Kredit und Kapital, 39. Jahrgang, Heft 2, Seiten 211–232
Abhandlungen

Restricted Export Flexibility and Risk Management with Options and Forward Contracts

By Axel F. A. Adam-Müller, Lancaster, and
Kit Pong Wong, Hong Kong*

I. Introduction

Exchange rate risk management has become increasingly important as more and more firms of all sizes and of all industries source and sell abroad. In face of the continuing incidence of exchange rate fluctuations, international firms need to devote themselves to devising various risk management strategies so as to cope with their exchange rate risk exposure. On the one hand, they can adopt an operational hedge by following flexible sales or input/output policies in response to volatile exchange rates. On the other hand, they can opt for a financial hedge by trading various types of currency derivatives. The interaction between operational hedging and financial hedging is crucial to the understanding of the behavior of international firms under exchange rate uncertainty.

The extant literature on the competitive exporting firm under exchange rate uncertainty typically assumes that the firm is risk averse and makes its production, export, and hedging decisions simultaneously (see, e.g., Benninga, Eldor, and Zilcha (1985), Broll and Zilcha (1992), Adam-Müller (1997), and Wong (2003a)). Two notable results emanate. First, the "separation theorem" states that the firm's optimal production decision depends neither on its risk attitude nor on the incidence of the underlying exchange rate uncertainty when there is a currency forward market. Second, the "full-hedging theorem" states that the firm should completely eliminate its exchange rate risk exposure by adopting a full

* We would like to thank Jan Beran, Günter Franke, Christian Gollier, Roland Jeske, Henri Loubergé and two anonymous referees for helpful comments and suggestions. Any remaining errors are our responsibility. We gratefully acknowledge financial support from the Germany/Hong Kong Joint Research Scheme provided by the German Academic Exchange Service (DAAD) and the Hong Kong Research Grants Council (RGC).

hedge if the currency forward market is unbiased. A corollary of this theorem is that other hedging instruments such as currency options appear to be redundant (see, e.g., Battermann, Braulke, Broll, and Schimmelpfennig (2000)).

To depart from the extant literature by incorporating operational hedging, we follow Ware and Winter (1988) (see also Sercu (1992), Broll and Wahl (1997), and Wong (2001)) to model export flexibility.[1] Specifically, we consider a competitive exporting firm which makes its export decision (i.e., sales allocation between the domestic market and a foreign market) only after it has observed the realized spot exchange rate.[2] The export flexibility enjoyed by the firm is, however, deemed to be restricted in that the firm has to maintain certain minimum levels of domestic sales and exports. The idea is that firms typically have explicit or implicit obligations to remain present in a market even under (temporarily) unfavorable conditions. These obligations may either be due to already signed contracts with existing customers, or be simply due to the necessity to maintain a minimum level of activity in a market so as to remain visible to future customers.[3] This minimum level of activity is the result of a longer-term consideration in which market exit and entry costs determine whether a firm is currently in the market with at least the minimum level of activity, or whether the firm is not in the market at all. Instead of attempting to analyze these longer-term market entry and exit decisions of the exporting firm, we take the minimum levels of domestic sales and exports as given and proceed from there.[4] This restriction on the firm's export flexibility is exactly the point where our model differs from those of Broll and Wahl (1997) and Wong (2001).

---

1 *Eldor* and *Zilcha* (1987) and *Wong* (2003b) model export flexibility in a similar fashion for the globally competitive but domestically monopolistic exporting firm.

2 *Ben-Zvi* and *Helpman* (1992) argue that international transactions are better described by such a sequence of moves.

3 *Bagwell* and *Staiger* (1989) and *Bagwell* (1991) show that export subsidies facilitate the entry of high-quality firms under asymmetric information. *Shy* (2000) goes one step further and argues that the decision to export is chosen to signal product quality, despite the fact that exporting is dominated by non-exporting under symmetric information.

4 *Franke* (1991) and *Sercu* and *Vanhulle* (1992) focus on the effect of exchange rate uncertainty on exit and entry decisions of exporting firms, but their models do not analyze financial hedging. However, their analyses can be interpreted as capturing a firm's long-term decisions on market presence in the foreign market whereas this paper focuses on a shorter horizon where these decisions are taken as given.

Given the restricted export flexibility, the firm's optimal sales allocation rule is state contingent: It exports more than the minimum level to the foreign market when the realized spot exchange rate is sufficiently favorable such that the foreign price (measured in units of the domestic currency) exceeds the domestic price; otherwise, it maintains the minimum level of exports and sells the rest in the domestic market. This operational hedge through the *ex post* sales allocation rule transforms the *ex ante* uncertainty to feature a call option-like pattern such that the firm's exchange rate exposure is piecewise linear.

To examine how the restricted export-flexible firm's production decision is affected by the availability of hedging opportunities, we consider two different scenarios. First, we allow the firm to avail itself of customized derivatives contracts so as to achieve the first-best risk-sharing outcome. We show in this first-best hedging environment that the firm optimally tailors its customized derivatives contract in a way that its hedged domestic currency profits are stabilized at the expected level. We further show that such a hedge position can be replicated by using plain vanilla derivatives, namely currency forward contracts and currency call options with a single strike price, which is set equal to the ratio of the domestic price to the foreign price.

Since currency options may not exist in countries where derivative markets are just starting to develop, it is of interest to study the second-best environment wherein the firm is restricted to use currency forward contracts as the sole hedging instrument.[5] Currency forward contracts, because of their simple linear specification, are by and large readily available. We show in this second-best environment that the firm's optimal output is unambiguously below the first-best level. In other words, introducing currency options so as to complete the incomplete hedging environment enhances the firm's incentives to produce. There is thus an output enhancement effect of currency options in the context of export flexibility.

In an extension, we generalize the firm's exchange rate exposure from the piecewise linear shape as dictated by the optimal response to restricted export flexibility. In its generalized form, the firm's exchange rate exposure may have any shape. We discuss a number of factors driving the shape of the firm's exposure and show that the existence of a

[5] See *Lien* and *Tse* (2002) for a recent overview of the use of futures contracts for hedging purposes.

complete market is sufficient to derive the separation property for a firm whose exchange rate exposure is state dependent.

Our paper is not the first to analyze the joint use of currency forward contracts and currency options. Lapan, Moschini and Hanson (1991) and Frechette (2001) consider the case of a producer facing price risk and basis risk; Moschini and Lapan (1995) incorporate production risk as an additional source of risk. Chang and Wong (2003) analyze the joint use of options and forward contracts in a cross hedging problem whereas Wong (2003a) focuses on a problem under joint price and quantity risk. Breuer and Gürtler (2001) as well as Lien and Wong (2004) consider the optimal currency hedge of international bidders. All these models have at least two different sources of risk. The present paper shows that the joint use of forward contracts and options can be rationalized even if there is only one single source of risk.

