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Towards a Theory of Central Bank Liquidity Management

Von Ulrich Bindseil*, Frankfurt/Main

I. Introduction

The provision of base money can be seen as the core practical task of any central bank and its main interface with the outside world. Short term interest rates follow directly from the interaction of the supply of base money with the demand for it, which is composed of the exogenous demand for banknotes and the banking sector's demand for reserves with the central bank. The "liquidity management" of a central bank is defined here as the framework, set of instruments and rules the central bank uses in steering the amount of bank reserves in order to control their price (i. e. short term interest rates) in consistency with its ultimate goals (e. g. price stability).

The central bank's balance sheet is a good starting point to explain the logic of liquidity management. When a central bank transacts with the rest of the world, it normally uses as a means of payment the currency it provides. The balance sheet therefore reflects the accumulated transactions of a central bank with the rest of the world. Table 1 displays a stylised balance sheet of a central bank.

From the point of view of liquidity management, the following four categories of items shall be distinguished:

- *Open market instruments* (balance sheet items A.1 and L.1). The open market instruments are used by the central bank in order to steer the quantity and therefore the scarcity of reserves.
- The *standing facilities* A.2 and L.2 may be considered as "autonomous monetary policy instruments" as their use is in principle under the discretion of banks: banks can at any time obtain liquidity through the

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Table 1

A Central Bank's Stylised Balance Sheet

Assets	Liabilities
1. Monetary policy instruments	*1. Monetary policy instruments*
1.1 Reverse operations	1.1 Reverse operations
1.2 Outright holdings of securities rel. to monetary policy	1.2 Time deposits
	1.3 Issuance of debt certificates
2. Marginal lending facility	*2. Deposit facility*
3. Autonomous liquidity factors	*3. Autonomous liquidity factors*
3.1 Net foreign assets[1]	3.1 Banknotes in circulation
	3.2 Government Deposits
	4. Current account holdings of banks (incl. Min. reserves)

marginal lending facility at a relatively high rate and deposit funds at the deposit facility for a relatively low rate.

- *Autonomous liquidity factors*. (items A.3 and L.3). They are related to central bank activities or services not determined by the central bank's liquidity management or by banks. However, as these transactions involve the same mean of payment, central bank money, transactions of this type have exactly the same liquidity providing or liquidity absorbing effect as monetary policy related transactions.
- The *current account holdings of banks (item L4)* at the central bank can be considered as a residual position balancing the central bank's balance sheet. All operations of the central bank with the rest of the world finally end in affecting the banks' current accounts (as long as they do not net out). By transforming the balance sheet identity, one sees that the current accounts (L4) are always determined by the following identity:

Current accounts = (asset items 1, 2, 3) – (Liability items 1, 2, 3)

Despite the crucial role the liquidity management of a central bank plays both as implementation of monetary policy and as determining the starting point of the term structure of interest rates, academic econo-

[1] Central banks may also use foreign assets as a monetary policy instrument.

mists have not paid much attention to this field. While more macro-economic aspects of central banking have witnessed veritable booms in recent years (e.g. the role of credibility, monetary vs. inflation targeting, the Taylor rule, etc.), the theory of the implementation of monetary policy mainly remained a domain of central bankers. While central banks have often motivated their liquidity management framework and strategy, these motivations are mostly more intuitive than rigorously theoretical.[2] The theory of central bank liquidity management has to be clearly distinguished from the macro-economic literature starting with the seminal paper by Poole (1970) (for a recent survey see Walsh (1998), Chapter 9). Central bank liquidity management as understood here concerns the shortest end of implementation of monetary policy, and assumes that macro-economic shocks that require some policy response do so only by affecting the central bank's operational target (overnight) rate, but in the very short run under consideration, these shocks never reveal themselves in the equilibrium value of the variable which is considered as the operational target. The Poole (1970) – literature also deals with "operating procedures" and "the instrument-choice" problem. However, this literature focuses in fact more on the shorter term macro-economic strategy of the central bank than on the actual day-to-day implementation of monetary policy, and does not deal with what central bankers normally understand under instruments and procedures of monetary policy. This orientation is also revealed by the fact that the attempts of calibration of the corresponding models are normally undertaken with monthly or quarterly data.

However, at least three strains of academic literature are nevertheless contributing directly to the theory of central bank liquidity management:

- The specification of the **reserve requirement** regime determines the liquidity management of a central bank in a pervasive way. While the literature on reserve requirements is extensive, it is in most cases macro-oriented, and does not focus on averaging which is the most relevant feature from the point of view of liquidity management. However, some studies also focus specifically on the role of reserve requirements with averaging for the market for reserves and short term rates (see Bindseil (1997) for a survey).

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[2] See for example *Deutsche Bundesbank* (1995) for the case of Germany and *European Central Bank* (1998) for the euro area. Furthermore, *Bank of International Settlements* (1997) contains a collection of descriptions of systems of implementation of monetary policy in the main industrialised countries prepared by central bankers.

- Several studies have examined empirical characteristics of overnight rates (e.g. the federal funds rate) in connection with liquidity effects and the role of the maintenance period for reserve requirements (e.g. Angeloni and Prati (1996), Ayuso, Haldane, Restoy (1994), Hamilton (1996), Bartolini, Bertola, Prati (1998)). The main empirical finding on which these studies focus is the increased volatility and mean of short term rates at the end of the maintenance period for fulfilling reserve requirements.
- Finally, several studies have provided models of the daily market for reserves held by banks with the central bank which rationalise the empirical findings of the literature mentioned in the previous bullet point (e.g. Baltensperger (1980), Ho and Saunders (1985), Spindt and Hoffmeister (1988) Kopecky and Tucker (1993), Hamilton (1996), Bartolini, Bertola, Prati (1998)).

The aim of this study is to build up on this literature to provide some elements of a basis of a normative theory of central bank liquidity management. Compared to the theoretical literature mentioned in the last bullet point, the objective of the approach taken here will be to (1) focus mainly on the point of view of the central bank and to discuss liquidity management from a normative point of view; (2) to distinguish between the choice of the operational framework (e.g. a reserve requirement system) and the choice of a liquidity management strategy, while integrating the analysis of both issues; (3) to put the information asymmetries between the central bank and the banking sector at the centre of the analysis; (4) to stress that, similarly to any other monetary policy analysis, the empirical analysis of liquidity management is subject to the Lucas critique.

Section II. will briefly examine the main parameters of the framework in which liquidity management takes place. This framework can mainly be shaped by the central bank and, since interacting with the liquidity management strategy chosen by the central bank, has to be integrated into the analysis. Section III. reviews the main factors shaping the overnight rate in an environment of reserve requirements with averaging. Section IV. turns to the liquidity management strategy of a central bank, completing the tools needed for a normative analysis of this central bank activity. Section V. outlines the approach to be taken by a normative theory of liquidity management and provides a basic example. Finally, section VI. draws conclusions.

II. The Framework of Central Bank Liquidity Management

1. Elements of the Framework

This section will review very briefly the main elements of the framework of the day-to-day implementation of monetary policy, limiting itself to mentioning the basic dimensions of this framework without going deeper into any analysis. Three main elements can be distinguished. Of these, the central bank directly determines two, namely the menu of monetary policy instruments and the reserve requirement system. The third element of the framework consists in the institutional features of the inter-bank money market.

A descriptive overview of the menu of monetary policy instruments is provided, for example, by the ECB in its document "The single monetary policy in Stage Three – General Documentation on ESCB monetary policy instruments and procedures". This note will not go into details concerning these aspects since it will mainly focus on the quantitative side of the supply of central bank money, simply assuming that efficient central bank instruments to withdraw or supply liquidity at any point in time are available to the central bank.

