Table 5

Common Averaging Operators[33]

Operator	Definition ($\gamma \in [0; 1]$)	Formula
Fuzzy-AND	$\gamma \min(p_1, p_2) + (1-\gamma)\frac{p_1 + p_2}{2}$	(1)
Fuzzy-OR	$\gamma \max(p_1, p_2) + (1-\gamma)\frac{p_1 + p_2}{2}$	(2)
min-max-Operator	$\gamma \min(p_1, p_2) + (1-\gamma)\max(p_1, p_2)$	(3)

erally speaking, the impact of the particular objective-related information of a PC on the objective function value of the PCC depends proportionally on the share of the PC within the PCC. That is why the objective function must allow for a weighting of the objective-related pieces of information.

(RA7) Proportionally increasing marginal impact of shares: The extent to which a PC l's objective-related information p_l influences the result of the sought objective function f_3 is determined by its share x_l.

Based on (RA7), the dimension of f is specified as:

f_3: $[0; 1]^{2^{r+1}} \rightarrow [0; 1]$.

In the given form, none of the Averaging Operators (Table 5) enables incorporating the shares of the PCs. Hence, (1) to (3) are adapted in such way that (RA1) to (RA7) are met. This can be done easily for (1) and (2), as the divisor 2 indicates that both pieces of information shall have equal impact on the result of the operator. Consequently, (1) and (2) are special cases for $x_1 = x_2 = 0.5$ of (1') and (2') in Table 6 below. In contrast, the min-max-Operator (3) does not contain a weighting of the pieces of information like Fuzzy-AND and Fuzzy-OR. One might argue that γ can be used as a weighting factor which represents the shares. This contradicts the original intention of the min-max-Operator, as γ shall express whether the minimum or the maximum shall have a greater impact on the result. Nevertheless, as it is basically possible to transform the min-max-Operator accordingly, it is listed along with the transformed Fuzzy-AND and Fuzzy-OR operator in Table 6.

[33] *Zimmermann* (2001), pp. 36–38.

Table 6

Averaging Operators Incorporating Shares

Operator	Definition ($\gamma \in [0; 1]$)	Formula
Fuzzy-AND	$\gamma \min(p_1, p_2) + (1-\gamma)(x_1 p_1 + x_2 p_2)$	(1')
Fuzzy-OR	$\gamma \max(p_1, p_2) + (1-\gamma)(x_1 p_1 + x_2 p_2)$	(2')
min-max-Operator	$\gamma \min(p_1, p_2) + (1-\gamma)\max(p_1, p_2)$ with $\gamma = \begin{cases} x_1 & if \quad p_1 \le p_2 \\ x_2 & if \quad p_1 > p_2 \end{cases}$	(3')

The operators listed in Table 6 meet all requirements (RA1) to (RA7), if $x_1 + x_2 = 1$. In the following, the min-max-Operator will be excluded for the following reasons: First, it can be shown that (3') is a special case of (1'). Second, the min-max-Operator is not extendable to more than two liquidity-related pieces of information; applying the minimum and the maximum operator would disregard these liquidity-related pieces of information which are neither maximum nor minimum, but have a positive share. In our context of an individualised asset allocation advisory, this problem usually consists of more than two PCs; hence, the aggregation of more than two liquidity-related pieces of information is crucial. One alternative to solve this problem is to require associativity from the operator:

(RA8) Associativity: f_5 is associative, i.e. it holds for each p_1, p_2, $p_3 \in [0; 1]$:
$f_3(p_1, f_3(p_2, p_3)) = f_3(f_3(p_1, p_2), p_3)$.

Associative Operators enable combining objective-related information to interim results; in combination with Commutativity, Associativity leads to Order Independence. Order Independence comes along with significant computational advantages: A function with r (for $r > 2$ PCs) input parameters can be replaced with applying a function with two input parameters $r - 1$ times.

However, counter examples demonstrate that none of the operators in Table 6 is associative, so it is not possible to simply include more parameters into these functions. Consequently, the operators Fuzzy-AND and Fuzzy-OR are adapted in the following in such way that they can be extended to more than two input parameters and meet the requirements (RA1) to (RA7).

Both operators consist of two summands: The first summand is an extremum operator (either minimum or maximum function), the second one is the weighted arithmetic mean. Both summands can be extended from 2 to r liquidity-related pieces of information:

- Extreme operators:
  - Minimum function:
    $r = 2$: $\min(p_1, p_2)$; $r \geq 2$: $\min(p_1, \ldots, p_r)$
  - Maximum function:
    $r = 2$: $\max(p_1, p_2)$; $r \geq 2$: $\max(p_1, \ldots, p_r)$
- Weighted arithmetic mean:
  $r = 2$: $x_1 p_1 + x_2 p_2$ $\quad r \geq 2$: $\sum_{l=1}^{r} x_l p_l$

Before combining these operators, it has to be assured that PCs with a share of 0 have no influence on the result (RA7). The weighted arithmetic mean includes the shares by default (and therefore PCs with a share of 0 have no impact). However, when computing the extreme operators, the signum function has to be applied to "sort out" the liquidity-related pieces of information of those PC which are not included in the PCC:

- Minimum function; $r \geq 2$: $\min\left({p_1}^{\operatorname{sgn}(x_1)}, \ldots, {p_r}^{\operatorname{sgn}(x_r)} \right)$
- Maximum function; $r \geq 2$: $\max\left(p_1 \operatorname{sgn}(x_1), \ldots, p_r \operatorname{sgn}(x_r) \right)$

The generalised Fuzzy-AND (4) and Fuzzy-OR (5) operators can be formalised as:

$$f_3(p_1,\ldots,p_r,x_1,\ldots,x_r,\gamma) = \gamma \min\left({p_1}^{\operatorname{sgn}(x_1)},\ldots,{p_r}^{\operatorname{sgn}(x_r)} \right) + (1-\gamma) \sum_{l=1}^{r} x_l p_l \tag{4}$$

$$f_3(p_1,\ldots,p_r,x_1,\ldots,x_r,\gamma) = \gamma \max\left(p_1 \operatorname{sgn}(x_1),\ldots,p_r \operatorname{sgn}(x_r) \right) + (1-\gamma) \sum_{l=1}^{r} x_l p_l \tag{5}$$

It can be formally shown that (4) and (5) also meet (RA1) to (RA7). Figure 2 illustrates that the generalised operators have the same properties as their counterparts for $r = 2$: The value range is limited by the extreme operator and the weighted arithmetic mean. The parameter γ determines which value in the resulting interval is the result of the operators.

The discussion so far reveals that the generalised operators Fuzzy-OR and Fuzzy-AND suit best with respect to the stated requirements. Yet, the questions remains, which of these two operators shall be used and how γ should be determined.

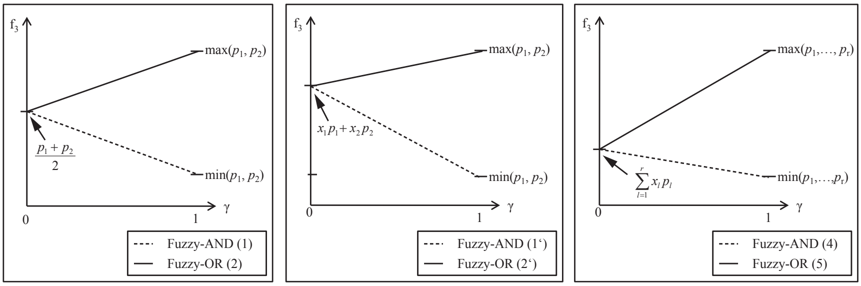

Figure 2: Value Ranges and Dependency on γ of the Different Fuzzy Operators

It seems plausible that the aggregation is more an AND-conjunction than an OR-conjunction, as an linguistic AND-conjunction implies that *all* information is represented by the result of the operator. Assuming $\gamma = 1$, (4) equals the min-operator and its result represents a threshold in terms of a function value of the PCC for liquidity which all liquidity-related information of the PCs exceed. In contrast, if (5) is chosen as objective function and we assume again $\gamma = 1$, the function value of the PCC for liquidity represents a threshold that only one liquidity-related information exceeds. The latter is not considered logical by the authors for the given application scenario. Hence, the Fuzzy-AND-operator (4) is proposed for the given application scenario.

Concerning the parameter γ, it is not possible to provide a general answer how it should be chosen. However, due to the reasoning above that the result of the aggregation shall be influenced by the objective-related pieces of information of all PCs (RA7) and not only of one, the authors consider it reasonable to be set as $\gamma = 0$: Thus, the compensatory effect is maximum and this special case of (4) is congruent with the existing approaches for aggregating liquidity as – for instance – proposed by *Amihud* (2002), p. 37.

b) Variability

Whereas liquidity addresses the possibility to access the full amount of invested money earlier than planned, another aspect of an investment is its variability: This aspect measures the extent to which a customer can change particular parameters of an investment during its duration, for instance the amount and the frequency of the saving payments (cf. Table 1). The relevance of liquidity and variability can differ significantly for a customer: Consider a customer whose income is constant and who plans to buy a house, but who has not found yet the right object and therefore wants to invest temporarily in a PCC. For this customer, liquidity of an investment is very important as he or she might need to turn his or her investment into cash for buying the house once he or she finds a suitable house. Varying the amount and the frequency of the investment's payments (and therefore its variability) is less relevant to the same customer, as he or she is able pay a constant amount at regular intervals.

Again, the value range of variability is defined as [0; 1]: 1 represents a perfectly variable PC, 0 a perfectly invariable one. Hence, a function f_4 has to be determined to assess the aggregated function value for the objective variability of a PCC.