The rest of the paper is organized as follows. Section II delineates a variant model of the competitive exporting firm under exchange rate uncertainty wherein the firm possesses restricted export flexibility. Section III characterizes the firm's optimal production and hedging decisions when complete hedging with customized derivatives contracts is accessible to the firm. Section IV examines the firm's optimal production and hedging decisions when currency forward contracts are the sole hedging instrument. Section V briefly addresses the firm's optimal decisions under a generalized type of exchange rate exposure. The final section concludes. All proofs are relegated to the Appendix.

II. The Model

Consider a one-period, two-date (0 and 1) model of the competitive exporting firm under exchange rate uncertainty.[6] The firm sells its single output in both the domestic country and a foreign country. Let P_d be the per-unit selling price in the domestic market, where P_d is denominated in the domestic currency. Likewise, let P_f be the per-unit selling price in the foreign market net of per-unit transaction costs of exporting, where P_f is denominated in the foreign currency. These two per-unit selling prices, P_d and P_f, are fixed and known to the firm *ex ante*. Due to the segmentation of the domestic and foreign markets, arbitrage transactions

[6] See *Broll* and *Eckwert* (2000) for a related multiperiod model.

are either impossible or unprofitable, thereby rendering the violation of the law of one price.[7]

At date 0, the firm has to commit to an output level, Q, according to a cost function, $C(Q)$, compounded to date 1, where $C(0) \geq 0$, $C'(Q) > 0$, and $C''(Q) > 0$. The then prevailing spot exchange rate at date 1, $\tilde{S}$, expressed in units of the domestic currency per unit of the foreign currency, is a non-negative random variable.[8] Let $G(S)$ be the firm's subjective cumulative distribution function of $\tilde{S}$, over support $[\underline{S}, \overline{S}]$, where $0 \leq \underline{S} < P_d/P_f < \overline{S} \leq \infty$.

The firm possesses export flexibility in that it makes its export decision after the exchange rate uncertainty is resolved. That is, the firm's sales allocation between the domestic and foreign markets is made contingent on the realized date 1 spot exchange rate S. The export flexibility enjoyed by the firm is, however, deemed to be restricted. Due to various explicit and implicit obligations, the firm has to maintain certain minimum levels of domestic sales and exports. These quantities are exogenously given and are denoted by Q_d for the domestic market and by Q_f for the foreign market.[9] The firm as such is flexible in allocating its sales only with respect to the amount of output exceeding the sum of these minimum levels of domestic sales and exports. In other words, the export flexibility prevails only for $Q - Q_d - Q_f > 0$.

The firm's optimal sales allocation rule under the restricted export flexibility is as follows. If $SP_f \geq P_d$, the firm sells the minimum amount, Q_d, in the domestic market and exports the rest, $Q - Q_d$, to the foreign country. On the other hand, if $SP_f < P_d$, the firm exports the minimum amount, Q_f, to the foreign country and sells the rest, $Q - Q_f$, in the domestic market. Thus, the firm's date 1 sales revenue denominated in the domestic currency is given by

$$R(S,Q) = \begin{cases} SP_fQ_f + P_d(Q - Q_f) & \text{if} \quad SP_f < P_d, \\ SP_f(Q - Q_d) + P_dQ_d & \text{if} \quad SP_f \geq P_d, \end{cases} \tag{1}$$

[7] The assumption of imperfect arbitrage among national markets is supported by a number of empirical studies of the law of one price such as *Engel* and *Rogers* (1996, 2001) and *Parsley* and *Wei* (1996).

[8] Throughout the paper, random variables have a tilde (˜) while their realizations do not.

[9] For simplicity, it is assumed that those realizations of $\tilde{S}$ where the firm will optimally exit either the domestic or the foreign market because of long-term considerations are not contained in the support of $\tilde{S}$. Without this assumption, it might be possible that the firm exits either market just in the period under consideration.

which can be compactly written as

$$R(\tilde{S}, Q) = \tilde{S}P_f Q_f + P_d(Q - Q_f) + P_f \max(\tilde{S} - P_d/P_f, 0)(Q - Q_d - Q_f). \tag{2}$$

Inspection of equation (2) reveals that the restricted flexibility in making the export decision after observing the realized date 1 spot exchange rate, S, bestows a valuable real (call) option on the firm. This real option is exercised whenever S is sufficiently high, i.e., $S \in (P_d/P_f, \overline{S}]$.

To examine the impact of complete and incomplete hedging on the firm's production decision, we consider two scenarios. In the next section, we allow the firm to use customized derivatives contracts to hedge against its exchange rate risk exposure. The payoff of a customized derivatives contract, net of its price compounded to date 1, is delineated by a function, $\Phi(S)$, whose functional form is to be chosen by the firm at date 0. The firm's date 1 profits denominated in the domestic currency are given by

$$\tilde{\Pi} = R(\tilde{S}, Q) - C(Q) + \Phi(\tilde{S}), \tag{3}$$

where $R(\tilde{S}, Q)$ is defined in equation (2). To serve the purposes of examining the hedging role and output effects of the customized derivative contract, it suffices to restrict our attention to the case where the derivative contract is fairly priced. That is, the functional form of $\Phi(S)$ is constrained by $\mathrm{E}[\Phi(\tilde{S})] = 0$, where $\mathrm{E}(\cdot)$ is the expectation operator with respect to $G(S)$.[10]

In Section IV, we restrict the firm to use currency forward contracts as the sole hedging instrument. Each of these contracts calls for delivery

---

[10] Our intention here is not to impose an ad hoc pricing theory on derivatives but to focus on the hedging role and output effects of the customized derivative contract. Thus, $G(S)$ is not the pricing kernel of the standard option pricing theory.

Relaxing the assumption of fairly priced securities implies the existence of a risk premium. It is well-known from *Arrow* ((1965), p. 39) that any risk averse decision maker will speculate on a risk premium. See also *Gollier* ((2001), Ch. 4). Hence, the firm will optimally enter into speculative derivatives positions on top of the hedging positions addressed in Propositions 1 to 3 below. Furthermore, these positions will generally be dependent. Should the firm face a situation with more than one separately tradable derivatives contract at least one of which exhibits a risk premium, the firm's problem resembles a portfolio problem that would have to be solved simultaneously with the production, export and hedging problem. See *Lapan, Moschini* and *Hanson* (1991). Since this paper focuses on the use of derivatives contracts for hedging purposes, the optimal derivatives positions resulting from both hedging and speculative motives are not analyzed.

of F units of the domestic currency per unit of the foreign currency. Thus, in the second-best (vis-à-vis the first-best) hedging environment, $\Phi(S) \equiv (F - S)H$, where H is the number of currency forward contracts sold (purchased if negative) by the firm at date 0. Since $\mathrm{E}[\Phi(\tilde{S})] = 0$, we have $F = \mathrm{E}(\tilde{S})$.

