

## **Alternative Monetary Policy Rules and the Specification of the Phillips Curve: A Comparison of Nominal Income with Strict Inflation Targeting**

By Alfred V. Guender, Christchurch/New Zealand\*

### **I. Introduction**

Over the past 10 years, the focus of monetary policy has changed quite dramatically in a number of countries.<sup>1</sup> In choosing among the available nominal target variables for monetary policy, policymakers in these countries have opted for formal inflation targets. The narrow focus of monetary policy on inflation may at first seem puzzling. After all, one would expect the policymaker to choose a target variable that is broadly consistent with the preferences of the public. Concern over real objectives such as full-employment should indeed lead central banks to adopt a nominal spending variable such as nominal GDP as the target of monetary policy. Yet inflation targeting has found wide appeal. Proponents of inflation targeting attribute the appeal of inflation targeting to the basic realization that monetary policy actions have no ultimate real effects on the economy. Hence monetary policy should focus on the variable that it affects most – inflation. Increased transparency in the conduct of monetary policy, greater accountability by policymakers for poor performance, and relative ease of communication with the public about the goals of monetary policy are often mentioned as additional benefits of a strategy of monetary policy centered on inflation targeting.

In the academic literature, several recent contributions discuss the merits of a number of different rule-based or target-based strategies of

---

\* I am deeply indebted to Graeme Guthrie and Andreas Irmen for help with using computer software. In addition, I wish to thank Arthur Benavie, David Black, Richard Froyen, and Bennett McCallum for making helpful comments. The suggestions of one referee are gratefully acknowledged. All errors are the sole responsibility of the author.

<sup>1</sup> Inflation has been designated to be the criterion shaping monetary policy action in New Zealand, Canada, the United Kingdom, Sweden, Australia, Finland, and other countries.

monetary policy.<sup>2</sup> Taylor (1993, 1994) designs a rule whereby the central bank adjusts the real interest rate in response to deviations in the rate of inflation and the level of real output from their targeted levels. Svensson (1997a) finds that it is optimal for a central bank to target the forecast of the inflation rate if price stability is the sole goal of monetary policy. Analyzing several different strategies of monetary policy in a simple stochastic macro model, Ball (1997) arrives at the conclusion that nominal income targeting is a disastrous strategy of monetary policy. This result is disputed by McCallum (1997b) who argues that Ball's findings are a direct result of the backward specification of the Phillips curve relation.

This paper shows that the alleged instability of nominal income targeting in the backward-looking model disappears if the policymaker chooses to adopt a hybrid nominal income target. This particular form of nominal income targeting requires the monetary authority to target the sum of the rate of inflation and the deviation of real output from capacity. The paper then goes on to examine the conditions under which a hybrid nominal income targeting strategy is preferable to a strict inflation target. Such a comparison is warranted as both strategies of monetary policy are efficient. We derive a policy frontier that divides the parameter space (weight on variance of inflation in loss function; sensitivity of inflation to excess demand) into two areas: one where strict inflation targeting is preferred to hybrid nominal income targeting and one where hybrid nominal income targeting is preferred to strict inflation targeting.

Next we examine the case where the backward-looking Phillips curve and IS curve are replaced by their forward-looking counterparts and proceed to trace out a policy frontier based on the strict inflation target and the hybrid nominal income target. Finally, using the forward-looking model as our baseline model, we compare and contrast the merits of strict inflation targeting to a strategy of nominal income growth targeting.

In each of the three comparisons the parameter measuring the response of inflation to deviations of real output from capacity is of critical importance. Drawing on reported parameter estimates for the United States, we attempt to estimate the weight the policymaker has to place

---

<sup>2</sup> The rule-based approach to monetary policy is not without its critics, however. *Friedman and Kuttner* (1996) and *Bernanke and Mishkin* (1997) voice their doubts about the effectiveness of strict rule-based monetary policy strategies. For a broad survey of recent research on monetary policy, see *Clarida, Gali, and Gertler* (1999).

on the variance of inflation in the loss function so that he prefers strict inflation targeting to nominal income targeting. The results for the backward- and forward-looking model indicate that in the United States a hybrid nominal income target would be preferred to a strict inflation target for most plausible values of the respective critical response parameter ( $\alpha$  or  $a$ ) as long as the weight on inflation variability relative to real output variability in the monetary authorities' loss function is not excessively high. A strict inflation target would become relatively more attractive if the alternative policy strategy were a nominal income growth target and the forward-looking model served as the baseline model.

The remainder of the paper proceeds as follows. Section II compares and contrasts the backward- and forward-looking models. In Section III we derive the variances of inflation and real output under hybrid nominal income targeting and strict inflation targeting in the context of a backward-looking aggregate model. In addition, we discuss the policy implications of adopting either rule and then derive the policy frontier. Section IV analyzes the two strategies of monetary policy based on a forward-looking aggregate model. In Section V we revisit the issue of nominal income targeting in terms of growth rates. Section VI concludes.

## II. The Strategy of Nominal Income Targeting: A Comparison of Two Simple Models

The model introduced by Ball (1997) consists of backward-looking IS and Phillips curve relations:

$$(1) \quad y_t = -\beta r_{t-1} + \lambda y_{t-1} + \varepsilon_t$$

$$(2) \quad \pi_t = \pi_{t-1} + \alpha y_{t-1} + \eta_t$$

where  $y$  is the deviation of real output from capacity

$r$  is the real rate of interest

$\pi$  is the rate of inflation

Both  $\varepsilon$  and  $\eta$  are white noise disturbances and  $\alpha > 0$ ,  $\beta > 0$ ,  $0 < \lambda < 1$ .

Using the above model, Ball makes the following three points. First, the simple Taylor rules currently in practice in a number of different countries are inefficient. The inefficiency arises as the estimated coefficient on real output in the Taylor rule reported for these countries is below the range prescribed by the model.<sup>3</sup> Second, both strict and flex-

ible inflation targeting are efficient strategies of monetary policy. Finally, nominal income targeting whether expressed in level or growth rate form is a disastrous strategy of monetary policy as it leads to instability in both the rate of inflation and the level of real output.

McCallum (1997b) refutes the proposition that nominal income targeting is an unsound strategy of monetary policy. His model takes the following form:

$$(1a) \quad y_t = -\beta r_t + E_t y_{t+1} + v_t$$

$$(2a) \quad \pi_t = E_t \pi_{t+1} + a y_t + u_t$$

Both  $u_t$  and  $v_t$  are white noise disturbances  $\beta > 0$   $a > 0$ .

This model is similar to Ball's but differs from it in two important respects. One alteration introduced by McCallum concerns the specification of the Phillips curve relation. The backward-looking Phillips curve employed by Ball is replaced by what McCallum calls a more plausible specification, one that includes expected future inflation. The attractiveness of a forward-looking Phillips curve derives primarily from theoretical considerations.<sup>4</sup> The other change relates to the control lag of monetary policy. In the original model proposed by Ball, a change in the rate of interest affects the level of output with a one period lag and the rate of inflation with a two period lag. In sharp contrast, McCallum employs specifications of the IS and the Phillips curve relation where a change in the interest rate in the current period affects both the level of real output and the rate of inflation in the same period. Put simply, McCallum does away with the control lags of monetary policy. The two changes introduced by McCallum have far-reaching implications: the instability in the rate of inflation and real output under nominal income targeting disappears.<sup>5</sup> It thus appears that McCallum's attempt at restoring the

<sup>3</sup> The coefficients on real output and inflation in the Taylor rule derived by Ball depend on the parameters that appear in the IS and the Phillips curve relation. The assumed values for  $\lambda$ ,  $\beta$ , and  $\alpha$  are .8, 1, and .4, respectively.

<sup>4</sup> The specification of the Phillips Curve proposed by McCallum (1997b) is due to Roberts (1995) who shows that the forward-looking Phillips curve is consistent with well-known theoretical models. Another specification of the Phillips curve considered by McCallum is one where the current price level is entirely predetermined, the P-bar model. The P-bar model is an attractive alternative to the forward-looking model as it satisfies the strict version of the natural rate hypothesis (McCallum (1994, pp. 259–61)).