2. The Reserve Requirement System

The size and time structure of the fulfilment of reserve requirements determines the market for bank reserves and the central bank's liquidity management in a fundamental way. The central bank has the choice to determine this time structure, and therefore, a normative theory of liquidity management has to encompass this question. The following four dimensions of the specification of the reserve requirement system can be distinguished:

1. *Discrete points in time which are relevant for the calculation of banks' fulfilment of (zero) reserve requirements*. Most central banks currently have a system in which end of day levels of reserve holdings are relevant for the calculation of reserve requirements. Intra-day holdings are in general not relevant. Call "reserve points" the points in time relevant for the fulfilment of (zero) reserve requirements.
2. *Averaging allowed in fulfilling reserve requirements*. Many central banks, especially those with a reserve requirement system, have opted for imposing certain reserve holdings only on *average* for a certain

sequence of the reserve points. For example, in the euro area, banks have to hold at minimum zero reserves at every reserve point (at 18.30 each day), but they have to fulfil positive reserve requirements on average within one month, encompassing all reserve points between the 24th of each month and the 23rd of the following month.

3. *Difference between the average reserve requirement in the maintenance period and the minimum reserve holding at each reserve point.* This determines how far it is relevant to distinguish reserve points from end of maintenance periods. The larger this difference, the more relevant it is to distinguish between them. Normally, this difference corresponds to the reserve requirement, since central banks impose a no-overdraft constraint on reserve points. However, other solutions were also adopted, for example, the Bank of Canada practised for some years a system of averaging around zero, and the Banca d'Italia allowed only a part of reserve holdings to be eligible for averaging.

4. *Overdrafts allowed (and possible) in between reserve points.* In between reserve points, banks may or may not be allowed to overdraft their accounts up to a certain limit. Even if they are entitled to unlimited overdrafts, the availability of collateral may set a limit to intra reserve point (normally: "intra-day") overdrafting, The availability of collateral can be influenced by the central bank which may accept more or fewer types of bank assets as collateral.[3]

In sum, the reserve requirement system can be described by a vector $W = (q, u, v, w)$, where q is length of the maintenance period (in days), u is the number of reserve points in the maintenance period (assuming that reserve points are always evenly distributed within the maintenance period for fulfilling reserve requirements), v is the difference between the reserve requirement and the no-overdraft threshold relevant at reserve points, and w is the aggregate overdraft limit of the banking sector in between reserve points.

Consider the following examples, assuming that sufficient collateral is available and that intra- reserve point overdrafts are allowed (w is very large).

(1) $(q, u) =$ *(1,1)*. This is for example the configuration of the Bank of England (and of other central banks without positive reserve

[3] In the US, banks may overdraft their accounts with the federal reserve system without providing collateral.

requirements and averaging[4]). There is one reserve point per day which coincides with the daily closing of the payment system (note that whenever $u = 1$, w is per definition = 0).

(2) $(q, u) = (1/86400, 1)$. In this system, every second is a reserve points, and the day is the maintenance period. Reserves would be scarce at every moment in time, and the relative scarcity within the day would be determined by the payment system activities in the different periods of the day. Intra-reserve point overdrafts are impossible due to the high frequency of reserve points. Inter-bank credits would have to have a precisely defined maturity, i.e. from 13.15 to 17.25. The cost of a credit would depend on the exact length of a contract in terms of number of reserve points it would encompass. Such a system has not yet been observed in reality. However, for instance in the US, intra-day overdrafts are not free and therefore the US model has some elements of this specification.[5]

(3) $(q, u) = (31,1)$. In this system, there is only one reserve point per month, allowing banks to maintain (collateralised) overdrafts of their current account for up to this period. Again, this system has not yet been observed in reality.

(4) (q, u, v) = *(31,31, "large").* This system of daily reserve points and a maintenance period of one month has been applied for example in Germany for many years and has been adopted by the European Central Bank. The current system of the Federal Reserve may be characterised by the vector (14,14, "medium").

(5) (q, u, v) = *(1, 86400, "large").* In this system, every second is a reserve point, and the maintenance period encompasses one day. As in option (2), intra-reserve point overdrafts are impossible under this setting since the period in between the reserve points is too short. This system also has never been observed.

(6) (q, u, v) = *(365, 12, "large").* Here, every end of month is a reserve point, and the maintenance period is one year long. Again, such a system has never been observed.

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[4] Note that the Bank of England imposes a small reserve requirement. However, no averaging is allowed. In any case, even a system with reserve requirements of zero is one with "reserve requirements" since reserve of at least zero have to be maintained at the end of the day. Under this terminology, "no reserve requirements" would mean that there is no demand for reserves at all, and therefore also no positive price for reserves.

[5] See for example *Summers* (1994).

What determines a central bank's choice of one of these options? The granting of averaging provisions in the fulfilment of reserve requirements to banks has been advocated in the literature as a means of smoothing short term interest rates (e.g. Poole (1968), Baltensperger (1980), Bindseil (1997), Davies (1998)). Options (2) and (5) have probably never been chosen due to their huge computational requirements. Option (6) may have been excluded since the controllability of short term rates and of the credit expansion in between reserve points is too low under this regime. The same is probably true for (3). The choice between (1) and (4) is an issue of ongoing debate between central bankers. Option (4) allows to achieve a high degree of stability of overnight rates with relatively few central bank operations. However, advocates of (1) stress that neither it is clear why the stability of overnight rates is an objective per se, nor why the central bank should refrain from conducting open market operations on a daily basis. This paper will not further discuss these questions, which are however core issues of a comprehensive theory of liquidity management.

3. *The Inter-bank Market for Reserves*

The organisation and efficiency of the interbank market for reserves has implications for the mapping of the aggregate reserve holding path in the maintenance period into overnight rates and is therefore also of importance for liquidity management. The following list of main features of the inter-bank market may constitute a starting point for further analysis:

- Number of institutions participating to the money market, their size structure and respective activities;
- Nature, size and stochastic structure of "exogenous" payment systems flows (e.g. customers transactions, longer term refinancing and investment strategy of the banks' treasuries), likelihood and stochastic structure of payment failures.
- Matching procedures (telephone trade vs. electronic matching systems, parallel systems).
- Structures developing between market participants: Trading relationships, limits, clusters of trading partners, asymmetry in services offered (for example specialisation of market participants to market makers).

- Instruments used (deposits, repos, outright operations in money market paper).
- Time structure of intra-day activity in the money market.
- The payment system in general.

Descriptions of money markets range from Bagehot's (1873/1962) *Lombard Street* to Pulli (1992) or Hamilton (1996). The current note does not further describe or analyse the institutional details of the interbank market for central bank money, and instead simply assumes its efficient working.

III. The Optimisation Behaviour of Banks and the Market for Reserves: the Case of Considerable Reserve Requirements with Averaging

The analysis is restricted now to the case of reserve requirements with averaging in a maintenance period of T days and one reserve point per day. Most of the tools employed in the following can however be transferred to other settings.

In case of a system of minimum reserves with averaging, banks face an inter-temporal optimisation problem when minimising the cost of holding required reserves over the maintenance period (the maintenance period is defined as the period encompassing all reserve points for which averaging applies). The opportunity cost of holding reserves on one reserve point is the over-night rate. Therefore banks will have an incentive to hold reserve surpluses (build up reserve deficits) whenever the market rate is low (high) relative to future expected reserve point-rates within the maintenance period. This behaviour by banks will tend to stabilise market rates as, in order for the market to clear, over-night rates will tend to be aligned with future expected overnight rates within the maintenance period.

