

# The Message in Daily West German Stock Prices: Empirical Evidence Using GARCH\*

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## I. Introduction

In the recent literature in financial economics, there is yet no agreement regarding the best stochastic process that describes stock returns. As early as 1963, *Mandelbrot* (1963) observed the puzzling fact that stock return series tend not to be independent over time, but characterised by succession of stable and volatile series, i.e. "... large changes tend to be followed by large changes – of either sign – and small changes tend to be followed by small changes ...". In addition to this volatility clustering he also observed that the distributions of returns are leptokurtic and therefore proposed the family of Paretian distributions as an alternative to the normality assumption. Such Paretian distributions with characteristic exponents of less than two indeed exhibit heavy tails and conform better to the distributions of stock return series.<sup>1</sup> While the approach of these studies is based on the empirical fit of observed stock return distributions, an alternative approach relies on describing the process that could generate distributions of returns having fatter tails than normal distributions. *Praetz* (1972), *Blattberg* and *Gonedes* (1974), for example, showed that the scaled- $t$  distribution, which can be derived as a continuous variance mixture of normal distributions, fits daily stock returns better than infinite variance stable Paretian distributions. Other models using different mixtures of normal distributions to generate distributions

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<sup>1</sup> *Fama* (1963) and (1965) contributed further evidence supporting *Mandelbrot's* hypothesis.

that would account for the higher magnitude of kurtosis, are, among others, the mixed diffusion-jump processes (Poisson mixtures) of Press (1967), Akgiray and Booth (1986) and Akgiray et al. (1989) and the discrete mixtures of Kon (1984). Furthermore, Epps and Epps (1976) and Tauchen and Pitts (1983) present models in which the distribution of variance is a function of the arrival of new information, the trading activity and the trading volume.

One of the most recently proposed classes of return generating processes in the literature that can capture the above distributional shape is the class of Auto-Regressive Conditional Heteroscedastic (ARCH) processes pioneered by Engle (1982).<sup>2</sup> As far as stock markets are concerned, the class of ARCH models has mainly been applied to American stock markets. Our contribution in this paper is to show whether such models can adequately describe the stock price behaviour of the West German capital markets which are much smaller and thinner than the American ones.

The plan for the rest of the article is as follows. Section II provides a description of the GARCH process. Section III describes the data and comments on outlying data observations. Section IV describes the empirical results and the final Section V concludes with suggested future research directions.

## II. Models with time-dependent conditional heteroscedasticity

The autoregressive conditional heteroscedasticity (ARCH) model introduced by Engle (1982) can basically be seen as an extension of the traditional linear model when the conditional variance of the error term is allowed to change over time. Allowing for different extensions of the basic ARCH model [Bollerslev (1986), Engle et al. (1987)], the general form of the univariate model can be represented as in the following two equations.

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<sup>2</sup> The ARCH process or the generalised ARCH (GARCH) process of Bollerslev (1986) obtained large-scale support in empirical studies. See, for example, French, Schwert and Stambaugh (1987), Akgiray (1989), Baillie and Bollerslev (1989), Hsieh (1989) and Baillie and De Gennaro (1990). A comprehensive survey of various applications and extensions is given in Engle and Bollerslev (1986). An alternative estimator which is based upon the robust estimation techniques proposed by Davidian and Carroll (1987) and which has properties similar to Engle's ARCH estimator has recently been developed by Schwert (1989).

$$(1) \quad y_t = x_t \beta + \delta h_t + \varepsilon_t \quad \varepsilon_t \mid \Omega_{t-1} \sim N(0, h_t)$$

$$(2) \quad h_t = \alpha_0 + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^p \phi_i h_{t-i}$$