<sup>5</sup> McCallum also employs the expected level of real output ( $E_t y_{t+1}$ ) instead of the lagged level of output in the IS relation. However, he argues that the instability

viability of nominal income targeting as a sensible strategy of monetary policy comes at the expense of sacrificing at least one attractive feature of Ball's model: the existence of control lags for monetary policy. But the property that real output responds to monetary policy before the rate of inflation changes imparts a more realistic flavor to Ball's model as it accords with both stylized facts.<sup>6</sup>

### III. Policy Analysis Based on the Backward-Looking Phillips Curve<sup>7</sup>

#### 1. A Hybrid Nominal Income Target

Ball's examination of the merits of nominal income targeting considers the case where the policymaker targets the growth rate of nominal income and the case where the policymaker attempts to achieve a fixed level of nominal income. It is important to realize that neither the *level* nor the *growth rate* version of nominal income targeting conforms to the optimal policy rule for monetary policy in Ball's model. There exists, however, a hybrid form of nominal income targeting which is, as explained below, consistent with the optimal monetary policy rule in the model proposed by Ball. Various forms of this operational strategy have been discussed in the literature.<sup>8</sup> The hybrid strategy involves setting a target value for the sum of inflation and the level of real output measured relative to capacity output. If the relevant time interval is one year,

---

result reported by Ball is a direct consequence of the specification of the Phillips curve.

<sup>6</sup> Empirical results favorable to the backward-looking Phillips curve specification have been reported by *Gordon* (1996) and *Fuhrer* (1996). Moreover, *McCallum* (1995) concedes that ... "prices evidently react more slowly than output in response to monetary actions, ...". It should be noted though that *McCallum* (1997b) invokes the empirical results reported by *Roberts* (1995) to back up his preference for the forward-looking specification of the Phillips curve relation.

<sup>7</sup> There are certain issues that this paper does not explicitly address. These issues pertain to the credibility of the monetary authorities and the possibility that the preferences of the monetary authorities differ from those of the government or society at large. We assume that the monetary rules announced by the monetary authorities are fully credible as is the case in *Ball* (1997) and *McCallum* (1997b).

<sup>8</sup> For empirical evaluations of hybrid nominal income targeting rules, see *Bryant, Hooper, and Mann* (1993) and *Henderson and McKibbin* (1993) and *Bryant* (1996). *Hall and Mankiw* (1994) assess the properties of an alternative hybrid targeting rule, one where the output gap enters explicitly. For a description of various forms of nominal income targeting, see *McCallum* (1997a). The adoption of the hybrid form of nominal income targeting is predicated on knowing the level of capacity output. Under level or growth rate nominal income targeting no such knowledge is required.

then the target value is formed by adding the expected rate of inflation (measured as a percentage) to the expected real output gap (measured as a percentage).

The hybrid form of nominal income targeting as described above is a special case of the optimal policy rule and also satisfies Ball's criterion for an efficient policy strategy. It is optimal because hybrid nominal income targeting is framed *solely* in terms of the ultimate goal variables, the (expected) output gap, and the (expected) rate of inflation, with the relative emphasis on the two goal variables in the optimal rule determined by the underlying preferences of the policymaker and the structural parameter in the Phillips curve. It is also efficient because the hybrid strategy of nominal income targeting imposes a unitary trade-off between the (expected) rate of inflation and the (expected) output gap ( $\theta = 1$ ).<sup>9</sup>

Let the target value be given by  $z^* = E_t[y_{t+1} + \pi_{t+1}] = 0$ . Combining the target with equations (1) and (2) yields the reaction function followed by the policymaker:

$$(3) \quad r_t = \frac{1}{\beta} \pi_t + \left( \frac{\alpha + \lambda}{\beta} \right) y_t$$

The policymaker follows a Taylor rule; the real interest rate is raised in response to a positive rate of inflation and a positive deviation of real output from capacity.<sup>10</sup> After substituting equation (3) into equation (1),

<sup>9</sup> Let the policymaker choose optimal policy on the basis of a weighted average of the output gap and the rate of inflation, the two variables the policymaker cares about:  $E_t[\theta y_{t+1} + \pi_{t+1}] = 0$ . The policymaker chooses  $\theta$  in such a way so as to minimize the loss function (consisting of the variance of inflation and the output gap, respectively). The solution to the minimization problem is given by

$\theta = \frac{\alpha\mu \pm \sqrt{\alpha^2\mu^2 + 4\mu}}{2\mu}$ . The size of  $\theta$  is a function of  $\mu$ , the preferences of the policy-

maker regarding the variability of inflation and the variability of the output gap and  $\alpha$ , the parameter on the output gap in the Phillips curve. Under hybrid nominal income targeting, the policymaker sets  $\theta$  equal to one.

The long version of the appendix (available upon request from the author) provides further details on the derivation of the optimal policy rule in the backward-looking model. Notice that the growth rate version of nominal income targeting is not optimal (and hence cannot be efficient) because the course of monetary policy depends in part on the current output gap  $y_t$  (i.e.  $z^{**} = E_t[y_{t+1} - y_t + \pi_{t+1}]$ ).

<sup>10</sup> The notion that policy ought to react to errors (in production) goes back to Phillips (1957).

Whether the Taylor rule embodied in equation (3) is actually operational is the subject of some controversy. The model assumes that the policymaker has full control over the setting of the policy instrument, the *real* rate of interest. In addition,

we obtain equation (4) below. Together, equation (4) and equation (2) characterize the time series processes for real output and the rate of inflation under hybrid nominal income targeting:

$$(4) \quad y_t = -\alpha y_{t-1} - \pi_{t-1} + \varepsilon_t$$

$$(5) \quad \pi_t = \pi_{t-1} + \alpha y_{t-1} + \eta_t$$

The variances of real output and the rate of inflation under the hybrid nominal income targeting strategy (NIT) are given by

$$(6) \quad V(y_t)^{NIT} = \frac{2\alpha\sigma_\varepsilon^2 + \sigma_\eta^2}{\alpha(2-\alpha)} \quad V(\pi_t)^{NIT} = \frac{\alpha^2\sigma_\varepsilon^2 + \sigma_\eta^2(1+2\alpha-\alpha^2)}{\alpha(2-\alpha)}$$

Both variances are positive and hence well defined as long as  $\alpha < 2$ .<sup>11</sup> Thus the conclusion that nominal income targeting is a disastrous strategy for monetary policy does not apply in the case of a hybrid target.<sup>12</sup>

An explanation for the apparent reversal of the instability result is warranted. In pursuing a hybrid target, the policymaker is no longer required to adhere to the constant marginal rate of substitution between the price level and real output imposed by the fixed nominal income target (or between inflation and real output growth in case of a nominal income growth target). But it is the strict adherence to maintaining a constant tradeoff between the price level (inflation) and real output (growth) that causes instability in the behavior of real output and inflation under nominal income (growth) targeting in Ball's model.<sup>13, 14</sup>

---

the set-up implies that in a given time period the policymaker observes the current rate of inflation and the current output gap. Important issues regarding the availability of contemporaneous feedback data and the extent of measurement error are thus ignored. A study that addresses these concerns is by *Croushore and Stark* (1999).

<sup>11</sup> The parameter  $\alpha$  is viewed as being structural.

<sup>12</sup> *Svensson* (1997b) suggests a staggered form of nominal income growth to avoid instability. However, this staggered form has only limited applicability in practice as it focuses on the *current* rate of inflation and *lagged* output gap growth.

<sup>13</sup> For a detailed description of how a positive shock to inflation causes instability in the real output gap and the rate of inflation under a nominal income growth target see *Svensson* (1997b). In essence, the positive shock to inflation, which causes the rate of inflation to ratchet up every period, requires offsetting declines in the output gap to keep the growth rate of nominal income in line with the target rate.

<sup>14</sup> Most analyses of the merits of nominal income targeting in a closed-economy framework emphasize its ability to insulate the economy from the effects of white

There is a further noteworthy result concerning the absence of symmetry in the effects of the disturbances. Under a hybrid nominal income target, the effects of demand side disturbances will fall disproportionately on real output. For  $0 < \alpha < 2$  the coefficient of  $\sigma_\epsilon^2$  in the expression for  $V(y_t)$  equals one plus the coefficient of  $\sigma_\epsilon^2$  in the expression for  $V(\pi_t)$ . In a similar vein, as long as  $0 < \alpha < 2$  the effect of shocks to the Phillips curve relation will fall disproportionately on the rate of inflation. The coefficient on  $\sigma_\eta^2$  in the expression for the variance of inflation always exceeds its counterpart in the expression for the variance of real output by a factor of one.<sup>15</sup>

## 2. A Strict Inflation Target

The existence of a control lag for monetary policy makes it impossible for the policymaker to affect the rate of inflation in the current or in the next period. Hence under a strict inflation target (where we assume the target rate  $\pi^*$  to be equal to zero), the policymaker sets the expected rate of inflation two periods into the future equal to zero.<sup>16</sup>

$$(7) \quad E_t \pi_{t+2} = 0 = \pi^*$$

Imposing the target value for the rate of inflation on the model (equations (1) and (2)) yields the reaction function followed by the policymaker under a strict inflation targeting regime:

$$(8) \quad r_t = \frac{1}{\alpha\beta} \pi_t + \left(\frac{1+\lambda}{\beta}\right) y_t$$

Compared to the Taylor rule under the hybrid nominal income targeting strategy, the Taylor rule under a strict inflation target reacts more

---

noise aggregate demand side disturbances (e.g. *Bean* (1983), *West* (1986), *Asako* and *Wagner* (1992), *Frankel* and *Chinn* (1995)). This insulating property does not carry over to the current framework – as evidenced by the presence of the variance of IS shocks in both the variance of real output and the variance of inflation. It should be added, however, that such clear-cut results obtain due to the assumption of white noise disturbances.