Combining PCs has analogous effects on the variability as it has on liquidity: Again, the resulting objective function value of a PCC shall lie between the minimum and the maximum objective-related information of the PCs: The impact of a less variable PC shall be compensated by a more variable PC (et vice versa). That is why Mean value property is required. For the same reasons as argued earlier for liquidity, Monotony, Commutativity, Continuity, Idempotence and Adaptivity are required.

Moreover, the share of a PC in a PCC shall have proportional effect on the impact of the PC's variability on the variability of the PCC, too. As with liquidity, adding the requirement Associativity leaves no operator left. Consequently, the argumentation outlined earlier for using the Fuzzy-AND-operator as the objective function searched for and preferably setting the parameter $\delta = 0$ also hold in this case; it can be generalised in analogy with (4):

$$(6) \qquad f_4(p_1,\ldots,p_r,x_1,\ldots,x_r,\delta) = \delta \min\left(p_1^{\operatorname{sgn}(x_1)},\ldots,p_r^{\operatorname{sgn}(x_r)} \right) + (1-\delta)\sum_{l=1}^{r} x_l p_l$$

c) Comprehensiveness

The next objective, comprehensiveness, deals with the complexity of a PCC: Criteria like "An investment I know everything about" or "An investment that I can deal with myself, without having to rely on experts" (cf. Table 2) indicate that customers want to understand "what is behind" a particular investment. For instance, they want to know about the transactions between the different parties involved, especially about the payments and which parameters the height of the payments depends on (such factors vary significantly for the PC listed in Table 3). Acquiring such knowledge requires effort from the customer: first, during the decision making process on investing in a particular PCC, and then after a PCC has been chosen for investment: the realised payments may be higher or lower than the planned ones due to several impact factors. Hence, a customer wants to understand the reasons for the deviation, which causes effort during the duration of the investment. Naturally, a customer wants to minimise this effort.[34] Therefore, he or she will prefer PCs which are easy to comprehend to complex ones.

As with the objectives liquidity and variability, no established measures for comprehensiveness exist. Therefore, the comprehensiveness of a PC has to be determined based on experts estimations (cf. Table 3, "familiarity of the investor"). For reasons of consistency, the value range for the information of a PC related to the objective comprehensiveness is defined as [0; 1]. 1 stands for PCs being perfectly comprehensible, PCs evaluated with 0 are perfectly incomprehensible. Consequently, a function f_5 is needed which assesses a function value for the objective comprehensiveness to a PCC.

The comprehensiveness of a PC has no influence on the comprehensiveness of a combination, if and only if the share of this particular PC in the combination is 0. Once the share of the PC is greater than 0, its comprehensiveness impacts the comprehensiveness of the combination. However, this impact is equally high for all shares $0 < x_l \leq 1$. For instance, if only a small fraction of the amount is invested in a life insurance, the customer faces the full complexity of this PC – no matter how high the actual share of the life insurance in the PCC is. Therefore, the contribution of a PC to the comprehensiveness of a PCC shall be independent from the PC's share. Hence, in contrast to liquidity and variability, comprehensiveness is computed based only on the comprehensiveness of those PCs which have a positive share in the considered PC combination:

(RB1) Constant marginal impact of shares: The impact of a PC l's objective-related information p_l on the result of the sought objective function f_5 is determined by the decision whether its share x_l is positive or not.

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[34] One valid objection to this argument is the fact that in reality, many customers invest in product categories that are not easy to comprehend. This behavior can however be represented by giving a low weight to the objective comprehensiveness.

As required for variability and liquidity, combining two little comprehensible PCs shall result in for a low function value for comprehensiveness (et vice versa). Such behavior is common for aggregation functions and achieved by requiring monotony:

(RB2) Monotony: In analogy with (RA2)

Concerning comprehensiveness, the act of combining two PCs does not give any justification that the order of the input parameters has impact on the result of the objective function. Consequently, commutativity is required:

(RB3) Commutativity: In analogy with (RA3)

Also here, the ability of aggregating comprehensive-related information to interim results is useful. Hence, the objective function shall be associative:

(RB4) Associativity: In analogy with (RA8)

As mentioned earlier, if both, Commutativity (RB3) and Associativity (RB4) hold, Order Independence, can be deduced, i.e.: $f_5(f_5(p_1, p_2), p_3)) = f_5(f_5(p_1, p_3), p_2)$.

Next, the extreme values of the defined value range for p have to be dealt with: A PC with a comprehensiveness-related information value of 0 is assumed perfectly incomprehensible. Consequently, if such a PC forms part of a PCC, the objective function value of the latter concerning comprehensiveness shall also be minimum. Thereby, the objective-related information of all other PCs which are part of the combination is disregarded. Moreover, this requirement regulates the behaviour of the operator at the lower bound of the value range and also expresses the intuitive imagination of an AND-conjunction[35] of the objective-related information: The lower value of two PCs for comprehensiveness shall determine the result of the objective function.

(RB5) Neutral Element 0: It holds for each $p \in [0; 1]$:
$f_5(p, 0) = f_5(0, p) = 0$.

A comprehensiveness-related information value 1 of a PC represents an assumed maximum comprehensiveness. In this case, the comprehensive

[35] *Zimmermann/Zysno* (1980), p. 38 f.

Table 7

Common Non-Parametrisable t-Norms[36]

Operator	Definition	Formula
Minimum	$\min(p_1,p_2)$	(7)
Algebraic Product	p_1p_2	(8)
Einstein Product	$\frac{p_1p_2}{1+(1-p_1)(1-p_2)}$	(9)
Bounded Difference	$\max(0, p_1 + p_2 - 1)$	(10)
Drastic Product	p_1, if $p_2 = 1$ p_2, if $p_1 = 1$ 0, if $p_1, p_2 = 1$	(11)

ness of the resulting PCC shall be determined by the other PC. This requirement regulates the behaviour of the objective function at the upper bound of the value range. Again, the intuitive expectation of an AND-conjunction[37] of the comprehensiveness-related information is represented: PCs which are perfectly comprehensible shall have no influence on the aggregated comprehensiveness of a PCC. Consequently, 1 shall be Identity Element:

(RB6) Identity Element 1: It holds for each $p \in [0; 1]$:
$f_5(p, 1) = f_5(1, p) = p$.

Functions meeting the requirements (RB2) to (RB6) are called t-norms[38]. T-norms are the function class for conjunctions in multi-valued logic. Their properties and preconditions have been analysed in literature[39] which particularly discusses the following non-parametrisable (Table 7) and parametrisable (Table 8) t-norms:

According to the requirements stated so far, many t-norms can be applied (as can be seen from Table 7 and Table 8). However, several requirements have not yet been considered. First, as with liquidity and variability, it seems reasonable that small changes in the input parameters shall

[36] *Zimmermann* (2001), p. 31 f.

[37] *Zimmermann/Zysno* (1980), p. 38 f.

[38] *Schweizer/Sklar* (1983), p. 73 f.

[39] Cf. *Schweizer/Sklar* (1983), *Dubois/Prade* (1980), and *Marichal* (2000).

Table 8

Common Parametrisable t-Norms[40]

Operator	Definition	Value range of ε	Formula
Yager	$1-\min\left[1,\sqrt[\varepsilon]{(1-p_1)^{\varepsilon}+(1-p_2)^{\varepsilon}}\right]$	$\varepsilon > 0$	(12)
Schweizer	$\sqrt[\varepsilon]{\max(0,p_1{}^{\varepsilon}+p_2{}^{\varepsilon}-1)}$	$\varepsilon > 0$	(13)
Hamacher	$\frac{p_1 p_2}{\varepsilon+(1-\varepsilon)(p_1+p_2-p_1 p_2)}$	$\varepsilon \geq 0$	(14)
Frank	$\log_{\varepsilon}\left[1+\frac{(\varepsilon^{p_1}-1)(\varepsilon^{p_2}-1)}{\varepsilon-1}\right]$	$\varepsilon > 0,\ \varepsilon \neq 1$	(15)
Dombi	$1-\frac{1}{1+\sqrt[\varepsilon]{\left(\frac{1-p_1}{p_1}\right)^{\varepsilon}+\left(\frac{1-p_2}{p_2}\right)^{\varepsilon}}}$	$\varepsilon > 0$	(16)

have small impact on the resulting function value for the objective comprehensiveness:

(RB7) Continuity: In analogy with (RA5)

The next requirement addresses the effect of combining two PCs and constitutes an important difference to the objectives variability and liquidity: If a PC is comprehensible to a certain extent, combining it with another PC (how comprehensible the latter may be) will make the result less comprehensible, as the effort for understanding the combination is higher in any case. Hence, the resulting objective function value of a PCC shall always be lower than the comprehensiveness-related pieces of information of the PCs. Consequently, Archimedean behaviour is added to the requirements stated so far:

(RB8) Archimedean property:
f_5 is an Archimedean t-norm, i.e. f_5 is continuous and it holds for each $p_1 \in (0; 1)$:
If $0 < p_1 < 1$, then $f_5(p_1, p_1) < p_1$.

[40] *Zimmermann* (2001), p. 34.

Another important aspect concerning the advisory process is the ability to isolate and trace back the effects of adding or removing a PC to explain such effects to the customer. Therefore the so far required Monotony (RB2) is replaced by Strict Monotony in addition to the other properties of the t-norms. This requirement makes the objective function bijective:

(RB9) Strict Monotony: It holds for each p_1, p_2, p_3, p_4, $p_5 \in (0; 1]$: If $f_5(p_1, p_2) = f_5(p_1, p_3)$, then $p_2 = p_3$ and if $f_5(p_1, p_2) = f_5(p_3, p_2)$, then $p_1 = p_3$,
and
If $p_1 < p_4$ and $p_2 < p_5$ then $f_5(p_1, p_2) < f_5(p_4, p_5)$.

T-norms being both continuous and strictly monotone are called strict t-norms and each strict t-norm is Archimedean.[41] That is why (RB8) can be left aside in the following.