The firm possesses a von Neumann-Morgenstern utility function, $U(\Pi)$, defined over its date 1 domestic currency profits, Π, with $U'(\Pi) > 0$ and $U''(\Pi) < 0$, indicating the presence of risk aversion.[11] Hence, our model directly relates firms with badly diversified owners managing their firm. In addition, it might also apply to firms with well-diversified owners for the following reasons: Stulz (1984) argues that the risk aversion of managers can cause firms to behave in a risk averse manner because this might make managers better off without creating addition costs to shareholders. According to DeMarzo and Duffie (1995), hedging might also serve as a means for managers to alter their perception on the labor market. As pointed out by Smith and Stulz (1985), the convexity of corporate tax schemes may also serve as a rationale for a concave objective function at firm level.[12] A concave objective function also applies if a market-value maximizing firm faces convex cost of financial distress.[13]

Anticipating its optimal contingent export decision, the firm chooses its output level, Q, and the functional form of its exotic derivative contract, $\Phi(S)$, at date 0 so as to maximize the expected utility of its date 1 domestic currency profits:

$$\max_{Q,\Phi(S)} \mathrm{E}[U(\tilde{\Pi})] \quad \text{s.t.} \quad \mathrm{E}[\Phi(\tilde{S})] = 0, \tag{4}$$

where $\tilde{\Pi}$ is defined in equation (3).

11 Futures hedging under more general preferences which include loss aversion and disappointment aversion is analyzed by *Lien* (2001a, b) and *Lien* and *Wang* (2002).

12 However, this argument is not supported by the evidence found by *Graham* and *Rogers* (2002). See also *Graham* and *Smith* (1999). *Eldor* and *Zilcha* (2002) analyze the interaction between optimal export production and futures hedging, given a convex tax scheme.

13 See *Froot*, *Scharfstein* and *Stein* (1993). Empirical evidence for firms hedging in order to reduce expected costs of financial distress is provided by *Graham* and *Rogers* (2002).

III. Complete Hedging with Customized Derivatives Contracts

The first-order conditions for program (4) are given by[14]

$$\mathrm{E}\{U'[\Pi^*(\tilde{S})][P_d + P_f \max(\tilde{S} - P_d/P_f, 0) - C'(Q^*)]\} = 0, \tag{5}$$

$$U'[\Pi^*(S)] - \lambda^* = 0 \quad \forall\, S \in [\underline{S}, \overline{S}], \tag{6}$$

where $\Pi^*(S) = R(S, Q^*) - C(Q^*) + \Phi^*(S)$, λ is the Lagrange multiplier, and an asterisk (*) indicates an optimal level. The second-order conditions for the unique maximum, Q^* and $\Phi^*(S)$, are satisfied given risk aversion and the strict convexity of $C(Q)$.

Proposition 1. If the restricted export-flexible firm is allowed to use customized derivatives contracts for hedging purposes, the firm's optimal output, Q^*, solves

$$C'(Q^*) = P_d + P_f \,\mathrm{E}[\max(\tilde{S} - P_d/P_f, 0)], \tag{7}$$

and its optimal hedge position, $\Phi^*(S)$, satisfies that

$$\begin{aligned}\Phi^*(S) = {} & [\mathrm{E}(\tilde{S}) - S]P_f Q_f \\ & + P_f\{\mathrm{E}[\max(\tilde{S} - P_d/P_f, 0)] - \max(S - P_d/P_f, 0)]\}(Q^* - Q_d - Q_f).\end{aligned} \tag{8}$$

The intuition of Proposition 1 is as follows. The availability of customized derivatives contracts offers actuarially fair "insurance" to the firm in the sense that trading these contracts does not affect the expected value of its domestic currency profits. However, reducing the variability of these profits increases expected utility. Therefore, the firm tailors its hedge position, $\Phi(S)$, in a way that its date 1 domestic currency profits, $R(S, Q) - C(Q) + \Phi(S)$, are stabilized at the expected level for all $S \in [\underline{S}, \overline{S}]$. The optimal output, Q^*, is then chosen to maximize $\mathrm{E}[R(\tilde{S}, Q)] - C(Q)$, thereby yielding equation (7). This equation also indicates that the existence of customized derivatives contracts allowing for the complete elimination of exchange rate risk is sufficient to derive the separation theorem for the restricted export-flexible firm since optimal

[14] We implicitly assume that the firm possesses some degree of export flexibility at the optimum, i.e., $Q^* > Q_d + Q_f$. This avoids the lengthy discussion of the Kuhn-Tucker conditions and corner solutions, which is not the focus of this paper.

output neither depends on the degree of risk aversion nor on the incidence of the exchange rate risk. It is important to notice that this separation result holds independently of whether the customized derivatives contracts are perceived as fairly priced or not.[15]

Proposition 2. The restricted export-flexible firm's optimal customized derivatives position, $\Phi^*(S)$, can be replicated by selling $P_f Q_f$ currency forward contracts and writing $P_f(Q^* - Q_d - Q_f)$ currency call options with the strike price set equal to P_d/P_f.

Proposition 2 describes how we can replicate the optimal customized derivatives contract characterized in equation (8) by trading forward contracts and plain vanilla call options.[16] Specifically, the short position of $P_f Q_f$ currency forward contracts is aimed at the exchange rate risk exposure associated with the minimum level of exports, Q_f. The option position of writing $P_f(Q^* - Q_d - Q_f)$ currency call options with the strike price P_d/P_f, on the other hand, is used to hedge against the conditional exchange rate risk exposure created by the restricted export flexibility: The existence of additional foreign currency revenue of $P_f(Q^* - Q_d - Q_f)$ is conditional on the date 1 spot exchange rate exceeding P_d/P_f. For all $S < P_d/P_f$, the call options are not exercised and the forward position provides a full hedge for the foreign currency revenue of $P_f Q_f$. For all $S \geq P_d/P_f$, the call options are exercised and the combined forward and call option position offers a full hedge for the foreign currency revenue of $P_f(Q^* - Q_d)$. The firm's date 1 domestic currency profits as such become invariant to the exchange rate.

If there is no export flexibility (i.e., $Q_d = 0$ and $Q_f = Q$), it is evident from equation (2) that the firm's date 1 domestic currency revenue is linear in S. In this case, currency forward contracts are the preferred hedging instrument since they are also linear in S (see Battermann, Braulke, Broll, and Schimmelpfennig, 2000). On the other hand, if the firm is fully export-flexible (i.e., $Q_d = Q_f = 0$), it is evident from equation (2) that the firm's date 1 domestic currency revenue is piecewise linear in S with a zero slope for all $S \leq P_d/P_f$. Broll and Wahl (1997) show that writing currency call options with the strike price at P_d/P_f

[15] A sketch substantiating this statement can be found in the Appendix, below the proof of Proposition 2.

[16] *Sercu* (1992) also describes how an export-flexible firm's profits can be hedged with options. However, he does not analyze the optimal hedging position since his analysis is based on the Modigliani-Miller assumptions.

eliminates all exchange rate risk, thereby making currency forward contracts redundant.

The restricted export flexibility allows the firm to implicitly hedge against its exchange rate risk exposure by the *ex post* sales allocation between the domestic and foreign markets. Specifically, for all $S < P_d/P_f$, the firm optimally allocates less, but still some, output to the foreign market and more output to the domestic market. This implicit real hedge has two effects on the firm's unhedged domestic currency profits at date 1. First, the unhedged profits are less volatile. Second, they become convex in S with strictly positive slope everywhere. The convexity calls for the use of currency options, similar to the case of a fully export-flexible firm. Due to the minimum export level, the slope is positive even at low realizations of $\tilde{S}$. This requires the use of currency forward contracts in addition to currency options, contrary to the case of a fully export-flexible firm.