<sup>15</sup> The finding that the effects of shocks on real output and inflation differ under a strategy of hybrid nominal income targeting stands in marked contrast to the symmetric results obtained under nominal income targeting in standard stochastic macro models (e.g. authors named in preceding footnote).

<sup>16</sup> As shown in the longer version of the appendix, under a strict inflation target the policymaker sets the weight on the output gap in the optimal policy rule equal to  $\alpha$ .

forcefully to both the current rate of inflation and the current output gap provided that  $\alpha < 1$ .

Inserting equation (8) into the IS equation yields equation (9) which describes the time series process for real output under strict inflation targeting:

$$(9) \quad y_t = -y_{t-1} - \frac{1}{\alpha} \pi_{t-1} + \varepsilon_t$$

$$(10) \quad \pi_t = \pi_{t-1} + \alpha y_{t-1} + \eta_t$$

The variances of real output and the rate of inflation under a strict inflation targeting regime (SIT) follow from equations (9) and (10) and are given by:

$$(11) \quad V(y_t)^{SIT} = 2\sigma_\varepsilon^2 + \frac{\sigma_\eta^2}{\alpha^2} \quad V(\pi_t)^{SIT} = \alpha^2 \sigma_\varepsilon^2 + 2\sigma_\eta^2$$

Even in case of a strict inflation target the variance of inflation is strictly positive as a consequence of the inability of the policymaker to exercise immediate and complete control over the rate of inflation. The variance of real output is inversely related to the size of  $\alpha$  while the variance of inflation varies positively with the size of  $\alpha$ .

### 3. Ranking the Two Policy Rules and Policy Implications

In this section we attempt to evaluate the circumstances under which the policymaker would prefer a strict inflation target to a hybrid nominal income target.

We begin by comparing the variances of real output and the rate of inflation under both regimes. The variance of real output is lower under NIT relative to SIT only for  $\alpha < 1$ . The variance of inflation under SIT is less than the variance of inflation under NIT for all values of  $\alpha$ . These results imply that the policymaker would always opt for a strict inflation target in case  $\alpha > 1$ . For  $0 < \alpha < 1$  the policymaker decides on the appropriate strategy on monetary policy by taking account of the emphasis placed on minimizing inflation variability relative to output variability.<sup>17</sup>

---

<sup>17</sup> In comparisons of alternative monetary policy strategies based on standard stochastic macro models involving rational expectations (see Footnote 14), the parameters of the aggregate demand schedule play an important role in determining the superiority of one rule over another. This is not the case in the current

The policymaker faces an expected loss function consisting of the variances of real output deviations and the rate of inflation:<sup>18</sup>

$$(12) \quad \Omega_t = V(y_t) + \mu V(\pi_t)$$

$\mu$  indicates the fixed weight the policymaker places on the variability of inflation relative to the variability of real output.  $\mu$  can take on any value between 0 and  $\alpha$ .<sup>19</sup>

After inserting into the loss function the variance of real output and the rate of inflation under either strategy of monetary policy, we obtain the following two expressions:

$$(13) \quad \Omega_t^{NIT} = \frac{2\alpha\sigma_\epsilon^2 + \sigma_\eta^2}{\alpha(2-\alpha)} + \mu \left( \frac{\alpha^2\sigma_\epsilon^2 + \sigma_\eta^2(1+2\alpha-\alpha^2)}{\alpha(2-\alpha)} \right)$$

$$(14) \quad \Omega_t^{SIT} = 2\sigma_\epsilon^2 + \frac{\sigma_\eta^2}{\alpha^2} + \mu(\alpha^2\sigma_\epsilon^2 + 2\sigma_\eta^2)$$

model as  $\beta$  does not figure in the calculation of the variance of real output and inflation, respectively.

<sup>18</sup> This is the ad-hoc loss function employed by *Ball* (1997). It is the standard objective function in the literature on monetary policy issues. *Woodford* (1999) shows that such loss functions have sound theoretical foundations (albeit in forward-looking models) in that they are second-order approximations to the appropriate welfare measure, the expected utility level of representative agents. Other specifications of the loss function could include the variance of the policy instrument,  $V(r_t)$ . However, this would necessitate choosing an arbitrary weight for  $V(r_t)$  as well as introducing the demand-side parameters  $\beta$  and  $\lambda$ , both of which would complicate the present analysis. See *Rudebusch* and *Svensson* (1999) for examples of this type of loss function. *Jensen* (1999) evaluates targeting regimes in terms of regime-specific loss functions. His specification calls for the inclusion in the loss function of the growth rate of nominal income under a nominal income growth regime. Alternatively, *Koenig* (1993) suggests that the performance of policy rules be evaluated on the basis of whether a weighted average of the price level and real output equals a pre-announced target. He finds both price level and inflation targeting to be suboptimal because both strategies put a zero weight on real output.

<sup>19</sup> *Rogoff* (1985) marks an early contribution to the debate on the “weight issue”. The approach taken in the present paper suggests that the variance of real output enter the policymaker’s expected loss function even under a strict inflation target. This stands in marked contrast to *Ball* (1997) and *Svensson* (1997a) where a strict inflation target implies an infinitely large weight on the variance of the rate of inflation. The objective in the current paper, however, is to establish the size of the weight the policymaker, i.e. the monetary authority, has to place on inflation variability in the expected loss function for the strict inflation target to dominate the hybrid nominal income target. In practice, even if the overriding goal of monetary policy is control of inflation, in the *short-run* central banks have stabilization goals other than inflation such as full employment.

For  $\alpha = 1$  the two loss functions are equal irrespective of the weight placed on  $\mu$ . In this particular case both strategies are equally preferred. For  $\alpha > 1$  a strict inflation target is always preferred to a nominal income target irrespective of the weight placed on  $\mu$ .

For  $0 < \alpha < 1$  we can depict the tradeoff between  $\alpha$  and  $\mu$  by constructing a policy frontier. This policy frontier divides the admissible parameter space into separate regions where one strategy of monetary policy dominates the other. Representative values for  $\alpha$  are gleaned from the literature and are arranged in Table 1. For a given value of  $\alpha$  we trace out the policy frontier by choosing the appropriate value for  $\mu$  so that the two loss functions are equal.<sup>20</sup> Figure 1 shows that the policy frontier is U-shaped and symmetric. Over the range  $0 < \alpha < 1$  values of  $\alpha$  close to 0 and 1 require high values of  $\mu$  for the policymaker to be indifferent between hybrid nominal income targeting and strict inflation tar-

Table 1  
Parameter Estimates of  $\alpha$  (Backward-Looking Model)

Source	Sample Period	Estimated Value of $\alpha$	Value of $\mu$ Required to Generate Indifference
Ball (1996) <sup>a</sup>	–	.4	8.33
Hall and Mankiw (1994) <sup>a</sup>	–	.05	42.11
Romer (1996) <sup>b</sup>	1952:1994	.37–.42	8.58–8.21
Gordon (1996) <sup>c</sup>	1955:2–1996:1	.3	9.52
Fuhrer (1995) <sup>c</sup>	1960:2–1993:4	.19	13

<sup>a</sup> Ball bases his choice for  $\alpha$  on the sacrifice ratio reported in Ball (1994). Hall and Mankiw choose  $\alpha$  so that it is consistent with the estimates of the output-inflation tradeoff reported in Ball, Mankiw, and Romer (1988).

<sup>b</sup> The equation estimated by Romer is based on the inclusion of the current deviation of output from capacity. Her results are based on annual data.