The requirements (RB1), (RB3) to (RB7) and (RB9) reduce the alternatives to one function: The parametrised Hamacher t-norm (cf. Table 8/Formula (14)) is the only function which meets all these requirements. Moreover, the Hamacher-operator is the only strict t-norm, which can be formulated as a rationale function.[42] This property provides advantages for the implementation: f_5 can be well-defined by stating a finite number of parameters and objective-related pieces of information respectively. Therefore, the Hamacher t-norm is proposed as aggregation function for the objective comprehensiveness.

The Hamacher t-norm includes a parameter ε which can be used to adjust the results to the particular decision situation: ε determines how strong a PC's deviation from the "ideal" comprehensiveness value 1 influences the comprehensiveness of the PCC.[43]

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41 *Schweizer/Sklar* (1983), p. 66.

42 *Mizumoto* (1989), p. 223.

43 The possibility of using the parameter ε is especially useful for a situation which is not directly addressed in this article, but which often occurs in practice: In the scenario, PCs are focused (as the ones listed in Table 2) which stand for a certain class of products; the imminent assumption is that these PCs differ significantly from each other in the pieces of information related to the different objectives and the aim is to combine two products of different categories. However, in reality, it might also be the aim to combine two products from the same category, e.g. two stock funds: In this case, their comprehensiveness-related information should be approximately equal (this consideration is similar to the one for (RA5) Idempotence). Hence, the resulting objective function value of the combination should also be closer to the objective-related pieces of information compared to

Formula (17) denotes the aggregation for two pieces of objective-related information. The signum function in combination with the Identity Element (RB6) is used to ensure that those PCCs with a share of 0 have no impact on the result. As discussed above, this function can (in contrast to the objective functions for liquidity and variability) and has to be recursively applied r–1 times, if r PCs are considered.

$$(17) \quad f_5\,(p_i,p_j,\varepsilon) = \frac{{p_i}^{\operatorname{sgn}(x_i)}{p_j}^{\operatorname{sgn}(x_j)}}{\varepsilon + (1-\varepsilon)({p_i}^{\operatorname{sgn}(x_i)} + {p_j}^{\operatorname{sgn}(x_j)} - {p_i}^{\operatorname{sgn}(x_i)}{p_j}^{\operatorname{sgn}(x_j)})} \exists i,j \in \{1,\ldots,r\}$$

d) Manageability

Manageability is the last objective for which an objective function is determined. It addresses criteria like "continuous information flow to the investor" and "involvement of/decisions by the investor" listed in Table 3. Manageability measures the effort required from a customer to deal with a PCC. This effort consists of two elements: First, the nonrecurring, initial effort when investing in the proposed PC. This initial effort consists – for instance – of the formality to open a bank/custodian account or activity during the process of searching for a real estate. After the decision, a PCC continuously causes effort for the customer as he or she has to control and eventually adjust the PCC during its duration. This effort includes e.g. receiving bank statements, reallocating, reinvesting of dividends, managing repairs of real estate etc. PCs hardly requiring a customer's effort are more manageable than PCs with frequent customer interaction. A customer wants to minimize this effort.

Again, let the PCs have a manageability-related information $p_6 \in [0; 1]$. 0 represents the maximum effort to manage a PC, 1 stands for the contrary. To be consistent, the value range of the function values of a PCC is also the interval [0; 1]. Consequently, the aim is to identify a function f_6 assessing a function value for the objective manageability to a PCC.

As the characteristics of a PCC concerning manageability are comparable to the ones above for comprehensiveness, the same properties shall

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combining two products from different categories. This can be achieved by choosing the parameter ε correspondingly: It can be shown that higher values for ε cause lower objective function values for $f_5(p_1, p_2)$ with $p_1, p_2 \in (0; 1)$. Hence, comprehensiveness of a PCC is evaluated the better, the lower ε is chosen. If $\varepsilon = 1$, the operator corresponds to the Algebraic Product (cf. Table 7/Formula (8)). Consequently, the aggregation behaviour can be adapted to the particular situation by adequately determining ε.

hold for manageability: First, the effort for managing a PC has to be taken from the first unit of money invested in that PC and can then be assumed as constant and independent from the actual invested amount ("fix effort"). For instance, the customer's effort for managing a life insurance is independent from the amount of money invested and therefore also independent from the share of the life insurance in the PCC chosen by the customer. That means, if the share x_l of a PC l is 0, its manageability has no influence on the manageability of a PCC. For $0 < x_l \leq 1$, the influence of PC l is constant. Consequently, a Constant marginal impact of shares is required (in analogy with (RB1)).

Next, as it is an intuitive property of aggregation functions, monotony shall also hold for the manageability function. In addition, associativity, and commutativity are imposed in analogy with comprehensiveness: The order in which the objective-related pieces of information go into the function shall have no impact on the results of the function and interims results are necessary. Besides, the following considerations hold:

A manageability-related information value 0 of a PC represents an assumed maximum managing effort. Consequently, if such a PC forms part of a PCC, the function value of the latter concerning manageability shall also be minimum (i.e. 0): It can't get less manageable than perfectly unmanageable. On the contrary, the influence of a perfectly unmanageable PC can't be compensated by a more manageable PC. Thereby, the objective-related information of all other PCs which might be part of the combination is disregarded and the behaviour of the operator at the lower bound of the value range is determined. This requirement expresses also the imagination of an AND-conjunction of the objective-related information: The lower value for manageability of two PCs shall determine the result of the function. Consequently, the objective function shall possess the Neutral Element 0.

The other end of the interval shall be treated as follows: If a PC l causes no managing effort at all, it is assigned the information value 1 (perfectly manageable). In this case, when combining l with another PCC m the manageability of the combination shall only be derived from the managing effort caused by m, as l does not affect the manageability of the combination. Hence, the objective function shall have the Identity Element 1.

Again, we require Continuity to ensure that small changes in input parameters have small impact on the results of the objective functions.

As illustrated, the impact of adding/removing a PC to/from a PCC on the manageability is necessary during an advisory session, hence, Strict Monotony is required. Finally, for computational reasons, f_6 shall be formulated as a rationale function. Based on these requirements, again the Hamacher t-norm (cf. Table 8/Formula (14)) is derived for f_6 as aggregation function (with ζ as a parameter for adjustments in analogy with ε for comprehensiveness).

$$(18)\quad f_6(p_i,p_j,\zeta) = \frac{{p_i}^{\mathrm{sgn}(x_i)}{p_j}^{\mathrm{sgn}(x_j)}}{\zeta + (1-\zeta)({p_i}^{\mathrm{sgn}(x_i)} + {p_j}^{\mathrm{sgn}(x_j)} - {p_i}^{\mathrm{sgn}(x_i)}{p_j}^{\mathrm{sgn}(x_j)})} \exists i,j \in \{1,\ldots,r\}$$

e) Interim Conclusion

So far, we axiomatically derived functions to evaluate PCCs with regard to the objectives liquidity, variability, comprehensiveness, and manageability. Together with established functions for the objectives return and risk, these objective functions can be used to evaluate combinations of product categories. These additional objective functions represent criteria which customers want a PCC to fulfil, but which could not be measured in an inter-subjectively verifiable manner so far. Consequently, these objective functions help to depict more of a customer's needs and are therefore a first contribution towards a more individualised advisory process in the financial services sector.

In our example, the shares of the PCs are assumed as $x_i \in \{0, 0.25, 0.5, 0.75, 1\}$. By using formulae (4), (6), (17), and (18), we derive the objective function values as listed in Table 9. For reasons of simplicity, we assume $\gamma = \delta = 0$ and $\varepsilon = \zeta = 1$.

However, evaluating the PCCs only is not sufficient within an individualised advisory process; a FSP should also support the customer in finding the "right" solution for him or her. This challenge is addressed in section V. Before, several other steps of the concept are outlined.

IV. Efficient PCCs, Normalisation, Weights

As can be seen in Figure 1, the final step in giving advice to a customer is to select a particular PCC suitable for him or her. Normally, there is not one PCC that is the optimum with respect to all objective functions (i.e., perfect). Therefore, a selection function is needed taking into account the customer's individual weights. Before this selection function is character-

Table 9

Example 1(ctd) – Product Categories Combinations and their Objective Function Values

PCC	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o
share PC_1	0.000	0.250	0.500	0.750	1.000	0.000	0.000	0.000	0.000	0.250	0.250	0.250	0.500	0.500	0.750
share PC_2	1.000	0.750	0.500	0.250	0.000	0.750	0.500	0.250	0.000	0.500	0.250	0.000	0.250	0.000	0.000
share PC_3	0.000	0.000	0.000	0.000	0.000	0.250	0.500	0.750	1.000	0.250	0.500	0.750	0.250	0.500	0.250
return	0.010	0.033	0.055	0.078	0.100	0.015	0.020	0.025	0.030	0.038	0.043	0.048	0.060	0.065	0.083
risk	0.010	0.043	0.092	0.141	0.190	0.023	0.048	0.074	0.100	0.032	0.041	0.060	0.080	0.076	0.129
liquidity	0.900	0.775	0.650	0.525	0.400	0.688	0.475	0.263	0.050	0.563	0.350	0.138	0.438	0.225	0.313
variability	0.900	0.713	0.525	0.338	0.150	0.688	0.475	0.263	0.050	0.500	0.288	0.075	0.313	0.100	0.125
compreh.	0.900	0.360	0.360	0.360	0.400	0.270	0.270	0.270	0.300	0.108	0.108	0.120	0.108	0.120	0.120
manageab.	0.900	0.270	0.270	0.270	0.300	0.045	0.045	0.045	0.050	0.014	0.014	0.015	0.014	0.015	0.015

ised by axiomatic considerations, the steps in the individualised advisory process are described and necessary assumptions are started.