This section will examine some basic relationships between the supply of reserves by the central bank, the evolution of autonomous liquidity factors (such as banknotes in circulation), and the overnight rate. This will form the basis to examine liquidity management strategies in the following section.

In the first subsection, the factors shaping the evolution of the overnight rate are examined under the assumptions that the overnight rate target of the central bank is fixed and well known and that the interbank market works perfectly. In subsection 2, the assumption that the overnight rate target of the central bank is known and constant is dropped. Finally, subsection 3 focuses on the relevance of imperfections of the inter-bank market.

Adopt the following notation in the remainder of this paper:

$t = 1 \ldots T$	Days of the maintenance period (the maintenance period is assumed to have T days)
$m_t;\ M = \sum_{t=1}^{T} m_t$	Amount of outstanding open market operations on day t of the maintenance period[6]; Sum of daily amounts of monetary policy operations outstanding
$a_t;\ A = \sum_{t=1}^{T} a_t$	Autonomous factors outstanding on day t of the maintenance period (e.g. banknotes); Sum of daily amounts of monetary policy operations outstanding in the maintenance period
ml_t	Recourse to marginal lending facility on day t
d_t	Recourse to deposit facility on day t
r_t	Reserve holdings of banks with the central bank on day t of the maintenance period;
$R = \sum_{t=1}^{T} r_t$	Sum of daily average reserve holdings in maintenance period
$v;\ V = Tv$	Daily reserve requirement; Total required reserves in the maintenance period (T times the average daily requirements)
$i,\ i_t^*,\ i_d, i_{ml}$	Overnight rate on day t; overnight rate target the central bank has on day t for the rest of the maintenance period; rate of deposit facility; rate of marginal lending facility
$E_t^{b[cb]}(X)$	Value expected for a variable X by the banking sector (the central bank) on day t
$f_t^{b[cb]}(X)$	(Homogenous) expectations of banks (the central bank) on day t concerning a random variable X as expressed through the density function $f(\,)$ they assume for this variable.

Furthermore, assume that the sequence of events on each business day has the following order:

1. At the start of the day, if any, the central bank conducts its open market operation (with same day settlement), the result of which (in the form of the then outstanding volume of operations m_t) becomes known immediately to the public.
2. the perfectly efficient inter-bank market takes place and the market clearing rate i_t is fixed.

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6 Assume, to keep the model as simple as possible, that all operations are outright operations, i.e. have an infinite maturity, such that the amount of outstanding monetary policy operations stays constant if no further operations take place.

3. the realisation of the daily amount of autonomous factor, a_t, takes place and becomes publicly known.
4. end of day settlement, including the recourse to the standing facility.

1. *Perfect Inter-bank Markets and a Well-known Operational Target*

Assume for the sake of simplicity that the central bank and the banking sector have the same expectations concerning the evolution of autonomous factors. Under the assumption of a perfect inter-bank market and risk neutral banks, the following equation (1) will determine the evolution of the overnight rate within the maintenance period:

$$i_t = E_t^b(i_{i+1}) = \ldots = E_t^b(i_T) = i_d\left(\int_{-\infty}^{V} f_t^b(M-A)\cdot d(M-A)\right) + i_{ml}\left(\int_{V}^{\infty} f_t^b(M-A)\cdot d(M-A)\right) \tag{1}$$

The subjective density function the banking sector assigns to its total reserve holdings in the maintenance period ($R = M - A$) determines the overnight rate since it determines the respective likelihood that one of the two standing facilities will be used at the end of the maintenance period. The more funds the central bank provides, the more likely it appears to banks that the *deposit* facility will have to be used at the end of the maintenance period and, thus, the lower overnight rates will be already at the moment where the expectations are formed. The less funds it provides, the more likely it appears to banks that the *marginal lending* facility will have to be used at the end of the maintenance period, and, thus, the higher overnight rates will be already at the moment where the expectations are formed.

In this abstract setting, a central bank can precisely steer the overnight rate until the day τ, the day of its last operation, by always adjusting the amount of outstanding open market operations on that day τ, such that the equation above returns the target rate of the central bank.

Under the described conditions, *it is irrelevant what the central bank does and how autonomous factors evolve within the maintenance period prior to* τ (the day of the last operation), since banks can anticipate i_τ, and, through intertemporal arbitrage, will always implement this rate also on prior days within the same maintenance period. If the central bank adopts the strategy to always conduct a fine-tuning operation on day T (the last day of the maintenance period), it can perfectly steer the overnight rate in the entire maintenance period. Generally, the average day-to-day volatility of overnight rates within the maintenance period would depend in such a setting only on when the central bank conducts on average its last monetary policy operation. The closer the last operation is on average to the end of the maintenance period, the lower would be the average volatility.

2. The Basic Signal Extraction Problem

It can generally be assumed that the central bank has superior information with regard to all relevant uncertain supply parameters: it has better forecasts of autonomous factors; it controls its open market operations; it knows more about its intentions with regard to the rates of the standing facilities. However, the actions of the central bank may reveal part of its superior information. This creates a signal extraction problem for counterparties: by observing the actions of the central bank (e. g. its allotment decisions in open market operations, statements of members of the Governing bodies with regard to intentions to change central bank rates, etc.), the banks will be able to extract some part of the Central Bank's superior information. However, this information will be noisy as long as, assuming a linear relationship between observed and unobserved variables, the number of observed variables is lower than the number of unobserved ones. To allow a very basic representation of the core signal extraction problem on the money market, take the following simplifying assumptions:

- The maintenance period consists of exactly one day ($T = 1$). On this day, the sequence of events is as follows: first, the central bank conducts its open market operations, the allotment amount being immediately published. Second, the inter-bank market takes place and the overnight rate is fixed that clears the market. Third, the realisation of autonomous factors takes place. Finally, the banks take recourse to standing facilities.
- Autonomous factors are white noise, i. e. $a = \varepsilon + \eta$, with ε, η being identically and independently distributed random variables with variance $\mathrm{var}(\varepsilon) = \sigma_\varepsilon^2 \in [0, 1]$, $\mathrm{var}(\eta) = 1 - \sigma_\varepsilon^2$, such that $\mathrm{var}(a) = 1$ (the index for the day of the maintenance period, t, is dropped). The central bank is assumed to have perfect forecasts of ε, but it has no prior

information on η. Banks are assumed to have no prior information on any of the two variables.

- The rate of the deposit facility is set to zero and the rate of the marginal lending facility to 1, for the sake of simplicity.
- In deciding on the allotment volume in its open market operation, the central bank takes into account its autonomous factor forecasts and to where, in the corridor set by standing facilities, it wants to steer the overnight rate. To simplify calculations, it is assumed that the central bank has no operational target in the form of the overnight rate, but in the form of a liquidity surplus or deficit at the end of the maintenance period, denoted γ. The higher the target liquidity deficit, the higher will be the likelihood of a recourse to the marginal lending facility, and the closer the overnight rate will be to the rate of this facility, at least to the extent anticipated by counterparties. As the central bank observes permanently news on the state of the macroeconomy and on dangers to price stability, the targeted γ may change. Assume that γ is white noise, i.e. a random variable with an expected value of zero and a variance of σ_γ^2.
- Excess reserves and reserve requirements are zero.