Loosely speaking, imposing some export flexibility onto the competitive exporting firm results in the optimality of using currency options for hedging purposes. Alternatively, restricting full export flexibility to some extent offers a hedging role for currency forward contracts. Hence, the restricted export-flexible firm has three appealing features: First, it seems to be more realistic than a fully export-flexible or an entirely export-inflexible firm. Second, it optimally uses a portfolio of currency forward contracts *and* currency options for hedging purposes, which is consistent with the observed risk management behavior (see, e. g., Bodnar, Hayt, and Marston, 1998, and Bodnar and Gebhardt, 1999). Third, it shows how to rationalize the joint use of forward contracts and options without the need to introduce a second source of risk into the model.

IV. Incomplete Hedging with Forward Contracts Only

In this section, we restrict the firm to use currency forward contracts as the sole hedging instrument such that $\Phi(S) \equiv (F - S)H$. We are particularly interested in examining the effects of such an incomplete hedging environment on the firm's optimal production decision. But before, we look at the optimal hedging position.

Proposition 3. If the restricted export-flexible firm is allowed to trade currency forward contracts only, its optimal forward position, H^{**}, satisfies that $P_f Q_f < H^{**} < P_f(Q^{**} - Q_d)$.

The intuition underlying Proposition 3 is as follows. Inspection of equation (2) reveals that the firm's unhedged domestic currency revenue at date 1 is piecewise linear and convex in S. For high realizations of $\tilde{S}$ (i.e., $S \geq P_d/P_f$), the optimal sales allocation rule is to export as much as possible, which generates the foreign currency revenue of $P_f(Q - Q_d)$. The firm could completely eliminate this exchange rate risk exposure by setting $H = P_f(Q - Q_d)$. For low realizations of $\tilde{S}$ (i.e., $S < P_d/P_f$), the foreign currency revenue only amounts to $P_f Q_f$. In this case, complete elimination of the exchange rate risk calls for setting $H = P_f Q_f$. It is thus evident that there is a conflict between hedging against the exchange rate risk for high and for low realizations of $\tilde{S}$. Proposition 3 states that the firm optimally opts for a compromise between these two forward positions.

The next result focuses on the impact of incomplete hedging on optimal production.

Proposition 4. The restricted export-flexible firm's output is lower when it is allowed to trade currency forward contracts as the sole hedging instrument than when it is also allowed to trade currency call options with the strike price P_d/P_f. That is, $Q^{**} < Q^*$.

The intuition behind Proposition 4 is as follows. Using equation (4), the firm's marginal domestic currency revenue at date 1 is given by $P_d + P_f \max(\tilde{S} - P_d/P_f, 0)$. If the firm can trade currency call options with the strike price equal to P_d/P_f, this uncertain marginal revenue can be stabilized at the expected level. Rewrite $P_d + P_f \max(\tilde{S} - P_d/P_f, 0) = \tilde{S}P_f + P_f \max(P_d/P_f - \tilde{S}, 0)$. If the firm can only trade currency forward contracts, the first risk component, $\tilde{S}P_f$, can be completely eliminated but not the second risk component, $P_f \max(P_d/P_f - \tilde{S}, 0)$, which is nonlinear. Comparing these two scenarios with the same expected marginal revenue, Proposition 4 states that the firm, given risk aversion, optimally produces less under uncertain marginal revenue than under certain marginal revenue. Thus, Proposition 4 is in line with other results showing the output enhancing effect of completing a previously incomplete market for currency derivatives, see Benninga, Eldor and Zilcha (1985) and Broll, Wahl and Zilcha (1995).

V. An Extension: State Dependent Exposure

As equation (2) shows, the firm's revenues in domestic currency and, hence, its exchange rate exposure is piecewise linear in the exchange rate due to restricted export flexibility. Proposition 2 shows that this exposure can be perfectly hedged by a portfolio of currency forward contracts and currency options that exactly matches the shape of the firm's exposure. Proposition 1 indicates that the availability of a perfect hedge leads to the separation theorem, determining the firm's optimal output. If these currency derivatives contracts are fairly priced, risk aversion ensures that a perfect hedge is also optimal as shown in Proposition 1. The purpose of this section is to briefly analyze the firm's optimal decisions under a more general type of exchange rate exposure and to discuss why such types of exposure may arise.

The motivation for the piecewise linear exposure was based on the firm's long-term optimization dictating at least a minimum level of activity in both the domestic as well as the foreign market. In a more general setting, the firm's export strategy will result from the strategic interaction with its current and potential future competitors. The shape of the firm's exchange rate exposure depends on the structure of the entire industry such that a large variety of exposures may arise.

As an example, consider a scenario where a number of export-flexible firms from the same country compete in an imperfect product market in another country. Unless the firms are identical, the critical exchange rate at which a particular firm starts exporting will be different across firms. In addition, the foreign sales price is likely to reflect the degree of competition and will, hence, be dependent on the exchange rate as well. This should lead to a more complex shape of the firm's exposure. One possibility is an exposure where there is a "kink" at each critical exchange rate, defined by either the firm or one of its competitors entering the foreign market, such that the exposure is piecewise linear with many different slopes. Intra-industry trade from the foreign country to the domestic market further complicates the strategic interaction and, thereby, is most likely to have an impact of the firm's exchange rate exposure as well. In sum, each industry equilibrium will dictate an optimal export strategy for the firm that, in turn, will give rise to a particular shape of the firm's exchange rate exposure.

Rather than motivating a particular shape of the firm's exposure, the remainder of this section focuses on a generalization from the piecewise

linear shape of the exposure to an exposure that is entirely state-dependent, i.e. that can basically have any shape. Consider a finite and discrete state space where S_i is the realization of the exchange rate $\tilde{S}$ in state i with $i, j = 1, \ldots I$. The market is complete if there are state contingent claims for all states i.[17] If $e_i(\tilde{S})$ denotes the payoff of a state contingent claim for state i, then $e_i(S_j) = 1$ if $i = j$ and $e_i(S_j) = 0$ otherwise. f_i is the exogenously given price of a claim relating to state i, compounded to date 1. Let $\hat{R}(Q, \tilde{S})$ denote the firm's state-dependent revenues, denominated in the domestic currency. $\hat{R}(Q, \tilde{S})$ captures the firm's exposure to the uncertain exchange rate $\tilde{S}$. It is important to notice that no further assumptions on the shape of $\hat{R}(Q, \tilde{S})$ have to be imposed.[18] Hence, $\hat{R}(Q, \tilde{S})$ is capable of representing state dependent exchange rate exposure.