<sup>c</sup> Based on quarterly data. Gordon includes the current unemployment gap in his model while Fuhrer derives his empirical estimate by including the lagged unemployment gap. The unemployment gap is defined as the current unemployment rate minus the natural rate. The respective value for  $\alpha$  is obtained by multiplying the response of the rate of inflation to deviations of unemployment from the natural rate by  $-5$ . This is the value of the parameter in the equation linking deviations of output from capacity to the deviation of the rate of unemployment from the natural rate of unemployment (Okun's Law) and is reported by Gordon (1996).

<sup>20</sup> The variances of real output and inflation are divided by  $\sigma_\epsilon^2$  so that either variance is only a function of  $\sigma_\eta^2/\sigma_\epsilon^2$ . The shape or location of the policy frontier is invariant to changes in  $\sigma_\eta^2/\sigma_\epsilon^2$ . More specifically, for a given  $\alpha$  the value of  $\mu$  that makes the two loss functions equal is independent of changes in  $\sigma_\eta^2/\sigma_\epsilon^2$ . Changes in the ratio of the variances of the disturbances merely cause equal changes in the numerical value of the loss function for both strategies. In addition, for  $\sigma_\eta^2/\sigma_\epsilon^2 \neq 1$  the coefficients on the variance of real output and the variance of the rate of inflation are not equal under hybrid nominal income targeting.

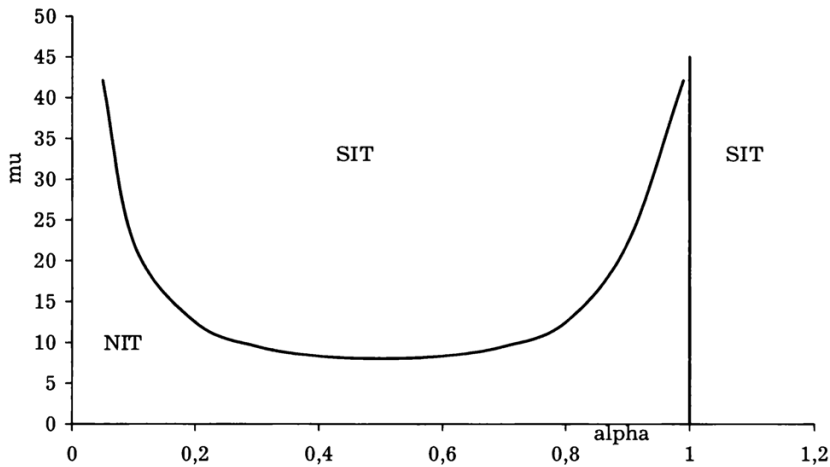


Figure 1: Backward-Looking PC – Policy Frontier: Hybrid NIT vs SIT

getting. For less extreme values of  $\alpha$ , corresponding lower values of  $\mu$  make the policymaker indifferent between pursuing either strategy. The indifference curve bottoms out at  $\alpha = .5$  and  $\mu = 8$ . The policymaker chooses to pursue a strict inflation target if the combination of  $\alpha$  and  $\mu$  lies above the policy frontier. Conversely, a nominal income target is preferred if the combination of the two parameters lies below the frontier.

The third column of Table 1 presents empirical estimates of the size of  $\alpha$  which have been reported in the literature. The values of  $\mu$  that generate indifference on the part of the policymaker between targeting hybrid nominal income and inflation for the estimated values of  $\alpha$  appear in the fourth column. The calculated values of  $\mu$  range from a high value of 42.11 to a low value of 8.21. According to Figure 2 all values lie on the downward sloping and flat portion of the policy frontier. This is a direct result of all estimated values of  $\alpha$  being less than .5.

These results point to the following policy implications. With all reported estimates of  $\alpha$  being considerably lower than 1, the area to the right of the U-shaped policy frontier where a strict inflation target is unambiguously superior to a hybrid nominal income target ( $\alpha > 1$ ) is of little practical relevance. As Figure 2 shows, four of the six empirical estimates of  $\alpha$  lie between .3 and .42 and imply corresponding values of  $\mu$  between 8.21 and 9.52. The policymaker would thus have to value inflation variability roughly 9 times more than real output variability to prefer a strict inflation regime to a hybrid nominal income target. Such

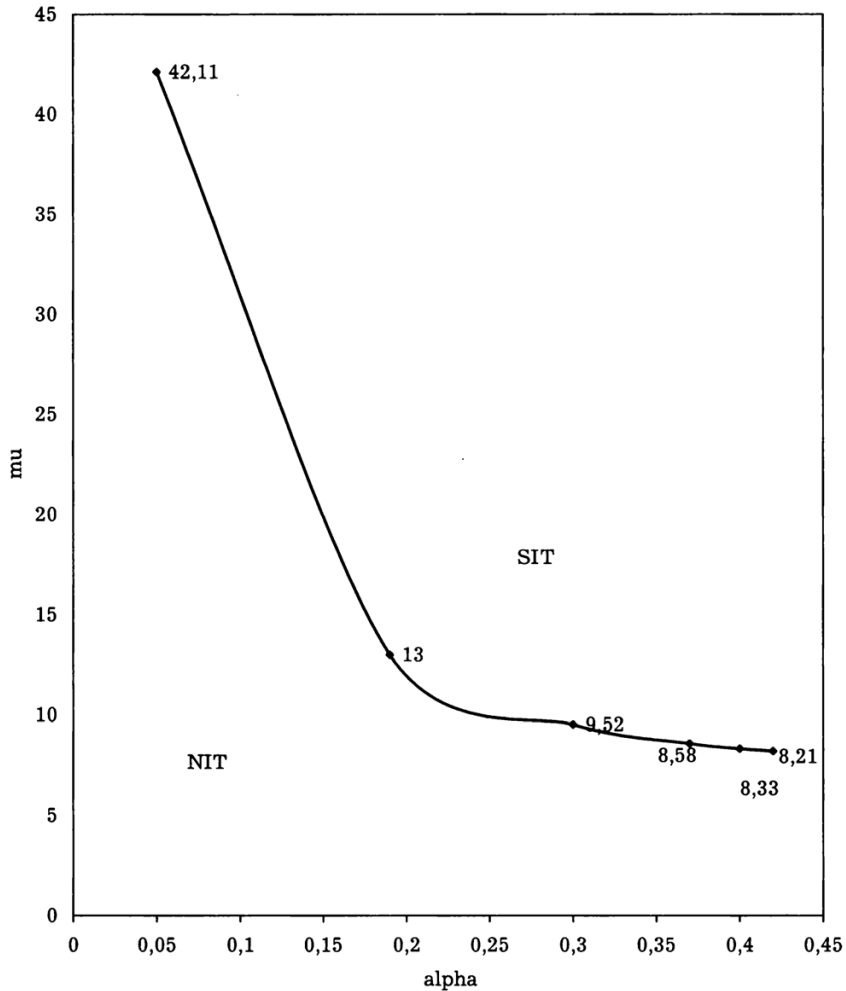


Figure 2: The Relationship Between  $\mu$  and the Empirical Values of  $\alpha$

strong emphasis on keeping inflation variability at bay is perhaps a bit unlikely in the United States.<sup>21</sup> For extremely low values of  $\alpha$  such as

<sup>21</sup> After all, the Fed is bound by the guidelines of the Humphrey-Hawkins Act of 1978 which would tend to lower the weight the Fed can place on  $\mu$ . However, a larger weight on inflation variability is more likely in countries like New Zealand where the overriding goal of monetary policy is to ensure price stability. While political considerations (such as the status of central bank) play some role in determining the size of  $\mu$ , the welfare costs associated with inflation matter as well.

the one reported by Hall and Mankiw, a strict inflation target can be safely ruled out.<sup>22</sup>

#### IV. Policy Analysis Based on the Forward-Looking Phillips Curve

In this section we again examine two strategies of monetary policy, one geared towards attaining an announced hybrid nominal income target and the other focusing solely on meeting a prespecified inflation target. However, we now adopt the model proposed by McCallum (1997b).<sup>23</sup> We replace the backward IS and Phillips curve relations with their forward-looking counterparts:

$$(1a) \quad y_t = -\beta r_t + E_t y_{t+1} + v_t$$

$$(2a) \quad \pi_t = E_t \pi_{t+1} + a y_t + u_t$$

##### 1. A Hybrid Nominal Income Target

Following McCallum (1997b), we specify the nominal income target in terms of current observable values. The hybrid nominal income target consists of the sum of the deviation of real output from capacity and inflation:  $z^* = [y_t + \pi_t]$ . Let  $z^* = 0$  for simplicity. As shown in the appendix, this rule constitutes an efficient form of monetary policy because the policymaker chooses a unitary tradeoff between real output and inflation.