The signal extraction problem of banks in this minimalist setting can be described as follows: When observing the allotment decision of the central bank, banks are aware that the allotment amount reflects two unobserved stochastic variables that are linked through a linear relationship to the observed amount. Knowing the two unobserved variables separately would be, however of crucial interest for counterparties since this would allow to know more about the marginal value of reserves at the end of the maintenance period, and therefore about their "fair" price in the inter-bank market session. For instance, an ample allotment decision may simply reflect the fact that the central bank has a high autonomous factor forecast of ε, the innovation in γ having been negligible. Then, the fair overnight rate would remain at its previous levels, reflecting the unchanged likelihood of recourse to the marginal lending facility. If however it has no information on autonomous factor flows, i.e. ε is close to zero, then the ample allotment decision would indicate a change of monetary policy intentions, and the fair overnight rate would move downwards. Formally, this simple signal extraction problem can be described as follows. Counterparties observe the allotment amount m and know the linear structure:

$$m = v + \gamma \tag{2}$$

Applying the standard signal extraction formula, one obtains the following estimators for the unobserved variables (the "b" in the superscript indicates that those are the expectations formed by banks):

$$(3) \qquad E^b(\gamma \mid m = m_0) = \frac{\sigma_\gamma^2}{\sigma_\gamma^2 + \sigma_\varepsilon^2}\, m_0, \qquad E^b(\varepsilon \mid m = m_0) = \frac{\sigma_\varepsilon^2}{\sigma_\gamma^2 + \sigma_\varepsilon^2}\, m_0$$

The variances of the errors of the estimates will be:

$$(4) \qquad E\left(\gamma - E^b(\gamma)\right)^2 = \sigma_\gamma^2 - \frac{\sigma_\gamma^4}{\sigma_\gamma^2 + \sigma_\varepsilon^2}, \qquad E\left(\varepsilon - E^b(\varepsilon)\right)^2 = \sigma_\varepsilon^2 - \frac{\sigma_\varepsilon^4}{\sigma_\gamma^2 + \sigma_\varepsilon^2}$$

The overnight rate in the inter-bank market will amount to:

$$(5) \qquad i = P((m - a \mid m = m_0) < 0)$$

Hence, using the Gaussian (standard normal) cumulative density function G(), we can write:

$$(6) \qquad i = 1 - G\left(\frac{\left(1 - \frac{\sigma_\varepsilon^2}{\sigma_\gamma^2 + \sigma_\varepsilon^2}\right) m_0}{\sqrt{1 - \frac{\sigma_\varepsilon^4}{\sigma_\gamma^2 + \sigma_\varepsilon^2}}}\right)$$

Since the signal extraction is noisy, the interest rate will in fact not correspond, day after day, to the interest rate compatible with γ, i.e. the interest rate that would reflect full knowledge of this variable. There are four potential ways out of this signal extraction problem, all of which consist in making the signal extraction trivial:

- The central bank may **publish its autonomous factor forecast** ε. Then, the observation of m can be mapped without noise into γ, the target of the central bank with regard to the end of maintenance liquidity deficit, and into an adequate overnight rate.

- The central bank may **ignore its autonomous factor forecasts**, i.e. it may simplify its allotment rule to $m = \gamma$. Then, again, the linear relationship would map only one unobserved into one observed variable, and extracting the unobserved one would again be trivial. However, this strategy would come at the price of a higher variance of the recourse to the standing facilities, since the central bank would not neutralise any longer the anticipated autonomous factor shocks.

- The central bank may **announce its target** end of maintenance liquidity surplus γ, or alternatively, the inter-bank rate compatible with this expected liquidity deficit at the end of the maintenance period. This solution was adopted for instance by the Federal Reserve System in 1995. Fixed rate tenders (as employed so far by the Eurosystem in its main refinancing operations) may also be viewed as an implicit announcement of the target average overnight rate.
- Finally, the central bank may stick to a strategy to always target the mid of the corridor, i.e. to always set $\gamma = 0$, or at least to never change its policy towards γ. Since it is possible to adjust the rates of the standing facilities at any degree of precision, one may argue that it is never required for monetary policy reasons to fine-tune the position of short term rates within the corridor through a specific liquidity policy.

IV. The Central Bank's Liquidity Management Strategy

Again, the analysis in this section is restricted to the case of reserve requirements and averaging in a maintenance period of T days. However, once more, the proposed tools could be applied in a similar way to a maintenance period of 1 day in which the intra-day evolution of expectations concerning reserve holdings at end of the day would play a similar role as the intra-maintenance period evolution of the expectations concerning the average reserve holdings in the maintenance period.

A *liquidity management strategy* of a central bank is defined here as a function mapping relevant information available to the central bank (on the evolution of autonomous factors within the maintenance period, on the evolution of reserve holdings, on its overnight rate target, etc.) into a liquidity supply through monetary policy instruments. The strategy may be distinct for different days of the maintenance period or for different days during the week. We continue to focus only on the liquidity supply path and not on different types of operations (e.g. outright vs. reverse operations).

The idea of a liquidity management "strategy" of the central bank does not imply a total pre-commitment of the central bank, i.e. to opt totally for "rules" instead of for "discretion" in the implementation of monetary policy. It reflects the concept that there are some systematic elements in each liquidity management approach and that if all these systematic components that relate the liquidity management decisions of the central bank to specific "information" variables are translated into a strategy, the residual components of the liquidity supply should be non-correlated (orthogonal) to those specific variables. Therefore, in the equation specifying the mapping of information variables into the liquidity supply path, the other components can be treated as "noise".

In the following, a series of possible liquidity management strategies will be proposed to illustrate the concept.

(1) One Operation at the Beginning of Maintenance Period

In this strategy, the central bank changes the amount of liquidity supplied only once per maintenance period, namely on the first day, in order to provide an amount of liquidity which on that day implements an expected overnight rate of i^*.

(7)
$$m_2 = \left(V + z\,(i_T^*) + \sum_{t=1}^{T} E_1^{cb}(a_t)\right)/T$$
$$\forall t \in [2, \ldots T] : m_t = m_{t-1}$$

The function $z()$ maps the overnight rate target into a liquidity surplus relative to the liquidity needs as determined by the sum of reserve requirements and autonomous factors. This function can be derived from the equations given in section III.

This strategy, which is purely *forward looking* (in the sense of never taking account past developments), allows to save costs of frequent monetary policy operations. However, if the central bank's forecast of autonomous factors are relatively bad, it does not allow to steer the interest rate closely during the whole maintenance period: if news on the autonomous factors emerge, rates will drift more and more away from the target rate and towards one of the standing facility rates.

(2) Weekly Regular Operations

This strategy foresees one regular operation per week and is both backwards and forward looking:

If day t is a weekday other than the settlement day, then:

$$m_t = m_{t-1}$$

If day t is a specific weekday, the settlement day, and if $t-7<1$:

$$m_t = v + z\,(i^*_T) + \left(\sum_{j=1}^{t-1}(a_j + v - m_j) + \sum_{j=t}^{t+6} E^{cb}_t(a_j)\right)/7 \tag{8}$$

If day t is a specific weekday, the settlement day, and if $0<t-7$ and $t+7<T$:

$$m_t = v + z\,(i^*_T) + \left(\sum_{j=t-7}^{t-1}(a_j + v - m_j) + \sum_{j=t}^{t+6} E^{cb}_t(a_j)\right)/7$$

If day t is a specific weekday, the settlement day, and if $t+7>T$:

$$m_t = v + \left(\sum_{j=t-7}^{t-1}(a_j + v - m_j) + \sum_{j=t}^{T} E^{cb}_t(a_j) + z\,(i^*_T)\right)/(T-t)$$

This strategy aims in each weekly operation to neutralise any surplus or deficit that accumulated due to errors in autonomous factor forecasting in the course of last week, and to target, in any operation except the last one of the maintenance period an accumulated reserve surplus of zero on the day preceding the day of the settlement of the next operation. In the last operation, the central bank would target a zero accumulated reserve surplus on the last day of the maintenance period, plus the surplus liquidity supply depending on the target overnight rate.