One precondition to selecting a particular PCC is to sort out dominated PCCs, so that the selection function is applied to the objective function values of the efficient PCCs only (cf. Figure 1). *Branke et al.* (2008) state two reasons why to do this: First, the customer has to weigh the objectives against each other and this is often hard to do without knowing the particular alternatives – as dominated PCCs are not a rational choice, they might distort the customer. Second, even if the customer's weights are known, FSPs want to offer their customers several alternatives and not only one particular solution. As the problem of selecting efficient PCCs turns out to be NP-complete in our case, a heuristic procedure has to be applied to reduce the number of PCCs accordingly.

A5: There is a set E of – with respect to the $n = 6$ objective functions – s efficient PCCs PCC_i, $i \in \{1, 2, \ldots, s\}$.

$$E = \{PCC_1, PCC_2, \ldots, PCC_s\}$$

For each of the efficient PCCs part of E, the assumption (A4) still holds.

Sorting out the dominated PCCs means in our example that the PCCs g, h, and i (crossed out in Table 10) are not taken into account anymore, as they are dominated by PCC b.

Another preparing step has to be taken due to the different value domains of the objective functions: Whereas the four objective functions derived in this article have all the same value domain [0; 1], return and risk have different value domains. In order to eliminate undesired effects, the objective functions are normalised. For objectives to be maximised, the following formula is applicable:

$$\bar{f}_{zi} = \frac{f_{zi} - \min vd_z}{\max vd_z - \min vd_z}(\max_{NI} - \min_{NI}) + \min_{NI} \tag{19}$$

Hereby, max vd_z/min vd_z stand for the maximum/minimum observable function value within the value domain of objective z. The variables $\max_{NI}$ and $\min_{NI}$ can be used for further adaptions of the normalisation and represent the desired upper and lower end of the normalisation interval.

In case objectives are to be minimised, they can be normalised via the following formula:

$$\bar{f}_{zi} = \frac{\max vd_z - f_{zi}}{\max vd_z - \min vd_z}(\max_{NI} - \min_{NI}) + \min_{NI} \tag{20}$$

Table 10

Example 1 (ctd) – Efficient and Dominated Product Categories Combinations

PCC	a	b	c	d	e	f	~~g~~	~~h~~	~~i~~	j	k	l	m	n	o
share PCC_1	0.000	0.250	0.500	0.750	1.000	0.000	~~0.000~~	~~0.000~~	~~0.000~~	0.250	0.250	0.250	0.500	0.500	0.750
share PCC_2	1.000	0.750	0.500	0.250	0.000	0.750	~~0.500~~	~~0.250~~	~~0.000~~	0.500	0.250	0.000	0.250	0.000	0.000
share PCC_3	0.000	0.000	0.000	0.000	0.000	0.250	~~0.500~~	~~0.750~~	~~1.000~~	0.250	0.500	0.750	0.250	0.500	0.250
return	0.010	0.033	0.055	0.078	0.100	0.015	~~0.020~~	~~0.025~~	~~0.030~~	0.038	0.043	0.048	0.060	0.065	0.083
risk	0.010	0.043	0.092	0.141	0.190	0.023	~~0.048~~	~~0.074~~	~~0.100~~	0.032	0.041	0.060	0.080	0.076	0.129
liquidity	0.900	0.775	0.650	0.525	0.400	0.688	~~0.475~~	~~0.263~~	~~0.050~~	0.563	0.350	0.138	0.438	0.225	0.313
variability	0.900	0.713	0.525	0.338	0.150	0.688	~~0.475~~	~~0.263~~	~~0.050~~	0.500	0.288	0.075	0.313	0.100	0.125
compreh.	0.900	0.360	0.360	0.360	0.400	0.270	~~0.270~~	~~0.270~~	~~0.300~~	0.108	0.108	0.120	0.108	0.120	0.120
manageab.	0.900	0.270	0.270	0.270	0.300	0.045	~~0.045~~	~~0.045~~	~~0.050~~	0.014	0.014	0.015	0.014	0.015	0.015

It is noteworthy that by applying (20), the objectives are to be maximised afterwards: In contrast to before, now higher values are preferable to lower ones.

A6: The value domains of all $n = 6$ objective functions are normalised:

$\bar{f}_{zi} \in]0;1] \, \exists z \in \{1,...,6\}, \exists i \in \{1,...,s\}$

Higher values for $\bar{f}_{zi}$ are preferable to lower ones.

For the example, Table 11 depicts the objective function values of the efficient PCCs, normalised to the interval [0.1; 1].

So far, this article mainly dealt with the product side; as a result, we currently have a set of efficient PCCs, which are evaluated according to the objective functions derived in section III. For reasons of comparability, the objective function values of the PCCs are normalised. This is the set of the product side, which is the same for all customers.

At this point, we address the question of representing the customer in the advisory process. In order to select a PCC which suits the customer, the selection function needs to take the importance of the objective functions to the particular customer into account. This is due to the mentioned fact that normally there is not one PCC that is best regarding all objectives. Consequently, the objectives have to be weighed against each other.

The problem of determining the set of weights W is beyond of the scope of this article. As mentioned in section II, results of early research on measuring attitudes in the field of psychology can be applied.[44] As Figure 1 depicts, the weights are originally derived from information on the customer.[45] How such information can be used in an individualised advisory process from a technical point of view, is analysed in *Fridgen et al.* (2000).

A7: The advisory process considers one individual customer at each instance. For this customer, a vector W exists containing $n = 6$ individual weights w_z, $z \in \{1, 2, \ldots, n\}$, which represent the importance of the objective functions for the customer:

$W = \{w_1, w_2, \ldots, w_n\}; \sum_{z=1}^{n} w_z > 0$

The importance of one objective for the customer increases with the value of the weight.

[44] Cf. e.g., *Thurstone* (1931).

[45] Cf. e.g., *Buhl et al.* (2003).

Table 11

Example 1 (ctd) – Product Categories Combinations and their Normalised Objective Function Values

PCC	a	b	c	d	e	f	j	k	l	m	n	o
return	0.100	0.325	0.550	0.775	1.000	0.150	0.375	0.425	0.475	0.600	0.650	0.825
risk	1.000	0.837	0.592	0.346	0.100	0.935	0.888	0.847	0.750	0.649	0.668	0.405
liquidity	1.000	0.852	0.705	0.557	0.410	0.749	0.602	0.351	0.100	0.454	0.203	0.307
variability	1.000	0.795	0.591	0.386	0.182	0.768	0.564	0.332	0.100	0.359	0.127	0.155
compreh.	1.000	0.386	0.386	0.386	0.432	0.284	0.100	0.100	0.114	0.100	0.114	0.114
manageab.	1.000	0.360	0.360	0.360	0.391	0.132	0.100	0.100	0.102	0.100	0.102	0.102

Table 12

Example 1 (ctd) – Customers Weights for the Objectives

	return	risk	liquidity	variability	comprehensiveness	manageability
z	1	2	3	4	5	6
w_z (*C1*)	5	1	1	1	1	1
w_z (*C2*)	1	4	1	1	3	3
w_z (*C3*)	2	2	3	3	4	1

For the example, we assume the weights in Table 12 for three different customers (C1, C2, and C3) who shall be advised.

V. Selection Function

The assumptions (A5) to (A7) define the multi-objective decision situation in which the selection function is applied in a more detailed manner. Against this background, the relevant literature for this context is analysed in the next section.

1. Axiomatic Considerations in Portfolio Selection

For the literature analysis conducted in this section, two main streams within the area of portfolio selection are focused. The first one is multi-objective decision making in portfolio selection in general (as the PCCs in our context are a structurally the same as a portfolio). Portfolio selection has always been a "multi"-objective problem – after the initial publication by Markowitz[46] introduced risk as an additional objective (besides return).[47] The additional objective functions derived in this article increase this "multitude" and constitute a completely new multi-objective decision situation in portfolio selection. This leads to the second stream, namely axiomatic consideration in portfolio selection: As the decision situation has not been dealt with before, consequently no axiomatic consideration can exist for this area. Nevertheless, axiomatic considerations were examined in order to transfer existing knowledge to the given problem.

[46] *Markowitz* (1952).

[47] For a general overview on multi-objective decision making in portfolio selection, cf. *Zopounidis/Doumpos* (2002) or *Steuer/Na* (2003).

After Markowitz' initial publication, literature in portfolio selection mainly considered the objectives return and risk.[48] Many publications refer to the axiomatic foundation of the Bernoulli-Principle (Expected Utility Theory) by *von Neumann/Morgenstern* (1947). Several authors discuss the effects of leaving aside one of the axioms defined by Neumann and Morgenstern: For instance, the independence axiom is weakened[49] and left aside.[50] *Dubra et al.* (2004) analyse effects dismissing the completeness axiom.[51] *Bamberg/Spremann* (1981) list several examples for axiomatic characterisations of the Bernoulli-Principle in different research areas. An overview on applications of the foundation by *Neumann/Morgenstern* in insurance economics is given by *Schmidt* (1998). However, none of these references provide selection functions based on axioms using return and risk, which can be considered superior to the Bernoulli principle. Consequently, we will discuss later the effects of applying one of the results for Expected Utility Theory to our given scenario.

It is in the late 70's of the past century that the portfolio selection problem was extended to more objectives than return and risk. The bibliographies by *Zopounidis/Doumpos* (2002) as well as *Steuer/Na* (2003) sum up the relevant literature for multi-objective decision making in portfolio selection until their publication and contain no reference with axiomatic considerations. After this time there are mainly two types of articles considering multiple objectives in portfolio selection: The first type focuses on the problem of determining efficient portfolios.[52] The second type treats the portfolio selection itself which is, for instance, done via selection functions chosen arbitrarily,[53] problem-specific procedures[54], or basing on a desired risk level.[55] Based on this review, there exist no axiomatic considerations for a multi-objective portfolio selection problem so far.