Imposing the above condition on the model consisting of equations (1a) and (2a) yields the following two equations for real output and the rate of inflation, respectively:<sup>24</sup>

---

The weight on the variance of inflation ought to reflect to some extent the actual cost to society of swings in the rate of inflation.

<sup>22</sup> At the same time it is not necessary for the policymaker to assign an infinitely large weight to the variance of the rate of inflation under strict inflation targeting for a strict inflation target to be preferable to a hybrid nominal income target. Rogoff (1985, p. 1187) also concludes that “society will not (in general) want the weight to be infinite.”

<sup>23</sup> For an explicit derivation of the forward-looking IS curve the reader is referred to McCallum and Nelson (1999).

<sup>24</sup> The appeal of the Taylor rule derives in part from the fact that the setting of the interest rate is sensitive to the *current* output gap and deviations of the *current* rate of inflation from target. Unlike in the backward-looking model, the policy instrument in the forward-looking model reacts to *expected* real output. The

$$(15) \quad (1+a)y_t = -E_t\pi_{t+1} - u_t$$

$$(16) \quad \pi_t = E_t\pi_{t+1} + ay_t + u_t$$

Two points are noteworthy. First, the hybrid nominal income target shields the economy from the effects of aggregate demand side disturbances as indicated by the absence of  $v_t$  from both equations. This result is in stark contrast to the model of section III which employs the backward-looking specification of the Phillips curve. The insulating property of the hybrid nominal income strategy exists in the current framework because there is no control lag, i.e. the policymaker can vary the instrument in a given period and affect both the level of real output and inflation contemporaneously. Second, we note the absence of any lagged variables such as  $y_{t-1}$ . Employing the method of undetermined coefficients, we pose the following putative solutions for  $y_t$  and  $\pi_t$ :

$$(17) \quad y_t = \tau_{11}u_t$$

$$(18) \quad \pi_t = \tau_{21}u_t$$

The solutions for the two undetermined coefficients are

$$\tau_{11} = -\frac{1}{1+a} \quad \tau_{21} = \frac{1}{1+a}$$

Substituting the solutions back into the expressions for real output and the rate of inflation, we obtain

$$(19) \quad y_t = -\frac{1}{1+a}u_t$$

$$(20) \quad \pi_t = \frac{1}{1+a}u_t$$

Notice the symmetric effect of the supply-side disturbance on real output and the rate of inflation, respectively. The variances of real output and the rate of inflation under a hybrid nominal income target are then given by

$$(21) \quad V(y_t)^{NIT} = \frac{1}{(1+a)^2} \sigma_u^2 \quad V(\pi_t)^{NIT} = \frac{1}{(1+a)^2} \sigma_u^2$$

---

Taylor rule implied by the hybrid nominal income targeting strategy is given by  $r_t = \frac{1}{\beta}(E_t y_{t+1} + \pi_t + v_t)$ . Thus the policymaker responds to the current rate of inflation in the same way as to the expected output gap or the demand shock.

The variance of real output is identical to the variance of inflation under the hybrid nominal income targeting scheme. This result is very different from the finding obtained for the backward-looking model where the effect of supply shocks is borne disproportionately by the variance of inflation. Moreover, the variances of real output and inflation in the forward-looking model are inversely related to the size of the parameter  $a$ .

## 2. A Strict Inflation Target

As the policymaker has the ability to affect real output and the rate of inflation contemporaneously, a strict inflation target would entail setting the current and the expected rate of inflation equal to zero:

$$(22) \quad \pi_t = E_t \pi_{t+1} = 0$$

Thus under a strict inflation targeting regime the variance of inflation reduces to zero. The strict inflation target implies further that real output observes the following process:<sup>25</sup>

$$(23) \quad y_t = -\frac{1}{a} u_t$$

The variance of real output is then given by

$$(24) \quad V(y_t)^{SIT} = \frac{\sigma_u^2}{a^2}$$

## 3. Ranking the Two Policy Rules and Policy Implications

Several noteworthy results emerge from our examination of the two strategies of monetary policy in the context of the forward-looking model. First, the variability of inflation is zero under the strict inflation target and hence lower than under the hybrid nominal income target. Second, the variance of real output is always lower under the hybrid nominal income targeting strategy.

The third noteworthy result concerns the shape of the policy frontier of the forward-looking model depicted in Figure 3. Unlike the U-shaped

---

<sup>25</sup> The reaction function of the policymaker under the strict inflation target is given by  $r_t = \frac{1}{\beta} \left( \frac{1}{a} u_t + E_t y_{t+1} + v_t \right)$ .

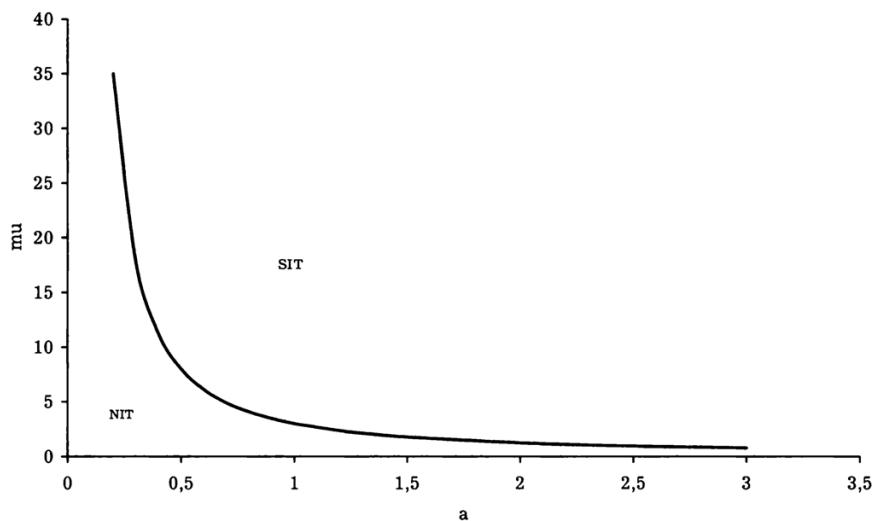


Figure 3: Forward-Looking PC – Policy Frontier: Hybrid NIT vs SIT

policy frontier that emerged from the backward-looking model, the current frontier involves a monotonic trade-off between  $a$  and  $\mu$ . As  $a$  increases in size lower values of  $\mu$  are required to maintain equality between hybrid nominal income targeting and strict inflation targeting as strategies of monetary policy. Initially, for low values of  $a$ , small increases in  $a$  are associated with large declines in  $\mu$  as we move along the frontier. Increasingly smaller declines in  $\mu$  are necessary to stay on the frontier as  $a$  continues to increase. The policymaker prefers strict inflation targeting (hybrid nominal income targeting) if combinations of  $a$  and  $\mu$  lie above (below) the frontier. Finally, it should be noted that there is only one value of  $\mu$  for  $a = 1$  where the two monetary policy strategies are equally preferred. This is in stark contrast to our previous finding in the context of the backward-looking model where for  $\alpha = 1$  the policymaker is indifferent between choosing a hybrid strategy of nominal income targeting or a strict inflation targeting irrespective of the value of  $\mu$ .

There is a clear and unambiguous policy implication. The greater the size of  $a$ , the more attractive a strict inflation target becomes. Drawing on the empirical estimates for  $a$  reported by Roberts (1995), .249 and .337, we find that the policymaker will have to assign a weight of approximately 24.16 or 14.74 to the variance of inflation in order to remain indifferent between strict inflation targeting and hybrid nominal income

targeting.<sup>26</sup> Should the parameter  $a$  increase in size to .75, then the weight on the variance of inflation would drop to 4.44. For the extremely large value of  $a = 3$  the value of  $\mu$  drops to .78 in which case the weight on the variance of the rate of inflation lies below the weight placed on the variance of real output.

## V. Nominal Income Growth Targeting vs a Strict Inflation Target in the Forward-Looking Model

In this section we first assess the implications of framing a monetary policy strategy aimed at reaching a nominal income growth *rate* target. Then we compare this strategy to the strict inflation targeting regime. Finally, we take a closer look at the implications of designing a strategy of monetary policy in terms of a nominal income growth rate target as opposed to a hybrid nominal income target in a setting where the alternative strategy is a strict inflation target. The forward-looking model is again our baseline model.