Furthermore, let ϕ_i denote the number of state contingent claims on state i that the firm purchases, $\phi_i > 0$, or sells, $\phi_i < 0$. Then, the firm's profits are given by $\hat{\Pi}(\tilde{S}) = \hat{R}(Q, \tilde{S}) - C(Q) + \sum_{i=1}^{I} \phi_i (e_i(\tilde{S}) - f_i)$. The firm's problem is to maximize its expected utility by optimally choosing its output and its portfolio of I different state contingent claims ϕ_i. The following result shows that the existence of a complete and competitive market for state contingent claims is sufficient to derive the separation theorem if the firm's hedging activities do not affect its competitors' activities.[19]

Proposition 5. If there is a complete and competitive market for state contingent claims and if the revenue function $\hat{R}(Q, \tilde{S})$ captures the firm's exchange rate exposure, then the firm's optimal output, Q^*, is determined by

$$C'(Q^*) = \sum_{i=1}^{I} f_i \frac{\partial \hat{R}(Q^*, S_i)}{\partial Q}. \tag{9}$$

[17] Another sufficient condition for the market to be complete is that every state contingent claim can be generated synthetically by some trading strategy in existing financial claims. Given a discrete state space, I different currency options with appropriately chosen strike prices or, alternatively, $I - 1$ such currency options and a currency forward contract will constitute a complete market.

[18] The revenue function $R(Q, \tilde{S})$ as derived in Section II is piecewise linear in S.

[19] So far, only very few attempts have been made to capture the effect of corporate hedging on product market equilibria in a rigorous model. See *Adam, Dasgupta* and *Titman* (2006) and *Mello* and *Ruckes* (2005).

The firm's optimal output equates marginal cost and the sum of all state-dependent marginal revenues, weighted with the price of the respective state-contingent claim. Neither the degree of risk aversion nor the incidence of the exchange rate risk affect the firm's optimal output. Proposition 5 shows that the existence of a complete market is sufficient to derive the separation theorem for an arbitrary dependence of the firm's revenue on the exchange rate. Like Proposition 1, it provides an example of the result that the existence of a set of hedging instruments that exactly matches the firm's exposure is sufficient for the separation theorem to hold. For the piecewise linear exposure outlined in Section II, currency forward contracts and call options with a particular strike price are sufficient as Propositions 1 and 2 show. For the arbitrary shape of the firm's exposure as given by $\hat{R}(Q, \tilde{S})$, a complete market for state contingent claims on $\tilde{S}$ is sufficient for the separation theorem.

Proposition 5 does not require the state contingent claims to be fairly priced. However, if they are, the risk averse firm will eliminate exchange rate risk from its profits completely by choosing a full hedge. To see why, notice that the first-order conditions for ϕ_i^* reduce to $-\text{Cov}(e_i(\tilde{S}), U'(\hat{\Pi}^*(\tilde{S}))) = 0$ since $f_i = \text{E}\big[e_i(\tilde{S})\big]$.[20] In order for these conditions to hold for all I state contingent claims, $\hat{\Pi}^*$ must be independent of $\tilde{S}$ in order to ensure that $U'(\hat{\Pi}^*(\tilde{S}))$ is a constant. This requires full hedging.

VI. Conclusions

Exchange rate risk management and its interaction with real operations play a significant role for an international firm's success. This paper has examined the optimal production and hedging decisions of the risk averse competitive exporting firm under exchange rate uncertainty. The firm possesses export flexibility in that it makes its export decision after the resolution of the exchange rate uncertainty. The export flexibility enjoyed by the firm is, however, limited by certain minimum sales requirements due to long-term considerations. As such, the firm's sales allocation between the domestic market and a foreign market is made state contingent on the realized spot exchange rate. The firm exports more than the minimum level to the foreign market when the realized spot exchange rate is sufficiently favorable; otherwise, it maintains the minimum level of exports and sells the rest in the domestic market. This

[20] Cov $(\cdot, \cdot)$ denotes the covariance operator with respect to $G(S)$.

operational hedge through the *ex post* sales allocation rule results in a call option-like feature in the *ex ante* uncertainty and, hence, in a piecewise linear exchange rate exposure.

We have considered two different hedging environments. In the first-best environment, the firm is allowed to avail itself of customized derivatives contracts for hedging purposes. We have shown that the firm optimally tailors its customized derivatives contract so as to stabilize its domestic currency profits at the expected level. Furthermore, we have shown that such a customized hedge position can be replicated by trading plain vanilla derivatives of currency forward contracts and currency call options with a single strike price, which is set equal to the ratio of the domestic price to the foreign price. In the second-best environment wherein the firm is restricted to use currency forward contracts as the sole hedging instrument, we have shown that the firm's optimal output is unambiguously below the first-best level. In other words, introducing currency options so as to complete the incomplete hedging environment enhances the firm's incentives to produce. Thus, we have established the output enhancement effect of currency options in the context of export flexibility.

In an extension, we have discussed why the firm's exchange rate exposure might have a shape other than the piecewise linear. The exporting firm's interaction with its competitors plays a particularly important role here. We have shown that a complete market for state contingent claims on the exchange rate is sufficient to preserve the separation property for an arbitrary shape of the firm's exposure. If these claims are fairly priced, the firm will fully hedge.

Appendix

Proof of Proposition 1

It is evident from equation (6) that $\Pi^*(S) = U'^{-1}(\lambda^*)$ for all $S \in [\underline{S}, \overline{S}]$. Since $U'^{-1}(\lambda^*)$ is a positive constant, $\Pi^*(S)$ must be independent of S, requiring $\partial R(S, Q^*)/\partial S = -\partial \Phi^*(S)/\partial S$ for all $S \in [\underline{S}, \overline{S}]$. Combining this with $E[\Phi^*(\tilde{S})] = 0$ directly leads to equation (8). Substituting equation (8) into equation (5) yields equation (7) since $U'[\Pi^*(\tilde{S})]$ is a constant. □

Proof of Proposition 2

This proposition follows immediately from equation (8). □

Armed with Proposition 2, it is straightforward to show that the (customized) derivatives contracts do not have to be perceived as fairly priced in order to derive the separation theorem. In fact, the existence of call options with strike price P_d/P_f alone implies the separation result. Let X denote the number of call options written with strike price P_d/P_f and call option premium P. Then, the firm's profits are given by $\tilde{\Pi} = R(\tilde{S}, Q) - C(Q) + (F - \tilde{S})H + X(P - \max(\tilde{S} - P_d/P_f, 0))$. The first-order condition for X^* is equivalent to $P\mathrm{E}\{U'[\Pi^*(\tilde{S})]\} = \mathrm{E}\{U'[\Pi^*(\tilde{S})]\max(\tilde{S} - P_d/P_f, 0)]\}$. Substituting this relation into (5) directly leads to the separation result, $C'(Q^*) = P_d + P_f P$, without imposing any additional assumptions on the forward rate F or the call option premium P. If the call option is perceived as fairly priced such that $P = \mathrm{E}[\max(\tilde{S} - P_d/P_f)]$, (7) follows directly.