The own conditional variance  $h_t$  enters in addition to the vector of explanatory variables,  $x_t$ , as regressor in the equation for the conditional mean. The parameters of the model (vector  $\beta$ ,  $\delta$ ,  $\alpha_0$ ,  $\alpha_i$  and  $\phi_i$ ) are obtained simultaneously using iterative maximum likelihood estimation techniques.<sup>3</sup> The variance  $h_t$  of the error term  $\varepsilon_t$  is obtained conditional on the information set  $\Omega$  available at time  $t - 1$ .<sup>4</sup> In the model  $\Omega_{t-1}$  is assumed to be a vector consisting of past error terms, i.e.  $\Omega_{t-1} = \{\varepsilon_{t-1}, \varepsilon_{t-2}, \dots, \varepsilon_{t-q}\}$ . The standard linear model can now be obtained as a special case of the model in equations (1) and (2), if the constraints  $\delta = 0$ ,  $\alpha_i = 0$  and  $\phi_i = 0$  are imposed. The original ARCH model suggested by Engle (1982) did not allow for a time varying risk premium to influence the conditional mean, i.e. the ARCH ( $q$ ) model assumed  $\delta = 0$ , and a less general specification of the conditional variance function, i.e.  $\phi_i = 0$ .<sup>5</sup> The GARCH ( $p, q$ ) model introduced later by Bollerslev (1986) imposes smoother behaviour on the conditional second moments and is actually an infinite order ARCH model with exponentially decaying weights for large lags. The sum of  $\Sigma \alpha_i + \Sigma \phi_i$  in the conditional variance equation measures the persistence of the volatility. Bollerslev (1986) has shown that if this sum is equal to one, the GARCH process becomes an integrated GARCH or IGARCH process.<sup>6</sup> If there is an allowance for a non-zero  $\delta$  the ARCH-in-mean (ARCH-M) or GARCH-in-mean (GARCH-M) model is obtained.<sup>7</sup> A further generalisation of the ARCH-M and GARCH-M model proposed by Hall (1990) is obtained by dropping the extreme assumption that the conditional variance given in equation (2) is an exact *non-stochastic* relationship. The alternative is

<sup>3</sup> Hall (1990) contains a survey of econometric issues related to the estimation of ARCH and GARCH models. The asymptotic theory for ARCH models is given in Weiss (1986).

<sup>4</sup> Note that by assumption equation (2) is a *non-stochastic* relationship.

<sup>5</sup> Note that  $\varepsilon_t$  are not autocorrelated. But the fact that the variance of  $\varepsilon_t$  depends on  $\varepsilon_{t-1}^2$  gives a misleading impression of there being a serial correlation. If we estimate (1) by OLS we will find a significant DW statistic because of the ARCH effects in (2).

<sup>6</sup> Nelson (1990) has shown that the IGARCH process is stationary and ergodic although the variance is unbounded.

<sup>7</sup> See Bollerslev, Engle and Wooldridge (1988).

the *stochastic* GARCH-in-mean model (SGARCH-M) in which the conditional variance is specified as

$$(3) \quad h_t = \alpha_0 + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^p \phi_i h_{t-i} + \omega_t \quad \omega_t \sim N(0, \sigma_t^2)$$

where  $\omega_t$  is a serially uncorrelated zero-mean disturbance with variance  $\sigma_t^2$ . Furthermore it is assumed that the two disturbances  $\varepsilon_t$  and  $\omega_t$  are independent of each other in all time periods. When  $\sigma_t^2 = 0$  the SGARCH-M specification collapses to the traditional GARCH-M model. In other words, the SGARCH-M specification is a fairly general model from which the more common alternative can be derived as a special case (“encompassing”).<sup>8</sup>

The usefulness of the general GARCH and SGARCH approach in areas such as financial market modelling now seems overwhelming. As already noticed above, it allows estimation of models containing time varying risk premiums, without having to use proxy variables as risk measures. Another reason for the GARCH models usefulness in applications to financial markets relates to the assumed probability distribution of the returns. Evidence indicates that stock returns are not normally distributed but leptokurtic. As shown in *Diebold* (1986) and *Milhoj* (1986) ARCH processes are leptokurtic compared to the normal distribution, which makes the ARCH models specially suitable candidates for the analysis of financial market data.