(3) Daily Operations

The following strategy foresees daily open market operations:

$$\begin{aligned} &m_1 = v + E^{cb}_t(a_t) + z\,(i^*_T)/T \\ &\forall\, t > 1: \quad m_t = v + a_{t-1} - E^{cb}_{t-1}(a_{t-1}) + E^{cb}_t(a_t) + z\,(i^*_T)/T \end{aligned} \tag{9}$$

(4) Fine Tuning Interventions on Day T Whenever Autonomous Factor Forecasting Errors Reach a Certain Threshold

Assume a strategy which is identical to strategy (2) expect that, if T is not the settlement day of the weekly operation, then:

$$\begin{aligned} &\text{if}\left(\text{abs}\left(\sum_{j=1}^{T-1}(a_j + v - m_j)\right) - z\,(i^*)\,(T-1)/T\right) > \zeta, \\ &\text{then:}\;\; m_T = v + \left(\sum_{j=1}^{T-1}(a_j + v - m_j)\right) + E^{cb}_T(a_T) + z\,(i^*_T)/T \\ &\text{else:}\;\; m_T = m_{T-1} \end{aligned} \tag{10}$$

This strategy foresees a fine tuning operation on the last day of the maintenance period whenever autonomous factor forecasts appear to have been too much out of line on the days preceding the last day of the maintenance period. In this strategy, fine tuning has a "real anchor" in terms of autonomous factor developments. This is not the case in strategy (6). Note that one could imagine variants of strategy (5) in which fine tuning would take place even earlier in the maintenance period, possibly with threshold that would decrease in the course of the maintenance period.

(5) Fine Tuning Whenever the Overnight Rate is out of a Narrow Corridor

Assume again a strategy which is identical to strategy (3) expect that, if T is not the specific day of the weekly operation, then:

$$\begin{aligned} &\text{if } \left(\text{abs}\left(i_{T-1} - i_T^*\right)\right) > \xi, \\ &\text{then: } m_T = v + \left(\sum_{j=1}^{T-1} (a_j + v - m_j)\right) + E_T^{cb}(a_T) + z\left(i_T^*\right)/T \\ &\text{else: } \quad m_T = m_{T-1} \end{aligned} \tag{11}$$

Again, one could imagine that thresholds for fine-tuning are applied already earlier in the maintenance period. Note that in the strategy suggested, the liquidity injection or reduction is again determined by autonomous factor flows relative to forecasts, which, in some cases, could even go in the wrong direction (i.e. interest rates are above the target rate, but liquidity is withdrawn through fine tuning). One could therefore imagine the following alternative:

$$\begin{aligned} &\text{if } \left(\text{abs}\left(i_{T-1} - i_T^*\right)\right) > \xi, \quad \text{then: } m_T = m_{T-1} + (i_{T-1} - i_T^*)\theta \\ &\text{else: } \quad m_T = m_{T-1} \end{aligned} \tag{12}$$

The factor θ could even be rather small: then, such fine tuning operations may still have the purpose to signal policy intentions of the central bank in case banks have no good estimate of the central bank's operational target in the current maintenance period. A liquidity injecting fine tuning operation would then simply be a signal that current overnight rates reveal that banks may have over-estimated the central bank's overnight rate target, and a liquidity absorbing one would simply be a way to tell banks the contrary.

Note that all of the strategies (1) to (5) have implicitly assumed that the strategy of the central bank expresses itself only in terms of the quantity path of liquidity supply through monetary policy instruments. While it is clear that at the end of the maintenance period, only quantity matters, signalling through other means than quantities (e.g. through the choice of certain instruments of certain tender procedures, through verbal messages) may also play a role within the maintenance period.

Obviously, a vast number of liquidity management strategies could be envisaged. However, depending on the assumptions taken concerning the inter-temporal arbitrage, many of these strategies turn out to be equivalent. For example, assuming perfect inter-temporal arbitrage and large reserve requirements, all strategies for which the equation

$$m_T = V + \sum_{t=1}^{T-1} a_t - \sum_{t=1}^{T-1} m_t + E_T(a_T) + z\,(i_T^*) \tag{13}$$

holds, should lead (for any realisation of autonomous factors that does not push banks to the no-overdraft constraint, and if i_T^* is well known) to an identical path of the overnight rate.

V. Towards a Theory of Liquidity Management

After having outlined some key elements needed to analyse liquidity management, this section will assemble these elements to describe briefly the approach a theory of liquidity management could take. It then proceeds with a basic example of the choice of a single parameter of a liquidity management strategy.

Putting together the tools provided in the former two sections allows to specify the mapping of exogenous variables (autonomous factors, overnight rate targets, etc.) into the main endogenous variable – the path of short term interest rates, or more generally, into time series properties of these short term interest rates. A normative theory of liquidity management requires that the different properties of the endogenous variables can be ranked through some social welfare function or through the preferences of a specific actor (e.g. the central bank) whose point of view we want to take. Nowadays, maintaining price stability is normally regarded as the primary ultimate objective of monetary policy. The link from this ultimate objective to the implementation of monetary policy is determined by the views taken on the transmission mechanism of monetary policy. Interest rates are normally seen to provide the main link between the central bank's monetary policy and macro-economic aggregates such as consumption, investment and prices: through controlling the supply of bank reserves, the central bank is able to control very short term interest rates. Expectations on short term interest rates determine via inter-temporal interest rate arbitrage also longer term rates, which are deemed to be more relevant for influencing the key macro-economic aggregates. Hence, the central bank can influence longer term rates (and steer its ultimate objective) the better, the better it can control expectations of short term interest rates. This implies for two reasons that the central bank will be keen on being able to control short term rates through an adequate set of instruments and an adequate liquidity management strategy: Firstly, the possibility to control short term rates seems to be a pre-condition for being able to influence expectations concerning short term rates; Secondly, since changes to short term rates are normally not purely transitory, they can have some impact on longer term rates (however, the extent of this naturally depends on the framework and on the liquidity management strategy). Therefore, the possibility to control short term rates will be a major element of the central bank's preferences which will determine its choice of an operational framework and a liquidity management strategy. This does not imply that the central bank will systematically steer overnight rates and avoid any volatility of short term rates. However, the central bank will always monitor the evolution of short term rates, and a volatility in short term rates which feeds into a volatility of longer term rates will trigger action.