Proof of Proposition 3

Substituting $\Phi(S) \equiv (F - S)H$ into program (4), where $F = \mathrm{E}(\tilde{S})$, the first-order conditions become

$$\mathrm{E}\{U'[\Pi^{**}(\tilde{S})][P_d + P_f \max(\tilde{S} - P_d/P_f, 0) - C'(Q^{**})]\} = 0, \tag{10}$$

$$\mathrm{E}\{U'[\Pi^{**}(\tilde{S})](F - \tilde{S})\} = 0, \tag{11}$$

where $\Pi^{**}(S) = R(S, Q^{**}) - C(Q^{**}) + (F - S)H^{**}$ and a double asterisk (**) indicates an optimal level. The second-order conditions for the unique maximum, Q^{**} and H^{**}, are satisfied given the strict concavity of $U(\Pi)$ and the strict convexity of $C(Q)$. Using the covariance operator, $\mathrm{Cov}(\cdot, \cdot)$, with respect to $G(S)$, equation (11) can be equivalently stated as

$$\mathrm{Cov}\{U'[\Pi^{**}(\tilde{S})], \tilde{S}\} = 0, \tag{12}$$

since $F = \mathrm{E}(\tilde{S})$. Partially differentiating $U'[\Pi^{**}(S)]$ with respect to S yields

$$\begin{aligned} \frac{\partial}{\partial S} U'[\Pi^{**}(S)] &= U''[\Pi^{**}(S)] \\ &\quad \times \left[P_f Q_f - H^{**} + P_f(Q^{**} - Q_d - Q_f) \frac{\partial}{\partial S} \max(S - P_d/P_f, 0) \right]. \end{aligned} \tag{13}$$

Notice that $\partial \max(S - P_d/P_f, 0)/\partial S \geq 0$. If $H^{**} \leq P_f Q_f$, equation (13) and $U''(\Pi) < 0$ imply that $U'[\Pi^{**}(S)]$ is decreasing in S everywhere; if

$H^{**} \geq P_f(Q^{**} - Q_d)$, then $U'[\Pi^{**}(S)]$ increases in S everywhere. Hence, $\mathrm{Cov}\{U'[\Pi^{**}(\tilde{S})], \tilde{S}\}$ is negative or positive, respectively. In either case, it contradicts equation (12). Thus, for equation (12) to hold, we must have $P_f Q_f < H^{**} < P_f(Q^{**} - Q_d)$. □

Proof of Proposition 4

The proof of Proposition 4 uses a Lemma that is stated and proven first.

Lemma. There exist two distinct points, $S_1 \in (\underline{S}, P_d/P_f)$ *and* $S_2 \in (P_d/P_f, \overline{S})$*, such that* $U'[\Pi^{**}(S)] \geq \mathrm{E}\{U'[\Pi^{**}(\tilde{S})]\}$ *for all* $S \in [S_1, S_2]$ *and* $U'[\Pi^{**}(S)] < \mathrm{E}\{U'[\Pi^{**}(\tilde{S})]\}$ *for all* $S \in [\underline{S}, S_1) \bigcup (S_2, \overline{S}]$*, where the equality holds only at* $S = S_1$ *and* $S = S_2$.

Proof of the Lemma. Using equation (13) and the fact that $P_f Q_f < H^{**} < P_f(Q^{**} - Q_d)$, we know that $U'[\Pi^{**}(S)]$ is strictly increasing for all $S \in [\underline{S}, P_d/P_f)$ and strictly decreasing for all $S \in (P_d/P_f, \overline{S}]$. In other words, $U'[\Pi^{**}(S)]$ is hump-shaped and attains a unique maximum at $S = P_d/P_f$. Since $\mathrm{E}\{U'[\Pi^{**}(\tilde{S})]\}$ is the expected value of $U'[\Pi^{**}(\tilde{S})]$, there must exist at least one and at most two distinct points in $(\underline{S}, \overline{S})$ at which $U'[\Pi^{**}(S)] = \mathrm{E}\{U'[\Pi^{**}(\tilde{S})]\}$.

Suppose first that $U'[\Pi^{**}(\underline{S})] \geq \mathrm{E}\{U'[\Pi^{**}(\tilde{S})]\}$. Then, there must exist a unique point, $\hat{S} \in (P_d/P_f, \overline{S})$, such that $U'[\Pi^{**}(S)] > \mathrm{E}\{U'[\Pi^{**}(\tilde{S})]\}$ for all $S \in (\underline{S}, \hat{S})$ and $U'[\Pi^{**}(S)] < \mathrm{E}\{U'[\Pi^{**}(\tilde{S})]\}$ for all $S \in (\hat{S}, \overline{S}]$. Thus, we have

$$\int_{\underline{S}}^{\overline{S}} \Big\{U'[\Pi^{**}(S)] - \mathrm{E}\{U'[\Pi^{**}(\tilde{S})]\}\Big\}(S - \hat{S})\, \mathrm{d}G(S) < 0. \tag{14}$$

However, the left-hand side of inequality (14) equals $\mathrm{Cov}\{U'[\Pi^{**}(\tilde{S})], \tilde{S}\}$. Thus, inequality (14) contradicts the first-order condition for H^{**} in equation (12).

Now suppose that $U'[\Pi^{**}(\overline{S})] \geq \mathrm{E}\{U'[\Pi^{**}(\tilde{S})]\}$. Then, there must exist a unique point, $\check{S} \in (\underline{S}, P_d/P_f)$, such that $U'[\Pi^{*}(S)] < \mathrm{E}\{U'[\Pi^{**}(\tilde{S})]\}$ for all $S \in [\underline{S}, \check{S})$ and $U'[\Pi^{**}(S)] > \mathrm{E}\{U'[\Pi^{**}(\tilde{S})]\}$ for all $S \in (\check{S}, \overline{S})$. Thus, we have

$$\int_{\underline{S}}^{\overline{S}} \Big\{U'[\Pi^{**}(S)] - \mathrm{E}\{U'[\Pi^{**}(\tilde{S})]\}\Big\}(S - \check{S})\, \mathrm{d}G(S) > 0. \tag{15}$$

Inequality (15) is contradictory to equation (12). Both $U'[\Pi^{**}(\underline{S})]$ and $U'[\Pi^{**}(\overline{S})]$ are thus strictly less than $\mathrm{E}\{U'[\Pi^{**}(\tilde{S})]\}$. In other words, the statement in the Lemma must be true. □

Proof of Proposition 4. Using the covariance operator, we can write equation (10) as

$$C'(Q^{**}) = P_d + P_f \mathrm{E}[\max(\tilde{S} - P_d/P_f, 0)] + P_f \frac{\mathrm{Cov}\{U'[\Pi^{**}(\tilde{S})], \max(\tilde{S} - P_d/P_f, 0)\}}{\mathrm{E}\{U'[\Pi^{**}(\tilde{S})]\}}. \tag{16}$$

Since $\mathrm{E}\{U'[\Pi^{**}(\tilde{S})]\} > 0$, if $\mathrm{Cov}\{U'[\Pi^{**}(\tilde{S})], \max(\tilde{S} - P_d/P_f, 0)\} < (>)\ 0$, the strict convexity of $C(Q)$ and equations (7) and (16) imply that $Q^{**} < (>)\ Q^*$.

