A Striking Result or just a Matter of Misinterpretation?

Comment on "Inflation Rates and Money Growth During High-Inflations"

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In a recent paper in this journal Gersbach (1993) uses a simple monetary model to analyze the implications of an increasing inflation rate – as observed during hyperinflation – on the money growth rate. Assuming rational expectations he derives a negative relationship between the current money growth rate and the future inflation rate instead of the well-known positive one. So "very high inflation requires strongly negative nominal money growth ..." (p. 234). According to the author this explains the observed low correlation between inflation and money growth rates. Unfortunately, his analysis has a major problem. While Gersbach's equations are algebraically correct, his "endogenization" of future expected inflation is not adequate.

We begin by rewriting his model in a slightly different form. Using the same notation as Gersbach we start from a money demand function for a closed economy (1), where m denotes the nominal money stock, p the price level, c and d are constants. For convenience the variables m and p are defined in logs. E represents the expectation formed in period t.

(1) $$m_t = c + p_t - d\,E(\pi_{t+1})$$

Equation (2) defines the expected inflation rate as the difference between the price level expected for the next period and the current price level.

(2) $$E(\pi_{t+1}) = E(p_{t+1}) - p_t$$

Equation (3) is the definition of the money growth rate.

(3) $$\mu_t = m_t - m_{t-1}$$

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Combining (1) with (3) and assuming that all variables up to period t are known to the market participants gives

$$\pi_t = \frac{1}{1+d}\,\mu_t + \frac{d}{1+d}\,E(\pi_{t+1}), \tag{4}$$

$$\text{with } E(\pi_{t+1}) = E(\pi_t) + \Delta E(\pi_{t+1}).$$

The actual inflation rate is a weighted sum of current money growth and the next period's expected inflation. Anticipation of higher future inflation increases the current inflation rate. In the steady state, which is characterized by constant money growth, $\Delta E(\pi_{t+1}) = 0$ and we have

$$E(\pi) = \mu, \tag{5}$$

hence

$$\pi = \mu. \tag{6}$$

Gersbach, however, solves for the expected inflation rate $E(\pi_{t+1})$. In our notation his solution corresponds to

$$E(\pi_{t+1}) = \frac{1+d}{d}\,\pi_t - \frac{1}{d}\,\mu_t. \tag{7}$$

Writing equation (4) this way conveys the impression that the rational expectation of future inflation is a positive function of previous inflation and a negative function of money growth. Nevertheless, this is false because the rational expectation of future inflation is a function of expected future money growth. We will derive the correct solution below. Gersbach's next step is to set $E(\pi_{t+1}) = \pi_{t+1}$, giving

$$\pi_{t+1} = \frac{1+d}{d}\,\pi_t - \frac{1}{d}\,\mu_t. \tag{8}$$

This implies that future inflation is a positive function of current inflation and a negative (!) function of current money growth. Thus Gersbach asserts that "for any inflation rate π_t, the lower μ_t, the higher becomes π_{t+1}" (p. 233). However, Gersbach's manipulation of equation (4) is seriously misleading. Equation (4) determines current inflation, not future inflation. Agents form expectations of future inflation on the basis of future expected money growth and adjust real balances accordingly. As a result the current inflation rate emerges which equilibrates the money market.

The correct determination of rational expectations is to solve for the expected inflation by forward substitution of equation (4).

$$E(\pi_t) = \frac{1}{(1+d)}\left[E(\mu_t) + \frac{d}{(1+d)}E(\mu_{t+1}) + \frac{d^2}{(1+d)^2}E(\mu_{t+2}) + \cdots + \frac{d^\infty}{(1+d)^\infty}E(\pi_{t+\infty})\right] \tag{9}$$

It can be seen that expected inflation is a positive function of future expected money growth, independently from assuming constant or varying money growth rates.

Gersbach then considers accelerating inflation defined by

$$\pi_{t+1} = k\pi_t, \ \ k > 1, \tag{10}$$

and investigates, which money growth can sustain continuously increasing inflation. He comes to the conclusion that "an accelerating inflation process, starting from low inflation will be only compatible with negative money growth rates, if agents anticipate the evolution of money growth rates" (p. 234). This is false. Under rational expectations expected inflation has to increase by the same factor k as actual inflation, i.e.

$$E(\pi_{t+1}) = kE(\pi_t). \tag{11}$$

As can be seen from equation (9) this is only the case if expected money growth increases by the identical rate k. With an increasing money growth rate $\mu_{t+1} = k\mu_t$ expected inflation is given by

$$E(\pi_t) = \frac{1}{(1+d)}E(\mu_t)\left[1 + \frac{kd}{(1+d)} + \frac{(kd)^2}{(1+d)^2} + \cdots\right] + \frac{d^\infty}{(1+d)^{\infty+1}}E(\pi_{t+\infty}). \tag{12}$$