Unfortunately, preferences of central banks in choosing their operational framework and liquidity management strategy are only rarely stated or analysed. A recent exception to this was the so-called "Framework Report" by the European Monetary Institute (1997, 14 - 15), in which the question of which general principles should guide the selection of the operational framework was explicitly raised. From this report and from more general considerations, it seems that the main elements of a modern central bank's preferences in choosing its operational framework and strategy can be summarised in the following three bullet points. Ceteris paribus, a central bank prefers

- to be able to control short term interest rates, which includes the possibility to steer the overnight rate precisely if deemed necessary;
- to be able to give signals of monetary policy intentions (and therefore to influence other rates along the yield curve)
- simple, transparent and cost-efficient arrangements, which includes a preference for a low frequency of monetary policy operations;

Translating those preferences into a concrete choice of an operational framework and a liquidity management strategy is, due to the number of parameters to be specified and the complexity of the mapping of these choice parameters into an achievement of the ultimate objectives, rather complex. The mapping to be specified by a normative theory of liquidity management is summarised in the following table 2:

Table 2

Mapping the Environment and the Central Bank's Preferences into a Liquidity Management Framework and a Strategy

Parameters of the environment: – time series properties of autonomous factors – time series properties of interest rate targets – institutional details of inter-bank money market – "natural" information asymmetry between the central bank and banks, for instance concerning autonomous factors.		**Liquidity management framework:** – the reserve requirement system incl. the time structure of fulfilling reserve requirements – the standing facilities – specification of banks, eligible collateral, types of instruments, tender procedures, etc. – measures affecting the organisation and efficiency of the inter-bank market
Preferences of the central bank: – aversion against lack of control of short term rates; – aversion against frequent monetary policy operations; – preferences to be able to give signals of monetary policy intentions (and therefore to influence other rates along the yield curve) – preference for simple, transparent and cost-efficient arrangements	$\Rightarrow$	**Liquidity management strategy:** – parameters of the mapping of information variables into (1) a liquidity provision path (2) an information policy (3) the choice of instruments & procedures in the different operations (4) other actions (e.g. adjusting rates of standing facilities, etc.)

The ultimate goal of a normative theory of liquidity management would be to construct and calibrate a model encompassing this entire mapping at once. To obtain a first idea of how the realisation of this program would look like, I will give in the following a simplistic example illustrating how one input parameter (preferences of the central bank towards the volatility of the overnight rate and the frequency of operations) is mapped into one parameter of its liquidity management strategy (namely an optimal threshold for fine tuning operations), for given values of all other input and output parameters.

Assume in this example that other input factors are specified as follows:

- perfectly efficient inter-bank market and payment systems
- time series properties of autonomous factors: no serial correlation between autonomous factor changes; on each day, autonomous factors are $N(0,\ \sigma_a^2)$-distributed
- Central banks and banks have no prior knowledge on autonomous factors
- The central bank assigns to each open market operation a social welfare cost of k.
- At the same time, it assigns a welfare cost of q to each basis point standard deviation of the deviation of the overnight rate from its target rate.

Furthermore, assume that the other choice parameters are pre-specified as follows:

- $(q, u, v) = (3, 3, v)$, i.e., the maintenance period is three days long, every end of day is a reserve point, overdrafting is not allowed and reserve requirements amount to v on average per day. The reserve requirement is assumed to be large in comparison to the autonomous factor volatility, such that the likelihood that the banking sector is pushed into overdraft can be ignored.
- A marginal lending and a deposit facility are available, at rates i_{ml}, i_d, respectively.
- The overnight rate target of the central bank is constant at $(i_{ml} + i_d)/2$
- On the first day, the central bank always conducts a "regular" operation, covering exactly the expected sum of autonomous factors (which equals zero) and the reserve requirement

Given these specifications, the central bank wants to specify a threshold X triggering a fine tuning intervention on the third day that maximises social welfare. The threshold is defined in terms of the autonomous factor forecasting error that occurred on the first two days of the maintenance period. Therefore, the liquidity management strategy of the central bank would be determined by the following equations:

$$
\begin{aligned}
& m_1 = v \\
& m_2 = m_1 \\
& \text{if } \left(|a_1 + a_2| < X\right) : m_3 = m_2 \\
& \text{if } \left(|a_1 + a_2| \geq X\right) : m_3 = m_2 + a_1 + a_3
\end{aligned}
\tag{14}
$$

First, the relationship between the threshold X and the volatility of overnight rates has to be calculated. It is useful to distinguish for this purpose between the three days of the maintenance period.

On **day 1,** the overnight rate is always $(i_{ml} + i_d)/2$. This follows from the fact that expectations are still totally symmetric, since no information on autonomous factor shocks has become known. The volatility of overnight rates is therefore zero.

Since the autonomous factor shock on day one contains information on $(A - M)$, the overnight rate on **day 2** will be determined by the experienced autonomous factor shock. To obtain the relation between the autonomous factor shock and overnight rates, the density functions $f(A - M \mid a_1)$ have to be determined. Unfortunately, these functions are no longer normally distributed once the central bank sets a threshold for intervention. If the sum of the first two autonomous factor shocks goes beyond the threshold, both will be neutralised totally by a fine tuning operation. Therefore, the likelihood of autonomous factor shocks going beyond the threshold is in fact reassigned to the outcome that the first two autonomous factor shocks are zero. Since such density functions are rather difficult to handle analytically, I took recourse to simulations, further specifying that $i_{ml} = 10$; $i_d = 2$; $\sigma_a = 1$. Figure 1 in the graphical annex provides for different thresholds, a graph displaying the different interest rates for the observed values of a_1. It is worth noting that for certain domains of a_1, $\partial i/\partial a_1 > 0$, i.e. the larger a_1 (i.e. the more liquidity is withdrawn through the autonomous factor shock), the lower the interest rate. The reason for this behaviour of the overnight rate is that large values of the first day shock make a fine tuning operation of the central bank likely, and the increase of likelihood of intervention when moving to larger absolute values of the observed autonomous factor shocks is more relevant than the increased absolute value of $E(A)$. Figure 3 of the graphical annex displays the relationship between the size of the threshold and the standard deviation of interest rates on day 2.

On **day 3:**

– if the central bank has intervened, which is the case with probability:

$$\int_{-\infty}^{-X} f(a_1+a_2)\,d\,(a_1+a_2)+\int_{X}^{\infty} f(a_1+a_2)\,d\,(a_1+a_2)=F\,(-X)+1-F\,(X), \tag{15}$$

then the overnight rate will be $(i_d + i_{ml})/2$.

– if the central bank has not intervened, the overnight rate will equal:

$$i_3 = i_d\left(\int_{-\infty}^{-(a_1+a_2)} f(a_3)\cdot da_3\right)+i_{ml}\left(\int_{-(a_1+a_2)}^{\infty} f(a_3)\cdot da_3\right) \tag{16}$$

Again, we take recourse to simulation for our specific example. Figure 2 of the graphical annex displays the relationship between observed autonomous factors and interest on day 3. Figure 4 of the graphical annex displays standard deviations of overnight rates on the third day as a function of the threshold for intervention. Figure 5 displays the standard deviation on all days of the maintenance period taken together, again as a function of thresholds.

For choosing the optimal level of the threshold, the central bank has to balance the advantages of operations (in the form of a reduced variability of overnight rates) against its aversion against fine tuning operations, i.e. the cost it assigns to such operations. Let $w(X)$ and $\sigma_i(X)$ be the functions mapping the size of the threshold into an average frequency of operations and a standard deviation of overnight rates, respectively. The cost of fine tuning per day will then equal $k.\ w(X)$, which is a monotonously falling function in X. The cost of interest rate volatility equals $q\,\sigma_i(X)$ and increases monotonously with X. Therefore, a unique equilibrium exists. In Figure 6 of the graphical annex, both cost functions are displayed for values of $k = 0.3$ and $q = 1$. In this case, the optimal threshold for fine tuning equals 2.2. This minimalist example reveals the potential complexity of a more realistic normative theory of liquidity management: Despite the fact that all environmental parameters were kept as simple as possible and the choice problem was reduced to one parameter, the solution of the problem required careful "manual" analysis of various effects.