Let $A = \mathrm{E}\{U'[\Pi^{**}(\tilde{S})] | \tilde{S} < P_d/P_f\}$ and $B = \mathrm{E}\{U'[\Pi^{**}(\tilde{S})] | \tilde{S} > P_d/P_f\}$, where $\mathrm{E}(\cdot|\cdot)$ is the conditional expectation operator with respect to $G(S)$. There are two mutually exclusive cases: (i) $A > B$ and (ii) $A \leq B$. Given the strict convexity of $C(Q)$, it follows from equations (5) and (10) that we have to show that $\mathrm{Cov}\{U'[\Pi^{**}(\tilde{S})], \max(\tilde{S} - P_d/P_f, 0)\} < 0$ in both cases.

Consider case (i) first. We write $\mathrm{Cov}\{U'[\Pi^{**}(\tilde{S})], \max(\tilde{S} - P_d/P_f, 0)\}$ as

$$\int_{P_d/P_f}^{\overline{S}} \Big\{U'[\Pi^{**}(S)] - \mathrm{E}\{U'[\Pi^{**}(\tilde{S})]\}\Big\}(S - S_2)\ \mathrm{d}G(S) + \int_{P_d/P_f}^{\overline{S}} \Big\{U'[\Pi^{**}(S)] - \mathrm{E}\{U'[\Pi^{**}(\tilde{S})]\}\Big\}(S_2 - P_d/P_f)\ \mathrm{d}G(S).$$

It follows from the Lemma that the first term of the above expression is negative. The second term of the above expression can be written as

$$(B - A)(S_2 - P_d/P_f)G(P_d/P_f)[1 - G(P_d/P_f)].$$

Since $S_2 > P_d/P_f$ and $0 < G(P_d/P_f) < 1$, this term is also negative by the supposition. Hence, we have $\mathrm{Cov}\{U'[\Pi^{**}(\tilde{S})], \max(\tilde{S} - P_d/P_f, 0)\} < 0$ in case (i).

Now, consider case (ii). Since $\tilde{S} - P_d/P_f = \max(\tilde{S} - P_d/P_f, 0) - \max(P_d/P_f - \tilde{S}, 0)$, equation (12) implies that $\mathrm{Cov}\{U'[\Pi^{**}(\tilde{S})], \max(\tilde{S} - P_d/P_f, 0)\}$ is equal to $\mathrm{Cov}\{U'[\Pi^{**}(\tilde{S})], \max(P_d/P_f - \tilde{S}, 0)\}$, which can be written as

$$\int_{\underline{S}}^{P_d/P_f} \Big\{U'[\Pi^{**}(S)] - \mathrm{E}\{U'[\Pi^{**}(\tilde{S})]\}\Big\}(S_1 - S)\ \mathrm{d}G(S) + \int_{\underline{S}}^{P_d/P_f} \Big\{U'[\Pi^{**}(S)] - \mathrm{E}\{U'[\Pi^{**}(\tilde{S})]\}\Big\}(P_d/P_f - S_1)\ \mathrm{d}G(S).$$

It follows from the Lemma that the first term of the above expression is negative. The second term of the above expression can be written as

$$(A - B)(P_d/P_f - S_1)G(P_d/P_f)[1 - G(P_d/P_f)].$$

Since $S_1 < P_d/P_f$ and $0 < G(P_d/P_f) < 1$, this term is non-positive by the supposition. Hence, we have $\mathrm{Cov}\{U'[\Pi^{**}(\tilde{S})], \max(\tilde{S} - P_d/P_f, 0)\} < 0$ in case (ii). □

Proof of Proposition 5

The first-order conditions for the optimal output and the set of optimal positions in state contingent claims are given by

$$\mathrm{E}\{U'[\hat{\Pi}^*(\tilde{S})][\partial \hat{R}(Q^*, \tilde{S})/\partial Q - C'(Q^*)]\} = 0, \tag{17}$$

$$\mathrm{E}\{U'[\hat{\Pi}^*(\tilde{S})][e_i(\tilde{S}) - f_i]\} = 0 \quad \forall i. \tag{18}$$

An asterisk (*) again denotes an optimal level. Let $\mathrm{prob}(S_i)$ denote the probability of state i and rewrite condition (17) as

$$\begin{aligned} C'(Q^*)\mathrm{E}\{U'[\hat{\Pi}^*(\tilde{S})]\} &= \mathrm{E}\{U'[\hat{\Pi}^*(\tilde{S})]\partial \hat{R}(Q^*, \tilde{S})/\partial Q\} \\ &= \sum_{i=1}^{I} \mathrm{prob}(S_i)\, U'(\hat{\Pi}^*(S_i))\, \frac{\partial \hat{R}(Q^*, S_i)}{\partial Q}. \end{aligned} \tag{19}$$

Using the properties of $e_i(\tilde{S})$, the conditions in (18) can be expressed as

$$f_i \mathrm{E}\{U'[\hat{\Pi}^*(\tilde{S})]\} = U'[\hat{\Pi}^*(S_i)]\ \mathrm{prob}(S_i) \quad \forall i. \tag{20}$$

Substituting the I conditions from (20) into (19) and dividing by $\mathrm{E}\{U'[\hat{\Pi}^*(\tilde{S})]\}$ results in (9). □

References

Adam, T. R., S. *Dasgupta* and S. *Titman,* 2006, Financial constraints, competition and hedging in industry equilibrium. Forthcoming in: Journal of Finance. – *Adam-Müller,* A. F. A., 1997, Export and hedging decisions under revenue and exchange rate risk: A note. European Economic Review 41, 1421–1426. – *Arrow,* K. J., 1965, Aspects of the theory of risk-bearing. Helsinki, Yrjö Jahnssonin Säätiö. – *Bagwell,* K., 1991, Optimal export policy for a new-product monopoly. American Economic Review 81, 1156–1169. – *Bagwell,* K. and R. W. *Staiger,* 1989, The role of