### III. Data and descriptive statistics

#### 1. Data

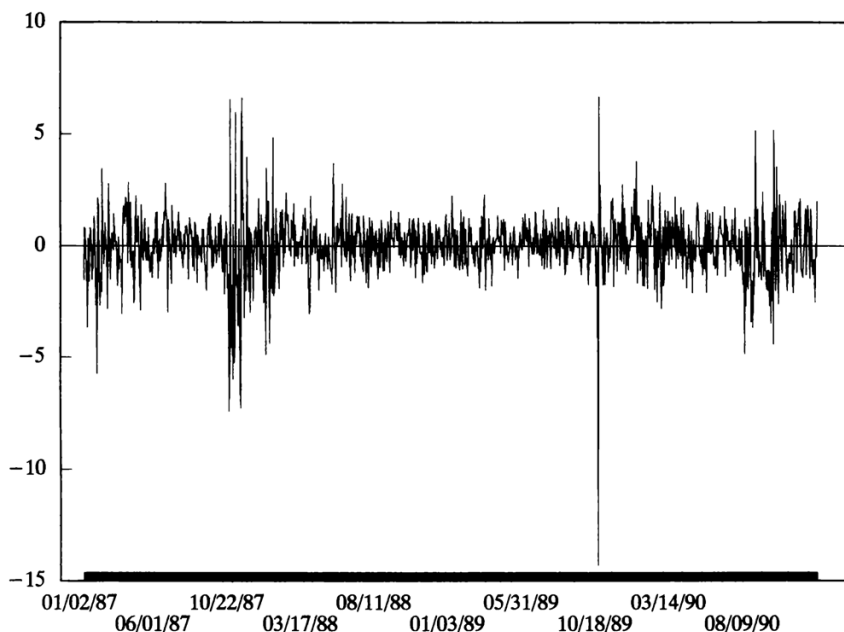
We study daily stock returns for the period 1987/1/2 to 1990/12/31.<sup>9</sup> The data are spot prices of the West German value-weighted all sector share price index of the *Frankfurter Allgemeine Zeitung* (FAZ). The

<sup>8</sup> Another generalisation of the traditional ARCH model has recently been presented in *Higgins* and *Bera* (1992).

<sup>9</sup> Most previous studies have used monthly data. The highly active capital markets might imply that the persistence of the effects of shocks to volatility can be expected to be short lived. Therefore, in addition to providing more information, daily data are more likely to allow detection of any conditional heteroscedasticity. Another reason for using fine frequency data is that under mild regularity conditions ARCH effects vanish under temporal aggregation as convergence to unconditional normality occurs.



index has been adjusted for splits and the issuance of new capital.<sup>10</sup> The daily returns for the market index  $P_t$ , 997 observations for the whole period, are continuously compounded returns. They are calculated as the difference in natural logarithm of the index value for two consecutive days,  $r_t = \ln(P_t) - \ln(P_{t-1})$ .<sup>11</sup>



*Figure I: Daily rates of change of the FAZ share price index over the period 1 January 1987 to 31 December 1990*

The evidence in Table 1 indicates that there is substantial variation in stock market volatility. Visual inspection of the  $r_t$  series reveals no evidence of serial correlation, although there did seem to be persistence in the conditional variances. Casual empiricism identified a number of out-

<sup>10</sup> The FAZ composite portfolio contains 100 stocks. The fact that the index has not been adjusted for dividends should have little effects on the estimates. Since the ex-dividend days are different for the 100 stocks in the FAZ portfolio, there are not large changes in the daily index due to dividend payments.