Specifying a nominal income growth target implies that the change in nominal income ( $\Delta z_t$ ) is set equal to a constant value.<sup>27</sup> For simplicity, let the constant be zero:

$$(25) \quad \Delta z_t = \pi_t + y_t - y_{t-1} = 0$$

Combining equation (25) with equations (1a) and (2a), we obtain again two expressions for real output and the rate of inflation:<sup>28</sup>

$$(26) \quad (1 + a)y_t = -E_t\pi_{t+1} - u_t + y_{t-1}$$

$$(27) \quad \pi_t = E_t\pi_{t+1} + ay_t + u_t$$

The variances of real output and the rate of inflation under the nominal income growth rate target are given by<sup>29</sup>

<sup>26</sup> The question of whether the parameter  $a$  (or  $\alpha$ ) can actually be interpreted as being structural arises. Roberts (1995, p. 982–83) argues that the [...] “New Keynesian Phillips Curve is structurally stable despite the substantial difference in average inflation in the two parts of the sample (before and after 1973).”

<sup>27</sup> The derivation of the processes for real output and the rate of inflation under a nominal income growth target follows McCallum (1997b).

<sup>28</sup> The reaction function under the nominal income growth target is given by  $r_t = \frac{1}{\beta} [E_t y_{t+1} - y_{t-1} + v_t + \pi_t]$ . Notice that the policymaker takes account of the output gap in time  $t-1$  in determining the setting for the policy instrument.

$$(28) \quad V(y_t)^{NITG} = \frac{\phi_{11}^2 \sigma_u^2}{\phi_{21}(2 - \phi_{21})} \quad V(\pi_t)^{NITG} = \left[ \frac{\phi_{21}^2}{\phi_{21}(2 - \phi_{21})} + (\phi_{11} + \phi_{21})^2 - \phi_{11} \right] \sigma_u^2$$

$$\text{where } \phi_{11} = \frac{2+a-\sqrt{a^2+4a}}{2} \quad \phi_{21} = \frac{-a+\sqrt{a^2+4a}}{2}$$

Recall that under a strict inflation target in the forward-looking model the policymaker can eliminate inflation. As a consequence, only the variance of real output deviations appears in the loss function.

$$V(y_t)^{SIT} = \frac{\sigma_u^2}{a^2}$$

In Figure 4 the solid line traces out the policy frontier for the two monetary policy strategies. The two important features of the policy frontier depicted in Figure 3 carry over to the policy frontier shown in Figure 4. The policy frontier again involves a trade-off between  $a$  and  $\mu$  and strict inflation targeting becomes a more attractive strategy of monetary policy as the size of  $a$  increases. Employing once more the empirical estimates of  $a$  reported by Roberts (1995), .249 and .337, we observe that a weight of 24.6 and 11.7, respectively, is required on the variance of inflation in the loss function for the policymaker to remain indifferent between strict inflation targeting and nominal income growth targeting.<sup>30</sup>

The relative attractiveness of specifying a nominal income target in terms of a growth rate as opposed to the hybrid form is brought out by comparing the two policy frontiers of Figure 4. The broken line represents the policy frontier shown in Figure 3 which is based on a comparison of the hybrid nominal income target with the strict inflation target. It appears that for very low values of the parameter  $a$  the growth rate specification of the nominal income targeting strategy does slightly better than the hybrid form in the direct comparison of nominal income targeting with strict inflation targeting. Conversely, the hybrid form of nominal income targeting is preferred to the growth rate targeting

<sup>29</sup> Using the method of undetermined coefficients, McCallum (1997b) derives final form equations for  $y_t$  and  $\pi_t$  under a nominal income growth target. The trial solutions that figure in the solutions for  $y_t$  and  $\pi_t$  are:  $y_t = \phi_{11}y_{t-1} + \phi_{12}u_t$  and  $\pi_t = \phi_{21}y_{t-1} + \phi_{22}u_t$ . McCallum argues that the negative root of the quadratic equation for  $\phi_{11}$  satisfies the conditions for dynamic stability.

A simple further step then produces the variances for real output and inflation reported in equation (28).

<sup>30</sup> Jensen (1999) also evaluates nominal income growth targeting and inflation targeting in a forward-looking model, albeit from a different angle. He finds inflation targeting superior to nominal income growth targeting in a setting where shocks do not involve monetary trade-offs for society, i.e. if shocks arise on the demand-side of the economy. The reverse holds for cost-push shocks.

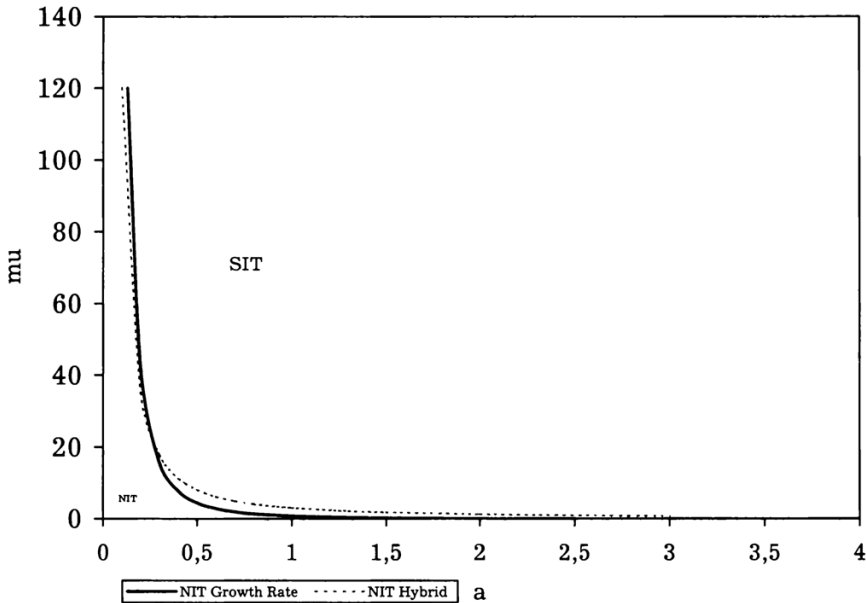


Figure 4: Forward-Looking PC – Policy Frontier: NIT (Growth Rate) vs SIT

scheme for values of  $a$  lying above approximately .254.<sup>31</sup> For instance, for  $a = .249$  the value of  $\mu$  on the policy frontier under the nominal income growth rate target (24.6) is slightly greater than under the hybrid target (24.16). In contrast for  $a = .337$  the associated value of  $\mu$  under the nominal income growth target (11.7) is lower than under the hybrid target (14.74). Another example highlights the difference between the two strategies of nominal income targeting relative to strict inflation targeting. Consider the case where the policymaker places a weight of .78 on the variance of inflation in the loss function. Under the hybrid form of nominal income targeting the associated value of  $a$  on the policy frontier is 3 while under the growth rate targeting scheme the implied value is much lower, namely 1.

## VI. Summary and Conclusion

This paper addresses the issue of whether nominal income targeting is a viable strategy of monetary policy in the simple backward-looking model suggested by Ball (1997). Our findings imply that a hybrid form of

<sup>31</sup> The two policy frontiers intersect at  $a = .254$  and  $\mu = 23.29$ .

nominal income targeting, one where the policymaker aims at achieving a pre-specified target consisting of the sum of the output gap and the rate of inflation, does not lead to instability in the rate of inflation or real output evident in Ball's model. Under hybrid nominal income targeting both the variance of real output and the variance of the rate of inflation are finite even though monetary policy affects real output and the rate of inflation at different lags.

In this paper we also examine the circumstances under which the policymaker prefers some type of a nominal income target to a strict inflation target as the fulcrum of monetary policy. The merits of both strategies of monetary policy are evaluated in the context of the backward- and the forward-looking model.

A comparison of hybrid nominal income targeting to strict inflation targeting in the backward-looking model yields a U-shaped policy frontier. For most coefficient estimates reported in the literature, hybrid nominal income targeting is likely to dominate strict inflation targeting as a strategy for monetary policy.

Carrying out a comparison of the two strategies of monetary policy in a forward-looking model of the type suggested by McCallum (1997b), we trace out a very different policy frontier. The shape of the policy frontier now suggests a monotonic trade-off between the weight placed on the variance of inflation in the loss function and the parameter  $a$  in the Phillips curve. As the parameter  $a$  increases in size the strict inflation target becomes a more attractive strategy of monetary policy relative to the hybrid form of nominal income targeting.

Finally, we match a strategy of targeting the growth rate of nominal income against a strict inflation target in the forward-looking model. The shape of the policy frontier again suggests a monotonic tradeoff between the weight placed on the variance of inflation and the parameter in the Phillips curve. A strict inflation target is more likely to dominate this form of nominal income targeting than the hybrid form for given values of  $a$  which are approximately greater than .25.