As long as

$$\frac{kd}{(1+d)} < 1, \tag{13}$$

the sum on the right hand side converges against a fixed limit[1] and the current inflation is

$$\pi_t = \frac{1}{1+d(1-k)}\mu_t. \tag{14}$$

1 If this condition is violated $E(\pi_{t+1})$ approaches infinity.

Instead of the steady state relation we get a constant relationship between accelerating inflation and accelerating nominal money growth depending on k. Inflation is the higher, the higher is the expected acceleration in money growth. Inflation and money growth both increase with factor k from period t onwards, as accelerating money growth induces period-to-period decreases in real money demand which is brought about by next period's expected inflation being higher than this period's. If k increases the inflation rate jumps to a new level because an immediate reduction in money demand is induced by higher expected inflation.

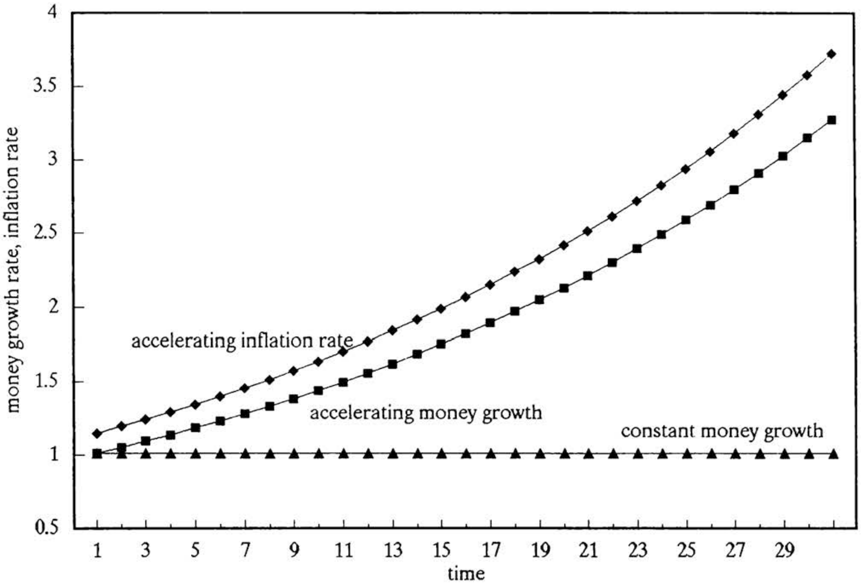

Fig. 1: Inflation paths with constant and increasing money growth

Fig. 1 illustrates the relation between inflation and money growth. With real income unchanged a constant money growth rate leads to an identical inflation rate while accelerating money growth causes increasing inflation which exceeds the money growth rate.

Gersbach's diverging result thus is due to a confusion concerning realized and expected inflation rates.

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References

Gersbach, Hans, 1993: "Inflation Rates and Money Growth During High-Inflations", Kredit und Kapital, Vol. 26, No. 2, 230 - 237.

Summary

Comment on "Inflation Rates and Money Growth During High-Inflations"

This comment refers to a paper by Gersbach (1993) on the relationship between money growth and inflation. Contrary to Gersbach's result it is shown that fully anticipated accelerating money growth leads to accelerating inflation. Starting from a money demand function of the Cagan-type, the forward looking rational expectations solution is derived and the relation between money growth and inflation is established.

Zusammenfassung

Die Vereinbarkeit von negativem Geldmengenwachstum mit Hyperinflation

In seinem Aufsatz kommt Gersbach (1993) zu dem Ergebnis, daß Hyperinflation mit negativem Geldmengenwachstum vereinbar sei. Im Gegensatz hierzu wird in unserem Kommentar gezeigt, daß nur akzelerierendes Geldmengenwachstum zu ständig steigender Inflation führt. Ausgehend von einer Geldnachfragefunktion vom Cagan-Typ wird die Lösung für rationale Erwartungen hergeleitet und der Zusammenhang zwischen Geldmengenwachstum und Inflation analysiert.

Résumé

Commentaire sur les taux d'inflation et la croissance monétaire en périodes de forte inflation

Gersbach (1993) conclut dans son article que l'hyperinflation est compatible avec une croissance monétaire négative. Nous montrons ici qu'au contraire, seule une croissance monétaire accélérée fait augmenter l'inflation de manière constante. A partir d'une fonction de demande monétaire de type Cagan, la solution pour des attentes rationnelles est déduite et le rapport entre la croissance monétaire et l'inflation est analysé.