VI. Conclusions

This paper outlines an approach to analyse the liquidity management of central banks. First it highlighted the relevance of the framework for liquidity management and in particular of the time structure of the fulfilment of reserve requirements. Since this framework can also be determined to a large extent by the central bank, it has to be analysed together with the actual implementation of monetary policy. Then, the note reviewed some elements of the optimisation calculus of banks towards the fulfilment of reserve requirements in a reserve requirement system with averaging. After this, the concept of a liquidity management strategy of the central bank was introduced and a series of examples of possible strategies was given. Finally, the different tools were assembled to analyse a simple example of the parameter choice by the central bank concerning its liquidity management strategy with pre-specified values of all other relevant parameters.

The discussion of all topics remained preliminary and incomplete. In the pursuit of the analysis, a more extensive treatment of the following issues would appear to be of priority:

- The role of the payment system and, more generally, of the non-perfection of inter-bank markets to understand the choices of central banks towards the setting of reserve points. Specifically, a better understanding why other time structures of fulfilling reserve requirements are not observed (e.g. several reserve points per day) seems in part to be related to these issues.
- The link of the liquidity management to the transmission mechanism of monetary policy and the central bank's pursuit of its ultimate objectives (e.g. price stability). How far is this pursuit affected by the choice of the operational framework? Certainly, the less reserve points and the longer the maintenance period, the less precisely the central bank will be able to steer the relevant instrument variables. Generally, the link from liquidity management to macro-economics seems to be closely related to the (endogenous) transmission of changes in short term rates to changes in longer term rates.
- A more rigorous analysis of the optimisation calculus of banks within the maintenance period in case that both aggregate and individual liquidity shocks play an important role.
- More refined liquidity management strategies of central banks, taking signalling considerations more explicitly into account.

- Integrating the different monetary policy instruments into the analysis (reverse transactions, outright operations, different tender procedures, etc.)

The basic example in section V. demonstrated that the analysis of optimal liquidity management strategies quickly becomes rather complex even under the strongest simplifying assumptions, suggesting to operate with models treating only a few elements at once. A series of partial models based on a common platform of basic assumptions may be the most productive way to reach a deeper understanding of the economic issues at stake and their interaction. The basic example suggested that simulation techniques are likely to play an important role in analysing the properties of these models.

Empirical work will also have to play a crucial role in progressing to a theory, which could be of use to rethink current arrangements from a new perspective:

- A broad comparative review of central bank's frameworks and strategies and their motivation would help detecting crucial factors shaping the central banks' choices and of possible "local optima" leading possibly to similar results despite a rather different appearance.
- The analysis of relevant observed time series to possibly calibrate models of liquidity management. This is obvious for exogenous variables (as for example autonomous factors), but raises the usual problems concerning the endogenous variables (this is illustrated for example by the relationship between autonomous factor shocks and interest rates in the basic example provided in section V.).

Central bank liquidity management has so far not been a topic extensively analysed by academic economists, which is in contrast to the wide literature on more macro-economic aspects of central banking. In so far, a theory of liquidity management would contribute to a more complete picture of central banking, encompassing the main link of central banks with the outside world. Furthermore, the development of a theory of central bank liquidity management raises a variety of questions of immediate theoretical interest for the understanding of the mechanics of money markets, payment system, and the nature of liquidity.

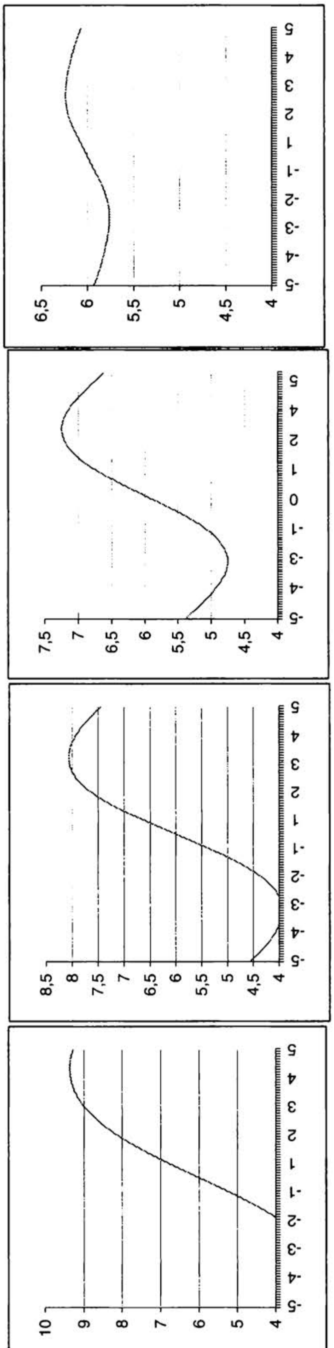

Figure 1: Interest Rates on Day 2 as a Function of Observed Autonomous Factors (a_1).

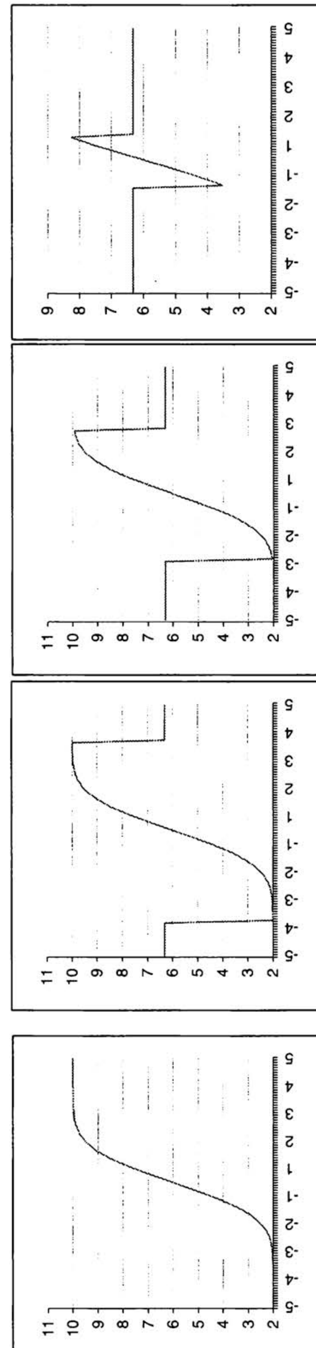

Figure 2: Interest Rates on Day 3 as a Function of Observed Autonomous Factors $(a_1 + a_2)$.

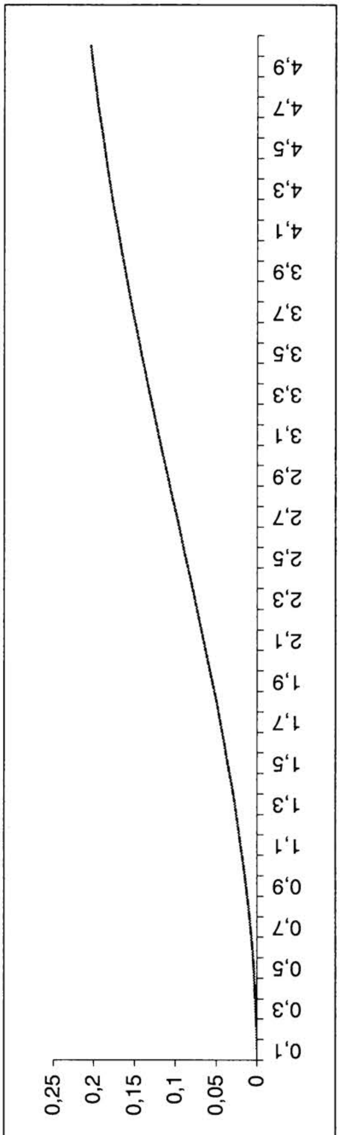

Figure 3: Standard Deviation of Interest Rate on Day 2 as a Function of Threshold of Intervention