In conclusion, while not establishing that different forms of nominal income targeting are superior to strict inflation targeting, this paper does rebut the argument that all forms of nominal income targeting are a disastrous strategy of monetary policy. We have seen that the relative attractiveness of either strategy depends on a number of factors, in particular on empirical estimates of the relevant parameters, and the specification of the baseline model. In view of these results further empirical

work on the appropriate specification of the Phillips Curve seems warranted. After all, only one of the two models or a combination thereof can be a fitting description of the actual economy.

## References

- Asako, Kazumi and Wagner, Helmut: "Nominal Income Targeting Versus Money Supply Targeting," *Scottish Journal of Political Economy*, 39 (May 1992), 167–87.
- Ball, Lawrence: "Efficient Rules For Monetary Policy," NBER Working Paper (5952), (1997).
- Bean, Charles R.: "Targeting Nominal Income: An Appraisal," *Economic Journal*, 93 (December 1983), 806–19.
- Bernanke, Ben S. and Mishkin, Frederic S.: "Inflation Targeting: A New Framework for Monetary Policy?," *Journal of Economic Perspectives*, 11 (Spring 1997), 97–116.
- Bryant, Ralph C.: "Alternative Rules for Monetary Policy and Fiscal Policy in New Zealand: A Preliminary Assessment of Stabilization Properties," Reserve Bank of New Zealand Discussion Paper (July 1996).
- Bryant, Ralph, Hooper, Peter, and Mann, Catherine, Eds.: *Evaluating Policy Regimes: New Research in Empirical Macroeconomics*, Washington, DC: The Brookings Institution (1993).
- Clarida, Richard, Jordi Gali, and Mark Gertler: (1999) "The Science of Monetary Policy: A New Keynesian Perspective," *Journal of Economic Literature*, 27 (December 1999), 1661–1707.
- Croushore, Dean and Tom Stark: "A Real-Time Data Set for Macroeconomists," Federal Reserve Bank of Philadelphia Working Paper no. 99–4 (June 1999).
- Frankel, Jeffrey and Chinn, Menzie: "The Stabilizing Properties of a Nominal GNP Rule," *Journal of Money, Credit, and Banking*, 27 (May 1995), 318–34.
- Friedman, Benjamin M. and Kuttner, Kenneth N.: "A Price Target for U.S. Monetary Policy: Lesson from the Experience with Money Growth Targets," *Brooking Papers on Economic Activity* (1996:4), 77–125.
- Fuhrer, Jeffrey C.: "The Phillips Curve is Alive and Well," *New England Economic Review*, (March/April 1995), 41–56.
- Hall, Robert E. and Mankiw, N. Gregory: "Nominal Income Targeting" in N. Gregory Mankiw, ed., *Monetary Policy*, Chicago, IL: University of Chicago Press (1994), 71–93.
- Henderson, Dale W. and McKibbin, Warren J.: "A Comparison of Some Basic Monetary Policy Regimes for Open Economies," *Carnegie-Rochester Conference Series on Public Policy*, 39 (1993), 221–317.
- Jensen, Henrik: "Targeting Nominal Income Growth or Inflation?," mimeo, University of Copenhagen, (June 1999).
- Koenig, Evan F.: "Optimal Monetary Policy in an Economy with Sticky Wages," *Economic Review*, Federal Reserve Bank of Dallas, Second Quarter (1995), 24–31.
- McCallum, Bennett T.: "Robustness Properties of a Rule for Monetary Policy," *Carnegie-Rochester Conference Series on Public Policy*, 29 (1988), 173–203.
- McCallum, Bennett T.: "A Semi-Classical Model of Price Level Adjustment," *Carnegie-Rochester Conference Series on Public Policy*, 41 (1994), 251–284.
- McCallum, Bennett T.: "New Zealand's Monetary Policy Arrangements: Some Critical Issues," Reserve Bank of New Zealand Discussion paper (June 1995a).
- McCallum, Bennett T.: "Panel Discussion" in *Goals, Guidelines, and Constraints Facing Monetary Policymakers*, Boston: Federal Reserve Bank of Boston (1995b).
- McCallum, Bennett T.: "Issues in the Design of Monetary Policy Rules," NBER Working Paper (6016), (1997a).
- McCallum, Bennett T.: "The Alleged Instability of Nominal Income Targeting," Reserve Bank of New

Zealand Discussion Paper (August 1997b). – *McCallum*, Bennett T., and Edward *Nelson*: “An Optimizing IS-LM Specification for Monetary Policy and Business Cycle Analysis,” *Journal of Money, Credit, and Banking*, 31 (1999), 296–316. – *Phillips*, A. W.: “Stabilisation Policy and the Time-Forms of Lagged Responses,” *Economic Journal*, 67 (1957), 265–277. – *Roberts*, John M.: “New Keynesian Economics and the Phillips Curve,” *Journal of Money, Credit, and Banking*, 27 (1995), 975–984. – *Rogoff*, Kenneth: “The Optimal Degree of Commitment to an Intermediate Monetary Target,” *Quarterly Journal of Economics*, 100 (1985), 1169–1189. – *Rudebusch*, Glenn D. and Lars *Svensson*, “Policy Rules for Inflation Targeting,” in John B. Taylor, ed., *Monetary Policy Rules*, Chicago, IL: University of Chicago Press (1999), 203–246. – *Svensson*, Lars: “Price Level vs. Inflation Targeting: A Free Lunch?,” NBER Working Paper (5719), (August 1996). – *Svensson*, Lars: “Inflation Forecast Targeting: Implementing and Monitoring Inflation Targets,” *European Economic Review*, 46 (1997a), 1111–1146. – *Svensson*, Lars: “Inflation Targeting: Some Extensions,” NBER Working Paper (5962), (March 1997b). – *Taylor*, John: “Discretion versus Policy Rules in Practice,” *Carnegie-Rochester Conference Series on Public Policy*, 39 (1993), 195–214. – *Taylor*, John: “The Inflation/Output Trade-off Revisited,” in *Goals, Guidelines, and Constraints Facing Monetary Policymakers*, Boston: Federal Reserve Bank of Boston (1995). – *West*, Kenneth D.: “Targeting Nominal Income: A Note,” *Economic Journal*, 96 December (1986), 1077–83. – *Woodford*, Michael: “Optimal Monetary Inertia,” NBER Working Paper (7261), (July 1999).

## Appendix

In the paper reference is made to the optimal monetary policy rule for the forward-looking model. The purpose of this appendix is to show how the optimal policy rule, the Taylor rule underlying it, and the time series processes for real output and the rate of inflation are derived. In addition, it is shown that hybrid nominal income targeting is a special case of the optimal monetary policy rule.

### *The Forward-Looking Model*

$$(1a) \quad y_t = -\beta r_t + E_t y_{t+1} + v_t$$

$$(1b) \quad \pi_t = E_t \pi_{t+1} + a y_t + u_t$$

The policymaker sets a fixed target for the real output gap and the rate of inflation. The parameter  $\theta$  indicates the weight the policymaker attaches to the output gap relative to the rate of inflation in the policy rule.

$$(2) \quad z^* = [\theta y_t + \pi_t] = 0$$

As shown below,  $\theta \geq 0$ . Hence the optimal value of  $\theta$  determines the trade-off between real output and the rate of inflation.

Inserting equations (1a) and (1b) into (2) and solving for  $r_t$  yields:

$$(3) \quad r_t = \frac{1}{\beta\theta} (E_t\pi_{t+1} + ay_t + u_t) + \frac{1}{\beta} (E_ty_{t+1} + v_t)$$

Substituting (3) into the IS relation (equation (1a)) results in:

$$(4) \quad (\theta + a)y_t = -E_t\pi_{t+1} - u_t$$

This equation shows how real output behaves after imposing the rule. Combine equation (4) with the evolution of the rate of inflation (equation 1b):

$$(5) \quad \pi_t = \frac{\theta}{\theta + a} (E_t\pi_{t+1} + u_t)$$

Next we pose putative solutions for the endogenous variables:

$$(6) \quad y_t = \tau_{11}u_t$$

$$(7) \quad \pi_t = \tau_{21}u_t$$

It therefore follows that

$$(8) \quad E_t\pi_{t+1} = 0$$

$$(9) \quad E_ty_{t+1} = 0$$

Inserting (7) and (8) into (5) and matching coefficients yields:

$$(10) \quad \tau_{21} = \frac{\theta}{\theta + a}$$

Hence the solution for the rate of inflation is

$$(11) \quad \pi_t = \frac{\theta}{\theta + a} u_t$$

Substituting equations (8) and (11) into equation (1b) and solving for  $y_t$  yields the expression for output:

$$(12) \quad y_t = -\frac{1}{\theta + a} u_t$$

It follows then that

$$(13) \quad \text{Var}(\pi)_t = \left(\frac{\theta}{\theta + a}\right)^2 \sigma_u^2 \quad \text{and} \quad \text{Var}(y_t) = \left(\frac{1}{\theta + a}\right)^2 \sigma_u^2$$

The objective of the policymaker is to minimize a loss function consisting of the variance of real output and the rate of inflation, respectively.

$$\text{Min}_{\theta} \text{Var}(y_t) + \mu \text{Var}(\pi_t)$$

The solution to the above is given by  $\theta = \frac{1}{\mu a} \geq 0$ . This setting represents the optimal choice for the policy parameter  $\theta$ .