48 Cf. e.g., *Ballestero/Romero* (1996) or *Ballestero* (1998).

49 Cf. *Dekel* (1986) or *Schmeidler* (1989).

50 Cf. *Machina* (1982).

51 Cf. *Dubra et al.* (2004).

52 Cf. e.g., *Steuer et al.* (2006) or *Branke et al.* (2008).

53 Cf. e.g., *Ehrgott et al.* (2004).

54 Cf. e.g., *von Polyashuk* (2005).

55 Cf. e.g., *Bana-e-Costa/Soares* (2004).

2. Design

As the section above reveals, there is still a gap concerning selection functions for multi-objective portfolio selection which are adequate for the given decision situation. Hence, this section formally notes desirable properties of the selection function.

The first property aims at avoiding unexpected results in order not to distort the customer: Given all objective functions being equal, there is no reason why the result of the selection function should differ. Hence, it seems quite intuitive that all objective function values being equal, the result of the selection function is equal to this function value[56] – disregard the set of customer weights W.

(RC1) Idempotence: For $\sum_{z=1}^{n} w_z > 0$ it holds:
$F(a, \ldots, a; w_1, \ldots, w_n) = a$.

Another property seems intuitive, too: When comparing two alternatives, the disadvantage of one alternative compared to another alternative with respect to one objective can be compensated by an advantage with respect to another objective, so that the utility[57] is the same for both alternatives.[58] The extent of the compensation is expressed by the values of the weights: The higher the weight, the harder it is to compensate a difference in the corresponding objective.

(RC2) Compensation: For $w_n \in [0;\infty)$ and $\sum_{z=1}^{n} w_z > 0$:

$$\sum_{z=1}^{n} w_z \left(u\left(\bar{f}_{zi} \right) \right) - \left(u\left(\bar{f}_{zj} \right) \right) = 0 \text{ for } (i, j)$$

$$\Leftrightarrow F\left(\bar{f}_{1i}, \ldots, \bar{f}_{ni}; w_1, \ldots, w_n \right) = F\left(\bar{f}_{1j}, \ldots, \bar{f}_{nj}; w_1, \ldots, w_n \right).$$

where $u\left(\bar{f}_{zi} \right)$ is assumed to be an increasing one-to-one function.

[56] Cf. (RA5).

[57] The term 'utility' denotes here and in the following the result of the selection function F(.).

[58] This property excludes decision principles like e.g., the lexicographical order, which mainly decide based on one objective. It also contradicts the principle of fixing other objectives at a certain level and maximising return which can be found in the literature (cf. e.g., *Bana-e-Costa/Soares* (2004)) The authors strongly argue for the compensation criterion, because the customer should be shown the trade-offs concerning the objectives. Nevertheless, approaches focusing on one objective can be incorporated into our model by choosing the customer's weights adequately.

Table 13

Example 1 (ctd) – Normalised Customer's Weights for the Objectives

	return	risk	liquidity	variability	compre-hensiveness	manageability
z	1	2	3	4	5	6
$\bar{w}_z$ (*C1*)	0.50	0.10	0.10	0.10	0.10	0.10
$\bar{w}_z$ (*C2*)	0.08	0.31	0.08	0.08	0.23	0.23
$\bar{w}_z$ (*C3*)	0.13	0.13	0.20	0.20	0.27	0.07

Theorem 1: For $F : \Re_+^n \times \Re_+^n \rightarrow \Re_+$ it holds: F satisfies (RC1) and (RC2) if and only if

(21) $$F\left(\bar{f}_{1i},\ldots,\bar{f}_{ni};w_1,\ldots,w_n\right) = u^{-1}\left(\sum_{z=1}^{n}\frac{w_z}{\sum_{z=1}^{n}w_z}u\left(\bar{f}_{zi}\right)\right).$$

A consequence of Theorem 1 is the normalisation of the weights. In the following we refer to the not normalised weights as w_z and to the normalised weights as $\bar{w}_z = w_z \big/ \sum_{z=1}^{n} w_z$.

Table 13 contains the normalised weights for our example.

By requiring properties (RC1) and (RC2), *Buhl* (1988) has characterised a class of functions that can be described in the form (21). Functions of this type can be written as (weighted) additive functions[59].[60] The assumption made in (RC2) on $u\left(\bar{f}_{zi}\right)$ being a strictly monotonically increasing function seems natural, as it requires – in accordance with utility theory – the following: For each pair of objectives and constant utility it holds that for increasing one objective function value we have to decrease the other objective function value. For example, consider the objectives return and risk. If we increase the objective function value for return, but want the overall utility to stay constant, it is common knowl-

[59] For (weighted) additive function, cf. *Keeney/Raiffa* (1976) or *Zeleny* (1982).

[60] By applying the results to the given application scenario, a particularity has to be considered: Whereas *Buhl* (1988) minimises formula (21) and thereby implicitly assumes that the objective functions to be aggregated are all also to be minimised, formula (21) is to be maximised in the given application scenario, because the utility of the *PCCi* is to be maximised. However, analogous to the proof in *Buhl* (1988), it can be shown that formula (21) leads also to reasonable results if the selection function and all objective functions are to be maximised.

edge that the objective function value for risk has to be reduced.[61] This implies a strictly monotonically decreasing slope of the indifference curve:

(RC3) Strictly monotonically decreasing indifference curves:
The indifference curves for each pair of objectives are strictly monotonically decreasing functions.

For all $z, k \in \{1, 2, n\}$ it holds:[62]

$$MRS\left(\bar{f}_{zi}, \bar{f}_{ki}\right) = -\frac{\partial F\left(\bar{f}_{zi}, \bar{f}_{ki}\right) / \partial \bar{f}_{zi}}{\partial F\left(\bar{f}_{zi}, \bar{f}_{ki}\right) / \partial \bar{f}_{ki}} < 0.$$

Theorem 2: For $F\left(\bar{f}_{1i}, ..., \bar{f}_{ni}; \bar{w}_1, ..., \bar{w}_n\right) = u^{-1}\left(\sum_{z=1}^{n} \bar{w}_z u\left(\bar{f}_{zi}\right)\right)$ characterised by properties (RC1) and (RC2) and $\bar{w}_z \in [0;1]$ it holds: F satisfies (RC3) if and only if $u\left(\bar{f}_{zi}\right)$ is a strictly monotonically increasing function:

$$u'\left(\bar{f}_{zi}\right) > 0. \tag{22}$$

Proof: see Appendix I.

Whereas (RC1) is a neutral requirement, (RC2) and (RC3) incorporate a particular judgement into the selection function. However, the judgement of both requirements is weak, as requiring a strictly monotonically increasing function $u'\left(\bar{f}_{zi}\right) > 0$ only says: "The more, the better" which is a well-agreed upon principle in utility theory.

As "the more, the better" is not a very strong requirement, the next property addresses the question "The more – how much better?" Looking first at one objective z separately, it seems intuitive that the utility gain from increasing the function value from $\bar{f}_{zi}^{\,before}$ to $\bar{f}_{zi}^{\,after}$ depends on the level of $\bar{f}_{zi}^{\,before}$. For example, an increase of the return by 5 % from 1 % to 6 % is usually valuated higher than an increase from 21 % to 26 % (representing an increase by 5 %, too). Literature terms such a rational a decreasing marginal utility. It means that an increase of the objective function by the same amount is valuated higher for lower starting function values than for higher ones.[63]

[61] Note that due to the normalisation, reducing the objective functions value for risk means a higher variance.

[62] Cf. *Varian* (1999) for a definition of strictly decreasing indifference curves.

[63] Note that there might be implicit connections between the objectives. For instance, it could be argued that a higher diversification (= lower risk) of a PCC in terms of a higher number of PCs it contains comes along with a lower compre-

What impact has the assumption of a decreasing marginal utility on the trade-off of two objectives? Assume the marginal utility for the two objectives manageability and comprehensiveness as decreasing and a customer faces an objective function value for manageability of 0.6 and comprehensiveness of 0.4. Furthermore, in order to get an additional 0.1 of comprehensiveness, he or she is willing to give in 0.2 of manageability so that his or her overall utility stays the same. If he or she has however a higher (e.g. 0.8)/lower (e.g. 0.4) objective function value for manageability, he or she is willing to give in more (e.g. 0.3)/less (e.g. 0.1) of manageability for the same increase of comprehensiveness (0.1) in order to remain on the same utility level. In utility theory, this exchange rate for a particular combination of objective function values is called marginal rate of substitution.

To the best of the authors' knowledge, no empirical investigations exist on the type of marginal rate of substitution for most of the objectives dealt with in this article. Only for the objectives return and risk, literature analyses the deciding behaviour of individual investors. Regarding the marginal utility of these two objectives, one might object that these results do not fully support a decreasing marginal utility: In fact, many publications discuss and report risk affinity[64], meaning that higher risk is sometimes preferred to lower risk as a higher risk comes also along with a higher return. Such customers do not only disregard the risk associated with the decision; they value a higher risk higher than a low one, because – if risk is measured by means of the variance, as suggested here – a higher risk comes along with a greater chance for a higher return. Consequently, for such customers, there is no trade-off between those two objectives.

It may well be the case, that customers act this way, however the question remains whether a FSP should advise its customers to do so. In the authors' opinion, a FSP should not implement such behaviour by default for the following reasons: First, as the results of Table 1 reveal, risk-avoiding behaviour seems very common. Second, laws and directives clearly emphasize the objective risk when regulating the advisory of customers. Hence, if the individual customer insists on a risk-seeking deci-

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hensiveness, a fact formula (17) depicts implicitly, as a higher number of PCs leads two a lower value for comprehensiveness. However, as a higher number of PCs can, but needs cause a lower risk value, this connection was not explicitly depicted.