export subsidies when product quality is unknown. Journal of International Economics 27, 69–89. – *Battermann,* H. L., M. *Braulke,* U. *Broll,* and J. *Schimmelpfennig,* 2000, The preferred hedge instrument. Economics Letters 66, 85–91. – *Benninga,* S., R. *Eldor,* and I. *Zilcha,* 1985, Optimal international hedging and output policies in commodity and currency forward markets. Journal of International Money and Finance 4, 537–552. – *Ben-Zvi,* S. and E. *Helpman,* 1992, Oligopoly in segmented markets. In: G. M. Grossman (ed.), Imperfect competition and international trade, Cambridge (Mass.), MIT Press. – *Bodnar,* G. M. and G. *Gebhardt,* 1999, Derivatives usage in risk management by US and German non-financial firms: A comparative survey. Journal of International Financial Management and Accounting 10, 153–187. – *Bodnar,* G. M., G. *Hayt,* and R. C. *Marston,* 1998, 1998 Wharton survey of financial risk management by US non-financial firms. Financial Management 27, 70–91. – *Breuer,* W. and M. *Gürtler,* 2001, Hedging in incomplete markets: An approximation procedure for practical application. Journal of Futures Markets 21, 599–631. – *Broll,* U. and B. *Eckwert,* 2000, Market structure and multiperiod hedging. International Review of Economics and Finance 9, 291–298. – *Broll,* U. and J. E. *Wahl,* 1997, Export flexibility and hedging. Bulletin of Economic Research 49, 205–211. – *Broll,* U., J. E. *Wahl* and I. *Zilcha,* 1995, Indirect hedging of exchange rate risk. Journal of International Money and Finance 14, 667–678. – *Broll,* U. and I. *Zilcha,* 1992, Exchange rate uncertainty, futures markets and the multinational firm. European Economic Review 36, 815–826. – *Chang,* E. C. and K. P. *Wong,* 2003, Cross-hedging with currency options and futures. Journal of Financial and Quantitative Analysis 38, 555–574. – *DeMarzo,* P. M. and D. *Duffie,* 1995, Corporate incentives for hedging and hedge accounting. Review of Financial Studies 8, 743–771. – *Eldor,* R. and I. *Zilcha,* 1987, Discriminating monopoly, forward markets and international trade. International Economic Review 28, 459–468. – *Eldor,* R. and I. *Zilcha,* 2002, Tax asymmetry, production and hedging. Journal of Economics and Business 54, 345–356. – *Engel,* C. and J. H. *Rogers,* 1996, How wide is the border? American Economic Review 86, 1112–1125. – *Engel,* C. and J. H. *Rogers,* 2001, Violating the law of one price: Should we make a federal case out of it? Journal of Money, Credit, and Banking 33, 1–15. – *Franke,* G., 1991, Exchange rate volatility and international trading strategy. Journal of International Money and Finance 10, 292–307. – *Frechette,* D. L., 2001, The demand for hedging with futures and options. Journal of Futures Markets 21, 693–712. – *Froot,* K. A., D. S. *Scharfstein,* and J. S. *Stein,* 1993, Risk management: Coordinating corporate investment and financing policies. Journal of Finance 48, 1629–1658. – *Graham,* J. R. and C. W. *Smith,* 1999, Tax incentives to hedge. Journal of Finance 54, 1969–1997. – *Graham,* J. R. and D. A. *Rogers,* 2002, Do firms hedge in response to tax incentives? Journal of Finance 57, 815–839. – *Gollier,* Ch., 2001, The economics of risk and time. Cambridge (Mass.), MIT Press. – *Lapan,* H. E., G. *Moschini* and S. D. *Hanson,* 1991, Production, hedging, and speculative decisions with options and futures markets. American Journal of Agricultural Economics 73, 66–74. – *Lien,* D., 2001a, A note on loss aversion and futures hedging. Journal of Futures Markets 21, 681-.692. – *Lien,* D., 2001b, Futures hedging under disappointment aversion. Journal of Futures Markets 21, 1029-1042. – *Lien,* D. and Y. K. *Tse,* 2002, Some recent developments in futures hedging. Journal of Economic Surveys 16, 357–396. – *Lien,* D. and Y. *Wang,* 2002, Risk aversion, disappointment aversion, and futures hedging. Journal of Futures Markets 22, 123–141.

– *Lien,* D. and K. P. *Wong,* 2004, Optimal bidding and hedging in international markets. Journal of International Money and Finance 23, 785–798. – *Mello,* A. S. and M. *Ruckes,* 2005, Financial hedging and product market rivalry. Working paper, University of Wisconsin-Madison. – *Moschini,* G. and H. *Lapan,* 1995, The hedging role of options and futures under joint price, basis, and production risk. International Economic Review 36, 1025-1049. – *Parsley,* D. C. and S. J. *Wei,* 1996, Convergence to the law of one price without trade barriers or currency fluctuations. Quarterly Journal of Economics 111, 1211–1236. – *Sercu,* P., 1992, Exchange risk, exposure, and the option to trade. Journal of International Money and Finance 11, 579–593. – *Sercu,* P. and C. *Vanhulle,* 1992, Exchange rate volatility, international trade, and the value of exporting firms. Journal of Banking and Finance 16, 155–182. – *Shy,* O., 2000, Exporting as a signal for product quality. Economica 67, 79–90. – *Smith,* C. W. and R. M. *Stulz,* 1985, The determinants of firms' hedging policies. Journal of Financial and Quantitative Analysis 20, 391–405. – *Stulz,* R. M., 1984, Optimal hedging policies. Journal of Financial and Quantitative Analysis 19/2, 127–140. – *Ware,* R. and R. *Winter,* 1988, Forward markets, currency options and the hedging for foreign exchange risk. Journal of International Economics 25, 291–302. – *Wong,* K. P., 2001, Currency hedging for export-flexible firms. International Economic Journal 15, 165–174. – *Wong,* K. P., 2003a, Currency hedging with options and futures. European Economic Review 47, 833–839. – *Wong,* K. P., 2003b, Export flexibility and currency hedging. International Economic Review 44, 1295–1312.

Summary

Restricted Export Flexibility and Risk Management with Options and Forward Contracts

This paper examines the interaction between operational and financial hedging in the context of a risk averse competitive exporting firm under exchange rate uncertainty. The firm is export-flexible in that it makes its export decision after observing the realized spot exchange rate. However, export-flexibility is limited by certain minimum sales requirements due to long-term considerations. This creates a piecewise linear exchange rate exposure. If the firm is allowed to use customized derivatives contracts, its optimal hedge position can be replicated by selling currency forward contracts and call options. If the firm is restricted to use forward contracts as the sole hedging instrument, optimal output is unambiguously smaller. Introducing currency call options thus stimulates production. An extension analyzes more general types of exchange rate exposure. (JEL F31, D21, D81)

Zusammenfassung

Eingeschränkte Exportflexibilität und Risikomanagement mit Devisenoptionen und Devisenterminkontrakten

Dieser Beitrag untersucht das Zusammenspiel von realwirtschaftlichen und finanzwirtschaftlichen Maßnahmen zur Risikosteuerung. Dies geschieht am Beispiel

eines risikoscheuen Unternehmens, das sein Produkt unter Wechselkursrisiko exportieren oder auf dem heimischen Markt verkaufen kann. Das Unternehmen ist flexibel in dem Sinne, dass die Entscheidung über einen Export erst dann getroffen zu werden braucht, wenn der relevante Devisenkassakurs bekannt ist. Allerdings ist diese Flexibilität dadurch eingeschränkt, dass aufgrund längerfristiger Bindungen sowohl im Ausland als auch im Inland bestimmte Mindestmengen verkauft werden müssen. Die optimale Exportpolitik erzeugt eine stückweise lineare Abhängigkeit des Unternehmensergebnisses vom Wechselkurs. Kann das Unternehmen maßgeschneiderte Wechselkursderivate handeln, so kann die optimale Position des Unternehmens in diesen exotischen Derivaten durch den Verkauf von Devisenterminkontrakten und Devisenkaufoptionen dupliziert werden. Kann das Unternehmen dagegen nur Devisenterminkontrakte handeln, so ist die optimale Produktionsmenge kleiner. Daher steigt die Produktionsmenge, wenn Devisenoptionen verfügbar werden. In einer Erweiterung werden allgemeine Formen der Abhängigkeit des Unternehmensergebnisses vom Wechselkurs untersucht.