We notice that the optimum value for  $\theta$  is a function of  $\mu$ , the weight on the variance of inflation in the loss function, and  $a$ , the parameter on the output gap in the forward-looking Phillips curve. The relationship between  $\theta$  and  $\mu$  is illustrated for three different values of  $a$  in Figure 1. An increase in the size of  $a$  shifts the curve downward, thus lowering  $\theta$ .

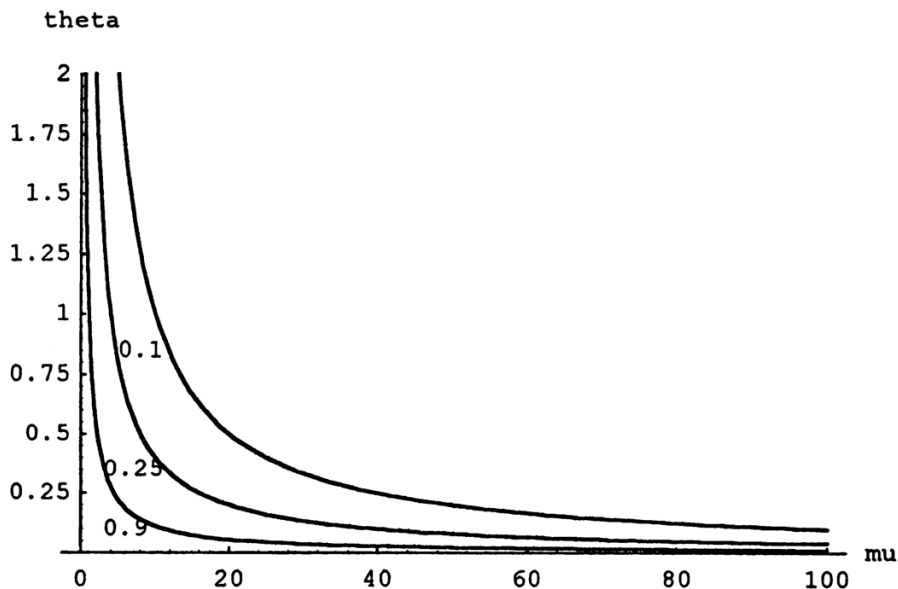


Figure 1: The relationship between  $\mu$  and  $\theta$  for  $a = 0.1, 0.25, 0.9$

*Efficient Monetary Policy Strategies*

## 1. Two Extreme Policies:

As  $\mu$  approaches infinity  $\theta$  goes toward zero. In this case the policy-maker would pursue a strict inflation target. Alternatively, as  $\mu$  approaches zero,  $\theta$  goes toward infinity, in which case the policymaker would pay attention only to deviations of output from capacity.

## 2. Hybrid Nominal Income Targeting:

This strategy imposes a unitary trade-off between deviations of real output from capacity and the rate of inflation. Hence the policymaker chooses  $\theta$  to equal one.

Letting  $\theta = 1$  in equation (13) produces the variances for real output and the rate of inflation reported in the main part of the paper.

Pursuing a hybrid strategy of nominal income targeting implies further that the policymaker places a weight of  $\mu = \frac{1}{a}$  on the variance of inflation in the loss function.

*Table 1*  
**The sensitivity of  $\mu$  to changes in  $a$**   
**under the hybrid nominal income targeting strategy ( $\theta = 1$ )**  
**in the forward-looking model**

$\mu = 10$	for $a = .1$
$\mu = 4$	for $a = .25$
$\mu = 1.11$	for $a = .9$

## Summary

### **Alternative Monetary Policy Rules and the Specification of the Phillips Curve: A Comparison of Nominal Income with Strict Inflation Targeting**

This paper shows that the instability of nominal income targeting in a simple backward-looking macro model disappears if the policymaker chooses to adopt a hybrid nominal income target, which is a special case of the optimal monetary policy. This form of nominal income targeting is compared to another form of optimal monetary policy, strict inflation targeting, so as to establish the conditions under which the former strategy is preferable to the latter. For most coefficient estimates reported in the literature hybrid nominal income targeting is likely to dominate strict inflation targeting as a strategy of monetary policy.

We also analyze the two strategies of monetary policy using a forward-looking specification as our baseline model. In contrast to the policy frontier based on the backward-looking model, this policy frontier is not U-shaped; instead it implies a monotonic trade-off between the relevant parameters. In this model the strict inflation target becomes more attractive relative to the hybrid nominal income target as the Phillips Curve parameter increases in size.

A strict inflation target is more likely to dominate a nominal income growth rate target than a hybrid nominal income target for certain values of the Phillips Curve parameter. (JEL E5)

## Zusammenfassung

### **Alternative Regeln der Geldpolitik und die Formulierung der Phillips-Kurve: Ein Vergleich zwischen der nominellen BSP-Zielsteuerung und der Inflationszielsteuerung**

Auf der Basis eines simplen „backward-looking“-Makromodells erläutert dieser Beitrag, wie eine auf BSP-Steuerung bedachte instabile Geldpolitik verhindert werden kann, indem die Notenbank ein Hybrid-BSP-Ziel anstrebt. Ein Hybrid-BSP-Ziel ist eine spezielle Form der optimalen Geldpolitik. Mittels eines Vergleichs wird dann belegt, unter welchen Voraussetzungen das Hybrid-BSP-Ziel einer anderen Form der optimalen Geldpolitik, einer die ein Inflationsziel anstrebt, überlegen ist.

Eine ähnliche Analyse der beiden geldpolitischen Strategien wird auch in einem „forward-looking“-Modell durchgeführt. Entgegen der U-förmigen Politikgrenzen (policy frontiers), die auf dem „backward-looking“-Modell basieren, implizieren die aus diesem Modell hervorgehenden Politikgrenzen einen monotonen Trade-off zwischen den relevanten Parametern des Modells. Je größer der Parameter der realen Outputlücke in der Phillips-Kurve, desto attraktiver gestaltet sich ein Inflationsziel im Vergleich mit dem Hybrid-BSP-Ziel.

Es ist eher wahrscheinlich, daß eine starre Geldpolitik, die auf einem Inflationsziel beharrt, einer Geldpolitik, die die *Wachstumsrate* des nominellen BSP – anstatt eines Hybrid-BSP – anstrebt, überlegen ist. Diese Schlußfolgerung hängt jedoch von der Größe des Parameters der Outputlücke in der Phillips-Kurve ab.

## Résumé

### **Règles alternatives de politique monétaire et la spécification de la courbe de Phillips: une comparaison entre les objectifs de revenu nominal et les objectifs d'inflation**

Sur base d'un simple modèle macro «backward-looking», l'auteur montre ici que l'instabilité de l'objectif de revenu nominal disparaît si la politique monétaire choisit d'adopter un objectif de revenu hybride qui est une forme spéciale de la politique monétaire optimale. Cette forme d'objectif de revenu nominal est comparée à une autre forme de politique monétaire optimale, l'objectif strict d'inflation, afin d'établir les conditions sous lesquelles la première stratégie est préférable à la seconde. Pour la plupart des estimations de coefficient rapportés dans la littérature, l'objectif de revenu nominal hybride semble être une meilleure stratégie monétaire que l'objectif strict d'inflation.

Une analyse similaire des deux stratégies de politique monétaire est également faite en se basant sur un modèle de «forward-looking». Contrairement à la frontière de la politique basée sur le modèle «backward-looking», cette frontière n'est pas en forme de U. Elle implique une alternance monotone entre les paramètres essentiels. Au plus le paramètre de la courbe de Phillips croît, au plus l'objectif strict d'inflation devient plus attrayant que l'objectif de revenu nominal hybride.

Un objectif strict d'inflation semble être une meilleure politique monétaire qu'une politique qui vise le taux de croissance du revenu nominal – au lieu d'un revenu hybride. Cette conclusion dépend cependant de la grandeur du paramètre de la courbe de Phillips.