64 Cf., e.g., *Crum et al.* (1981).

sion, it can be incorporated by choosing the weights adequately; in the given case, the weight for risk should be chosen as $\bar{w}_2 = 0$.

As a consequence, we require a decreasing marginal rate of substitution for each pair of objectives, a property which is also in accordance with standard utility theory. Therefore the indifference curves for each pair of objectives resulting from formula (21) have to satisfy (RC4):

(RC4) Strictly convex indifference curves:
The indifference curves for each pair of objectives are strictly convex functions. For all $z, k \in \{1, 2, \ldots, n\}$ it holds:[65]

$$\frac{\partial MRS\left(\bar{f}_{zi}, \bar{f}_{ki}\right)}{\partial \bar{f}_{zi}} > 0.$$

Theorem 3: For $F\left(\bar{f}_{1i}, \ldots, \bar{f}_{ni}; \bar{w}_1, \ldots, \bar{w}_n\right) = u^{-1}\left(\sum_{z=1}^{n} \bar{w}_z u\left(\bar{f}_{zi}\right)\right)$ characterised by properties (RC1) and (RC2) and $\bar{w}_z \in [0;1]$ it holds: F satisfies (RC4) if and only if $u\left(\bar{f}_{zi}\right)$ is a strictly monotonically increasing and strictly concave function:

$$u'\left(\bar{f}_{zi}\right) > 0 \wedge u''\left(\bar{f}_{zi}\right) < 0. \tag{23}$$

Proof: see Appendix II.

The last property we require aims at the elasticity of substitution of the considered selection function. The elasticity of substitution measures the extent of how easily a decrease of the function value of one objective can be compensated by an increase of the other objective's function value. It can be shown that functions of type (21) have a constant elasticity of substitution (see Appendix III for a proof). By requiring a particular value for the substitution elasticity we can restrain the possible functions for $u\left(\bar{f}_{zi}\right)$.

Again, no empirical findings on the elasticity of substitution between the objectives analysed in this article exist. A possible requirement for the indifference curve resulting from formula (21) for each pair of objectives might be the following: For each pair of objectives and constant utility it holds that by multiplying one objective function value with factor m, we have to multiply the other objective function value with factor $1/m$. Keeping in mind that we are dealing with relative, not absolute function values, we assume an exchange relationship as described above

[65] Cf. *Varian* (1999) for a definition of strictly convex indifference curves.

as intuitive. Therefore we require for each pair of objectives that – for the utility being constant – their function values behave to each other indirect proportionally. This restrains the allowed values for the elasticity of substitution from constant values to 1:[66]

(RC5) Elasticity of substitution = 1:
For all z, $k \in \{1, 2, \ldots, n\}$ it holds:

$$SE = \frac{\partial\left(\frac{\bar{f}_{ki}}{\bar{f}_{zi}}\right)}{\partial MRS\left(\bar{f}_{zi}, \bar{f}_{ki}\right)} \frac{MRS\left(\bar{f}_{zi}, \bar{f}_{ki}\right)}{\frac{\bar{f}_{ki}}{\bar{f}_{zi}}} = 1.$$

Theorem 4: For $F\left(\bar{f}_{1i}, \ldots, \bar{f}_{ni}; \bar{w}_1, \ldots, \bar{w}_n\right) = u^{-1}\left(\sum_{z=1}^{n} \bar{w}_z u\left(\bar{f}_{zi}\right)\right)$ characterised by properties (RC1) and (RC2) it holds: F satisfies (RC5) if and only if

$$u\left(\bar{f}_{zi}\right) = \ln \bar{f}_{zi}. \tag{24}$$

Proof: see Appendix IV.

So using formula (21) and (24) we derive

$$F\left(\bar{f}_{1i}, \ldots, \bar{f}_{ni}; \bar{w}_1, \ldots, \bar{w}_n\right) = \prod_{z=1}^{n} \bar{f}_{zi}^{\bar{w}_z}. \tag{25}$$

In case of a specific advisory situation – where a different substitution elasticity might be more adequate – we can nevertheless employ the results above: (RC1) through (RC4) remain reasonable properties of a selection function and, based on them, another property specifying the substitution elasticity can be defined.

By requiring properties (RC1), (RC2) and (RC5), a selection function that is appropriate in the given multi-objective decision situation of selecting a particular PCC was derived: the Cobb-Douglas function.

Table 14 illustrates the result of evaluating the PCCs via the designed Cobb-Douglas selection function, if the normalised weights of Table 13 are used.

In the given example, PCC *a* would be proposed to customers C2 and C3, as it has the highest selection function value. PCC *a* consists of PC 2 only, which has the lowest return, but the highest function values for all

[66] Cf. *Gravelle/Rees* (1981).

Table 14

Example 1 (ctd) – Product Categories Combinations and their Selection Function Values

PCC	a	b	c	d	e	f	j	k	l	m	n	o
return	0.100	0.325	0.550	0.775	1.000	0.150	0.375	0.425	0.475	0.600	0.650	0.825
risk	1.000	0.837	0.592	0.346	0.100	0.935	0.888	0.847	0.750	0.649	0.668	0.405
liquidity	1.000	0.852	0.705	0.557	0.410	0.749	0.602	0.351	0.100	0.454	0.203	0.307
variability	1.000	0.795	0.591	0.386	0.182	0.768	0.564	0.332	0.100	0.359	0.127	0.155
compreh.	1.000	0.386	0.386	0.386	0.432	0.284	0.100	0.100	0.114	0.100	0.114	0.114
manageab.	1.000	0.360	0.360	0.360	0.391	0.132	0.100	0.100	0.102	0.100	0.102	0.102
F(C1)	0.550	0.486	0.538	0.591	0.651	0.362	0.413	0.385	0.354	0.466	0.446	0.521
F(C2)	0.931	0.582	0.497	0.411	0.343	0.512	0.438	0.392	0.332	0.355	0.331	0.273
F(C3)	0.880	0.612	0.539	0.465	0.406	0.533	0.435	0.339	0.240	0.363	0.279	0.293

other objectives. Consequently, based on their weights, C2 and C3 weigh the low risk and the four "soft" objectives higher than the disadvantage of a low return. In contrast, customer C1 is mainly focused on a high return – hence, he is offered PCC *e*, which corresponds to PC 1. It can be seen that combinations of product categories have a disadvantage with regard to the objectives manageability and comprehensiveness compared to PCCs which consist only of one PC. This is due to the fact that combining one PC with another PC always results in the resulting combination being less comprehensible and less manageable than each of the two PCs alone.

a) Return and Risk – Separated or Integrated Consideration

So far, we have considered return and risk as unconnected objectives and have used separated objective function values as input factors for the selection function. A common approach in the existing economic literature, especially Expected Utility Theory, regarding these objectives is to integrate the two objective function values via a so-called preference function. In this section we want to examine whether a commonly used representative of these preference functions is appropriate in our application scenario. Assuming the return being a normal variable with μ as the expected return and σ^2 as the corresponding variance, a decision according to the following preference function is consistent with the Bernoulli principle:[67]

$$\Phi(\mu,\sigma) = \mu - \frac{a}{2}\sigma^2 \tag{26}$$

The variable a is called Arrow-Pratt risk aversion parameter and expresses the attitude of the decision maker towards risk ($a < 0$ means risk affinity, $a = 0$ risk neutrality and $a > 0$ risk aversion). In the preference function (26), absolute objective function values are used. To evaluate the appropriateness of (26), consider the following example: Tables 15 and 16 depict the absolute objective function values of two PCC_i for the objectives return and risk. While return is measured in percentage, the measure for risk is the variance.

The dark shadowed rows in the tables mark the PCC_i most decision makers preferred to the other alternative in the example considered.

[67] Cf. e.g., *Bamberg* (1986) or *Eeckhoudt et al.* (2005).

Table 15

Example 2 – Absolute Objective Function Values

	return [f_{1i} (in %)]	risk [f_{2i} (variance)]
PCC_1	6 %	1
PCC_2	7 %	10

Table 16

Example 3 – Absolute Objective Function Values

	return [f_{1i} (in %)]	risk [f_{2i} (variance)]
PCC_1	10 %	1,000
PCC_2	20 %	1,100

Clearly, in Table 16 the increase of the return was valued higher than the increase of the risk – in contrast to Table 15.

Theorem 5 applies the principle of formula (26) in our application scenario and provides interesting insights:

Theorem 5: For the preference function

(27) $$F(f_{1i}, f_{2i}, a) = f_{1i} - \frac{a}{2} f_{2i}$$

there exists no a, such that in Example 2 PCC_1 is valuated better than PCC_2, and in Example 3 PCC_2 is valuated better than PCC_1 for the not normalised values (Tables 15 and 16).

Outline of the proof: For the Examples 2 and 3 the "intuitively" preferred PCC shows a better preference function value if and only if

- in Example 2 (Table 15), we have $a > 10 / 45$ and
- in Example 3 (Table 16), we have $a < 9 / 45$.

It can be seen that it is impossible to find a value a, so that the "intuitively" preferred PCC is chosen. Thus, it is shown that in the given application scenario the use of the preference function (26) with a constant absolute risk aversion conception of risk is questionable. This problem still occurs when normalising the objective function values, which means that also a relative view does not change the results.[68] Besides this argument, using preference function (27) requires the determination of the Arrow-Pratt risk aversion parameter which is hardly comparable to the weights for the other objectives with regard to meaning and scale. As a consequence, it is proposed to consider the objectives return and risk separately and not via an integrating preference function for the given application scenario.

b) Preferential Independence

Functions of type (25) can be written as (weighted) additive functions.[69] Along with this kind of selection functions comes the requirement that the objective functions to be aggregated are mutually preferentially independent.[70] With the following examples we want to examine whether the objective functions in our application scenario meet this requirement.