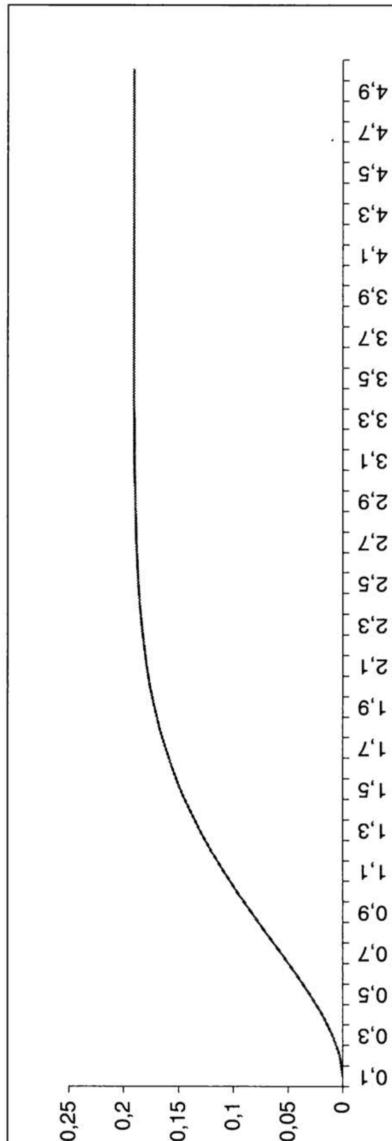

Figure 4: Standard Deviation of Interest Rate on Day 3 as a Function of Threshold of Intervention

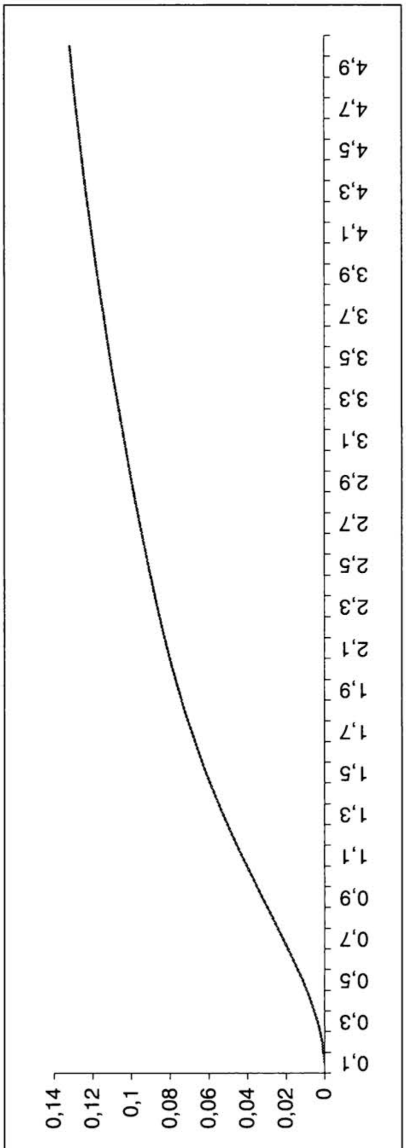

Figure 5: Standard Deviation of Interest Rate in Entire Maintenance Period as a Function of Threshold

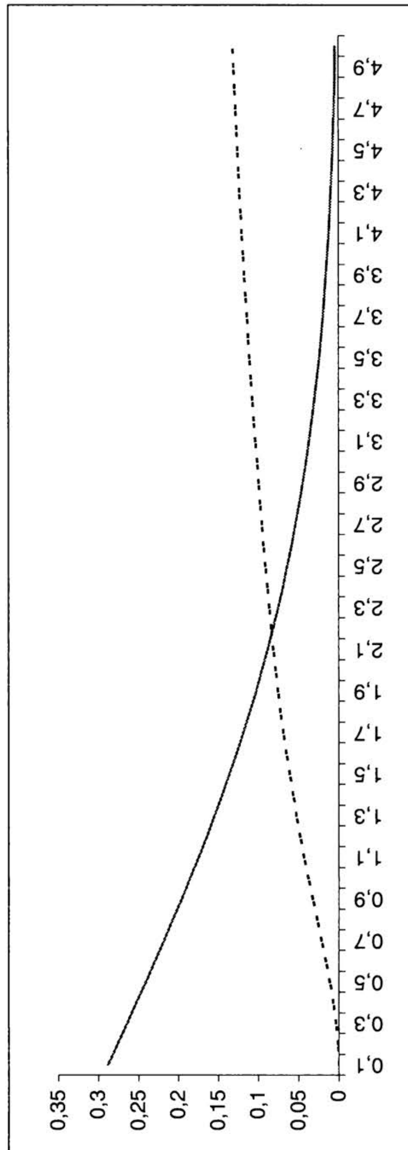

Figure 6: Costs of Fine Tuning Operations per Day vs. Cost of Volatility per Day ($k = 0.3$ and $q = 1$)

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Summary

Towards a Theory of Central Bank Liquidity Management

The "liquidity management" of a central bank is defined as the framework, set of instruments and rules the central bank uses in steering the amount of bank reserves in order to control their price (i.e. short term interest rates) in consistency with its ultimate goals (e.g. price stability). This paper discusses the basic tools of a tentative theory of liquidity management. The time-structure of fulfilling reserve requirements is identified as the crucial dimension of the "framework" for liquidity management. The paper then gives an overview of key elements explaining how the optimisation behaviour of banks shapes overnight rates in a system with reserve requirements and averaging. The paper continues by specifying the concept of "liquidity management strategy" and by providing examples of such strategies. Finally, based on the preceding elements, it outlines the approach that a normative theory of liquidity management could follow and provides a basic example as illustration. (JEL E 52)

Zusammenfassung

Elemente einer Theorie der Liquiditätssteuerung von Notenbanken

Die "Liquiditätssteuerung" einer Notenbank wird definiert als der Rahmen, die Instrumente und Regeln, die die Notenbank zur Steuerung des Angebots an Bankreserven, und damit zur Steuerung von deren Preis (d.h. insb. des Tagesgeldsatzes) einsetzt. Der vorliegende Beitrag diskutiert einige Elemente einer Theorie der Liquiditätssteuerung. Die Zeitstruktur bei der Erfüllung der Reservehaltung wird als Schlüsselgröße des Rahmens der Liquiditätssteuerung identifiziert. Anschließend wird die Rolle des Optimierungskalküls der Banken innerhalb der Reserveerfüllungsperiode verdeutlicht und das Konzept der "Strategie" der Liquiditätssteuerung eingeführt. Zuletzt werden die Grundzüge einer normativen Theorie der Liquiditätssteuerung entworfen und mit einem einfachen Beispiel illustriert.

Résumé

Eléments d'une théorie de la gestion des liquidités des banques centrales

La "gestion des liquidités" d'une banque centrale est définie comme le cadre, les instruments et les règles que la banque centrale met en place pour contrôler l'offre de réserves bancaires et par-là, pour réguler leurs prix (c'est-à-dire avant tout le taux de l'argent au jour le jour). Cet article discute quelques éléments d'une théorie de la gestion des liquidités. La structure dans le temps en cas de la réalisation des réserves est identifiée comme grandeur-clé de la gestion des liquidités. Ensuite, le rôle du calcul d'optimisation des banques au cours de la période de réalisation des réserves est expliqué et le concept de "stratégie" de gestion des liquidités est introduit. Finalement, les bases d'une théorie normative de la gestion des liquidités sont posées et illustrées à l'aide d'un exemple simple.