In the following two examples, we consider one decision maker comparing each time two PCC_i, $i = 1, 2$ with respect to $n = 3$ objective functions (return, risk and liquidity) in two independent decision situations. The difference between the two situations is the objective function value for the liquidity of the PCC_i. All other values remain the same.

Tables 17 and 18 show the absolute objective function values for return, risk and liquidity. Return and risk are measured in the same way as in the former examples, while liquidity is measured in the amount of the investment that is repaid in the case of liquidation. There exists an underlying set of efficient PCC_i which determines the empirical value ranges listed in the tables.

In this example, without taking liquidity into account, the decision maker prefers PCC_1 to PCC_2, because the increase of risk (+15) seems too high compared to the increase of return (+6 %). If we include liquidity in the consideration, in situation 1 the decision maker still prefers PCC_1, while in situation 2 his or her choice might turn around (note that this does not necessarily hold for all decision makers): In situation 1, we have a reasonable value for the liquidity. So, in situation 1, the decision only

[68] Besides, the established Expected Utility Theory mostly addresses absolute and not relative values.

[69] Cf. e.g., *Keeney/Raiffa* (1976) or *Zeleny* (1982).

[70] Cf. e.g., *Keeney/Raiffa* (1976).

Table 17

Example 4 – Absolute Objective Function Values

	PCC_1				PCC_2		
	return $f_{11} \in$ [2 %;25 %]	risk $f_{21} \in$ [1;50]	liquidity $f_{31} \in$ [2,000;9,000]		return $f_{12} \in$ [2 %;25 %]	risk $f_{22} \in$ [1;50]	liquidity $f_{32} \in$ [2,000;9,000]
Situation 1	9 %	15	8,000	>	15 %	30	8,000
Situation 2	9 %	15	3,000	<	15 %	30	3,000

Table 18

Example 5 – Absolute Objective Function Values

	PCC_1				PCC_2		
	return $f_{11} \in$ [2 %;25 %]	risk $f_{21} \in$ [1;50]	liquidity $f_{31} \in$ [2,000;9,000]		return $f_{12} \in$ [2 %;25 %]	risk $f_{22} \in$ [1;50]	liquidity $f_{32} \in$ [2,000;9,000]
Situation 3	9 %	15	8,000	>	5 %	11	8,000
Situation 4	9 %	15	3,000	>	5 %	11	3,000

depends on the objective function values for return and risk leading to the decision for PCC_1. In contrast, in situation 2, the objective function value of liquidity is quite low. So the decision maker might now value the same risk/return position differently, because PCC_1 seems now less attractive as it offers only reasonable or bad objective function values while PCC_2 contains at least a quite good value for return. If so, the decision maker values the risk/return position depending on the objective function value for liquidity, i.e. the objective functions are preferentially dependent.

In Example 5, PCC_1 has the same objective function values as in Example 4, while PCC_2 has the same values for liquidity, but differing values for return and risk compared to Example 4: In this example – without considering liquidity – the decision maker chooses PCC_1, because 5 % is usually considered only as an average objective function value for return and for obtaining a return of 9 %, risk increases only slightly. Including liquidity in the decisions, in contrast to Example 4, in Example 5 the decision maker prefers PCC_1 in both situations: While in situation 2, coming from the same low level of liquidity, the increase in return was the reason for a switch of the preferences, the decrease of the objective function value for risk does not lead to such a switch in situation 4. So in these situations the objective functions may not be preferentially dependent.

It seems that we can't assure that the objective functions are considered as mutually preferentially independent by all decision makers. But apparently it depends on the values of the objective function values whether a preference dependency exists. So the question comes up whether additive functions are appropriate in our application scenario. *Zeleny* (1982) mentions a pragmatic reason why the application of an additive selection function is acceptable: "From a purely practical viewpoint, the additive [… selection] functions are both simple and robust approximations, and they are the only practical options for cases with more than four attributes."

Furthermore interesting in this context are the following examples. In contrast to above we now consider the empirically normalised objective function values for return, risk and liquidity. The normalisation interval is [0.1; 1].

In this example the decision maker prefers – independently of the value of the liquidity – PCC_1 to PCC_2. By changing to a relative view it becomes obvious that a decision maker who chooses PCC_2 instead of PCC_1 in situation 2 does not only remain with the relatively bad value for liquidity; in addition, PCC_2 comes along with a smaller utility: This is due to the fact that the difference between PCC_1 to PCC_2 for the empirically normalised objective function values for return is smaller than the difference between the empirically normalised objective function values for risk.

Also in this example a decision makers prefers – disregarding the value of the liquidity – PCC_1 to PCC_2, because the difference between PCC_1 to PCC_2 for the empirically normalised objective function values for return is larger than the difference between the empirically normalised objective function values for risk.

Tables 19 and 20 represent the same objective function values as Tables 17 and 18. The only difference is that we now consider empirically normalised (relative) in contrast to absolute values. So it seems that it depends on the normalisation whether the objective functions can be

Table 19

Example 4 (ctd) – Empirically Normalised Objective Function Values

	PCC_1				PCC_2		
	return $\bar{f}_{11}$	risk $\bar{f}_{21}$	liquidity $\bar{f}_{31}$		return $\bar{f}_{11}$	risk $\bar{f}_{21}$	liquidity $\bar{f}_{31}$
Situation 1	0.37	0.74	0.87	>	0.61	0.47	0.87
Situation 2	0.37	0.74	0.23	>	0.61	0.47	0.23

Table 20

Example 5 (ctd) – Empirically Normalised Objective Function Values

	PCC_1				PCC_2		
	return $\bar{f}_{11}$	risk $\bar{f}_{21}$	liquidity $\bar{f}_{31}$		return $\bar{f}_{11}$	risk $\bar{f}_{21}$	liquidity $\bar{f}_{31}$
Situation 3	0.37	0.74	0.87	>	0.22	0.82	0.87
Situation 4	0.37	0.74	0.23	>	0.22	0.82	0.23

considered as preferentially dependent or not by the decision makers: If we show to the decision maker empirically normalised objective function values, he or she can decide more “intuitively” about his or her preferences for the single objectives because he or she is not influenced by strongly different values of the objective function values. In Tables 19 and 20 the value of an empirically normalised objective function value for all objectives means the same, while in Tables 17 and 18 it is very difficult to compare the different values. This effect of normalisation is in accordance with *von Nitzsch/Weber* (1993).

Based on these considerations, it seems acceptable to assume for the given application scenario that mutually preferentially independent objective function values are given and that formula (25) is appropriate.

VI. Summary

The article provides solution approaches for two aspects of an individualised asset allocation advisory for retail customers: On the one hand, it extends the knowledge base by deriving functions which allow the evaluation of PCCs (portfolios) with respect to the objectives liquidity, variability, manageability, and comprehensiveness. To achieve this, empirical literature on customers' objectives concerning investments was analysed and the four objectives liquidity, variability, comprehensiveness, and manageability were derived as relevant (besides return and risk). In addition, procedures were presented, how PCs (assets) can be evaluated with respect to these objectives. Finally, for each objective one particular function was defined in order to evaluate a PCC with regard to the particular objective. The designed objective functions are listed in Table 21.

As scientific literature has not addressed these objectives in a formal way so far, these results help FSPs to increase the quality of their advisory services: By including these objectives, the solution can be better customized to the customers' needs.

The same advantage provides the other proposed artefact of the article, namely a selection function to choose one PCC out of a set of efficient PCC which fits the customers' needs best. The selection function was also derived via an axiomatic approach:

$$F\left(\bar{f}_{1i},...,\bar{f}_{ni};\bar{w}_1,...,\bar{w}_n\right)=\prod_{z=1}^{n}\bar{f}_{zi}^{\,\bar{w}_z} \tag{25}$$

It uses the normalised objective function values and the normalised weights (which represent the customer's preferences) as input parameters. Hence, by using the selection function, the efficient PCCs can be ranked according to the customer's needs. Also here, scientific literature has not yet provided a requirements-based approach to deal with this problem.

The results of the article at hand may support FSPs in two ways concerning the quality of their advisory services: First, they can now incorporate more objectives than return and risk into their advisory process. Second, as the derived functions are standardised, they can be used in software applications to support the advisory process which can then be offered even to retail customers. By offering an individualised advisory service also to retail customers, FSPs can regain some of the trust lost during the credit crunch.

Table 21

Objective Functions to Measure Liquidity, Variability, Comprehensiveness, and Manageability

Objective	Objective function	Formula
Liquidity	$f_3(p_1,\ldots,p_r,x_1,\ldots,x_r,\gamma) = \gamma \min\left(p_1^{\operatorname{sgn}(x_1)},\ldots,p_r^{\operatorname{sgn}(x_r)}\right) + (1-\gamma)\sum_{l=1}^{r} x_l p_l$	(4)
Variability	$f_4(p_1,\ldots,p_r,x_1,\ldots,x_r,\delta) = \delta \min\left(p_1^{\operatorname{sgn}(x_1)},\ldots,p_r^{\operatorname{sgn}(x_r)}\right) + (1-\delta)\sum_{l=1}^{r} x_l p_l$	(6)
Comprehensiveness	$f_5(p_i,p_j,\varepsilon) = \frac{p_i^{\operatorname{sgn}(x_i)} p_j^{\operatorname{sgn}(x_j)}}{\varepsilon + (1-\varepsilon)(p_i^{\operatorname{sgn}(x_i)} + p_j^{\operatorname{sgn}(x_j)} - p_i^{\operatorname{sgn}(x_i)} p_j^{\operatorname{sgn}(x_j)})} \exists i,j \in \{1,\ldots,r\}$	(17)
Manageability	$f_6(p_i,p_j,\zeta) = \frac{p_i^{\operatorname{sgn}(x_i)} p_j^{\operatorname{sgn}(x_j)}}{\zeta + (1-\zeta)(p_i^{\operatorname{sgn}(x_i)} + p_j^{\operatorname{sgn}(x_j)} - p_i^{\operatorname{sgn}(x_i)} p_j^{\operatorname{sgn}(x_j)})} \exists i,j \in \{1,\ldots,r\}$	(18)

The concept for an individualised advisory service including the results of this article has already been implemented in form of a prototype which was presented at several scientific and business conferences. In addition, parts of the concepts found their way into software applications used by financial services providers. Nevertheless, further validation of the concept might be necessary, for instance with regard to empirical data on a consistently defined set of objectives (evaluation of product categories with regard to the objectives) as well as on the determination of customer's weights.

Appendix I

Proof:

[⇒]. For

(I) $$MRS\left(\bar{f}_{zi},\bar{f}_{ki}\right)\overset{(21)}{=}-\frac{\bar{w}_z}{\bar{w}_k}\frac{u'\left(\bar{f}_{zi}\right)}{u'\left(\bar{f}_{ki}\right)}$$

it follows from property (RC3) that

(II) $$u'\left(\bar{f}_{zi}\right)>0.$$

(Note that (I) in combination with (RC3) leads in general to (II) or to $u'\left(\bar{f}_{zi}\right)<0$. This case is however excluded by property (RC2) assuming $u'\left(\bar{f}_{zi}\right)>0$.)

[⇐]. Because of formula (22) we have

(III) $$MRS\left(\bar{f}_{zi},\bar{f}_{ki}\right)\overset{(21)}{=}\underbrace{-\frac{\bar{w}_z}{\bar{w}_k}}_{<0}\frac{\overbrace{u'\left(\bar{f}_{zi}\right)}^{>0}}{\underbrace{u'\left(\bar{f}_{ki}\right)}_{>0}}<0.$$

This procedure can be adopted for each pair of objectives.

q.e. d.

Appendix II

Proof:

[⇒]. For

(IV) $$\frac{\partial MRS\left(\bar{f}_{zi},\bar{f}_{ki}\right)}{\partial\bar{f}_{zi}}\overset{(21)}{=}-\frac{\bar{w}_z}{\bar{w}_k}\frac{u''\left(\bar{f}_{zi}\right)}{u'\left(\bar{f}_{ki}\right)}$$

it follows from property (RC4) that

$$\text{(V)} \qquad u''\left(\bar{f}_{zi}\right) < 0 \wedge u'\left(\bar{f}_{ki}\right) > 0.$$

(Note that (IV) in combination with (RC4) leads in general to (V) or to $u''\left(\bar{f}_{zi}\right) > 0 \wedge u'\left(\bar{f}_{ki}\right) < 0$. This case is however excluded by property (RC2) assuming $u'\left(\bar{f}_{zi}\right) > 0$.)

[⇐]. Because of formula (23) we have

$$\text{(VI)} \qquad \frac{\partial MRS\left(\bar{f}_{zi}, \bar{f}_{ki}\right)}{\partial \bar{f}_{zi}} \overset{(21)}{=} -\underbrace{\frac{\bar{w}_z}{\bar{w}_k}}_{<0} \frac{\overbrace{u''\left(\bar{f}_{zi}\right)}^{<0}}{\underbrace{u'\left(\bar{f}_{ki}\right)}_{>0}} > 0.$$

This procedure can be adopted for each pair of objectives.

q.e. d.

Appendix III

Theorem 6: *For* $F(\bar{f}_{1i}, \ldots, \bar{f}_{ni}; \bar{w}_1, \ldots, \bar{w}_n) = u^{-1}\left(\sum_{z=1}^{n} \bar{w}_z u\left(\bar{f}_{zi}\right)\right)$ *characterised by properties (RC1) through (RC4) it holds: F has a constant elasticity of substitution if*

$$\text{(VII)} \qquad \begin{aligned} & SE \mid MRSE \mid \overset{!}{=} \mid IE \mid +1 \\ & \Leftrightarrow \partial\left(\frac{\bar{f}_{ki}}{\bar{f}_{zi}}\right) \frac{\bar{f}_{zi}}{\bar{f}_{ki}} \frac{MRS}{\partial MRS} \frac{\mid \bar{f}_{zi} \mid}{\mid \partial F(\bar{f}_{zi}, \bar{f}_{ki}) / \partial \bar{f}_{ki} \mid} \frac{\mid \partial MRS \mid}{\mid MRS \mid} \overset{!}{=} \mid -MRS \mid \frac{\mid \bar{f}_{zi} \mid}{\mid \bar{f}_{ki} \mid} + 1. \end{aligned}$$

(SE: elasticity of substitution, MRSE: elasticity of marginal rate of substitution, IE: elasticity of isoquants)

(Cf. *Gravelle/Rees* (1981); *Allen* (1960); *Hicks* (1968)).

Proof:

[⇒]. For

$$\text{(VIII)} \qquad \partial\left(\frac{\bar{f}_{ki}}{\bar{f}_{zi}}\right) = \frac{\bar{f}_{zi} \partial F\left(\bar{f}_{zi}, \bar{f}_{ki}\right) / \partial \bar{f}_{zi} - \bar{f}_{ki} \partial F\left(\bar{f}_{zi}, \bar{f}_{ki}\right) / \partial \bar{f}_{ki}}{\bar{f}_{zi}^2}$$

it follows from property (RC3) and (RC4)

$$\text{(IX)} \qquad \begin{aligned} & SE \mid MRSE \mid = \\ & = -\frac{\bar{f}_{zi} \partial F\left(\bar{f}_{zi}, \bar{f}_{ki}\right) / \partial \bar{f}_{zi} - \bar{f}_{ki} \partial F\left(\bar{f}_{zi}, \bar{f}_{ki}\right) / \partial \bar{f}_{ki}}{\bar{f}_{zi}^2} \frac{\bar{f}_{zi}}{\bar{f}_{ki}} \frac{\mid \bar{f}_{zi} \mid}{\mid \partial F\left(\bar{f}_{zi}, \bar{f}_{ki}\right) / \partial \bar{f}_{ki} \mid}. \end{aligned}$$

Because of the normalisation $\left(\bar{f}_{zi} > 0\right)$ we have

$$\text{(X)}\quad \begin{aligned} SE \mid MRSE \mid &= -\frac{\bar{f}_{zi}\partial F\left(\bar{f}_{zi},\bar{f}_{ki}\right) / \partial\bar{f}_{zi}}{\bar{f}_{ki} \mid \partial F\left(\bar{f}_{zi},\bar{f}_{ki}\right) / \partial\bar{f}_{ki} \mid} + \frac{\bar{f}_{ki}\partial F\left(\bar{f}_{zi},\bar{f}_{ki}\right) / \partial\bar{f}_{ki}}{\bar{f}_{ki} \mid \partial F\left(\bar{f}_{zi},\bar{f}_{ki}\right) / \partial\bar{f}_{ki} \mid} = \\ &= MRS * \frac{\bar{f}_{zi}}{\bar{f}_{ki}} + 1 = \mid IE \mid + 1. \end{aligned}$$

This procedure can be adopted for each pair of objectives.

q.e. d.

Appendix IV

Proof:

[⇒]. For

$$\text{(XI)}\qquad SE \overset{(21)}{=} \frac{\partial\left(\frac{\bar{f}_{ki}}{\bar{f}_{zi}}\right)}{\partial\left(-\frac{\bar{w}_z}{\bar{w}_k}\frac{u'\left(\bar{f}_{zi}\right)}{u'\left(\bar{f}_{ki}\right)}\right)} \cdot \frac{-\frac{\bar{w}_z}{\bar{w}_k}\frac{u'\left(\bar{f}_{zi}\right)}{u'\left(\bar{f}_{ki}\right)}}{\frac{\bar{f}_{ki}}{\bar{f}_{zi}}} = \frac{\partial\left(\frac{\bar{f}_{ki}}{\bar{f}_{zi}}\right)}{\partial\left(\frac{u'\left(\bar{f}_{zi}\right)}{u'\left(\bar{f}_{ki}\right)}\right)} \cdot \frac{\frac{u'\left(\bar{f}_{zi}\right)}{u'\left(\bar{f}_{ki}\right)}}{\frac{\bar{f}_{ki}}{\bar{f}_{zi}}},$$

because of (RC5) we have

$$\text{(XII)}\qquad u\left(\bar{f}_{zi}\right) = \ln\bar{f}_{zi}\,.$$

[⇐]. It follows from formula (24)

$$\text{(XIII)}\qquad SE \overset{(25)}{=} \frac{\partial\left(\frac{\bar{f}_{ki}}{\bar{f}_{zi}}\right)}{\partial\left(-\frac{\bar{w}_z}{\bar{w}_k}\frac{\bar{f}_{ki}}{\bar{f}_{zi}}\right)} \cdot \frac{-\frac{\bar{w}_z}{\bar{w}_k}\frac{\bar{f}_{ki}}{\bar{f}_{zi}}}{\frac{\bar{f}_{ki}}{\bar{f}_{zi}}} = \frac{\partial\left(\frac{\bar{f}_{ki}}{\bar{f}_{zi}}\right)}{-\frac{\bar{w}_z}{\bar{w}_k}\partial\left(\frac{\bar{f}_{ki}}{\bar{f}_{zi}}\right)} \cdot \frac{-\frac{\bar{w}_z}{\bar{w}_k}\frac{\bar{f}_{ki}}{\bar{f}_{zi}}}{\frac{\bar{f}_{ki}}{\bar{f}_{zi}}} = 1.$$

This procedure can be adopted for each pair of objectives.

q.e. d.

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