<sup>11</sup>  $(1 - L) \ln P_t$  has the convenient interpretation of approximate percentage change.

liers which were associated with news and/or policy interventions.<sup>12</sup> Because it is reasonable to assume that the market expects such outliers to be clustered, i.e. once one outlier has been observed, the market would attach a far higher probability than the long run probability of observing another one in the near future, it was decided as a general rule not to remove outliers from the dataset. The one exception is the extreme outlier without any longlasting effect upon  $P_t$  on Monday 16 October 1989. Unless this observation is specifically modelled the following GARCH specifications are rejected.<sup>13</sup>

While the graph gives some indication of changing conditional variances, it does not, however, constitute a formal test. The next sections therefore show the results from carrying out statistical tests on the underlying stock market data.

## 2. Data description and sample distribution

In order to assess the distributional properties of the  $r_t$  series, various descriptive statistics are reported in Table 1, including mean, standard deviation, coefficient of skewness, coefficient of kurtosis, Jarque-Bera's test for normality, Ljung-Box test for a random white noise series and Engle's (1982) Lagrange Multiplier (LM) ARCH test.<sup>14</sup> Two sets of results are presented. First, the full data set consisting of  $n = 997$  daily returns, and second, the subset for which the October 16, 1989 outlier has been eliminated ( $n = 996$ ).<sup>15</sup>

In particular, the hypothesis of normality is rejected for the stock market returns. The distribution of  $r_t$  is negatively skewed, indicating that it is non-symmetric.<sup>16</sup> Furthermore, it exhibits a severe level of kurtosis meaning a distributional property which is more peaked or has fatter tails than normal distributions. In addition, the Jarque-Bera test

<sup>12</sup> An observation was classified as an outlier if the value of the daily holding period yield  $r_t$  did not lie within the interval  $(x - 2\sigma, x + 2\sigma)$ , where  $x$  is the sample mean and  $\sigma$  is the standard deviation. The potential explanations for outlying observations are discussed at some length in Dickens (1986). The most plausible explanation for their existence would appear to be that they were generated by a secondary process from the rest of the sampled data.

<sup>13</sup> The parameter  $\delta$  turns out insignificant and  $\alpha_1 + \phi_1$  exceed one.

<sup>14</sup> Throughout the paper, *kurtosis* refers to *excess kurtosis*, so that the value of zero corresponds to normality.

<sup>15</sup> Since daily data are used, observations for weekends and bank holidays are missing.

<sup>16</sup> Negative skewness indicates a longer left hand than right hand tail on the sample distribution.

Table 1  
Sample distribution of daily stock market returns

	Full data set	Outlier excluded
Mean	-0.00008	0.00006
Standard deviation	0.02	0.01
Skewness	-1.19	-0.33
Kurtosis (centered on 3)	15.43	7.75
Jarque-Bera test	6655.61	954.91
Ljung-Box test [4th-order]	3.19	6.00
Ljung-Box test [8th-order]	13.87	17.41
Ljung-Box test [16th-order]	27.71	34.05
ARCH [1st-order]	66.86	120.61

Notes: Under the null hypothesis of identically, independently normal distribution of returns, the coefficients of skewness and excess kurtosis are both zero. Under the assumption of normality their sample estimates have asymptotic standard deviations of  $\sqrt{(6/n)}$  and  $\sqrt{(24/n)}$ , respectively. Jarque-Bera test:  $n[(\text{skewness}^2/6) + (\text{excess kurtosis}^2/24)] \sim \chi^2(2)$ .

of normality also rejects the null hypothesis of normality at an extremely high level of significance.<sup>17</sup> Table 1 also presents the Ljung-Box test statistic. The objective is to test if daily stock returns can be represented by a random walk, i.e. if returns  $r_t$  are independently distributed random variables with expectation  $E(r_t) = 0$ . The Ljung-Box (LB) test statistics up to lag sixteen ( $k = 16$ ) were calculated and some of them are reported in the Table. While the joint test that the first four autocorrelation coefficients are zero is not rejected at the five percent level, the LB (8) and LB (16) statistics are both significant, which means that the null hypothesis of strict white noise is rejected, reflecting a rather long range of dependency in the daily return series.<sup>18</sup> Finally, Engle's (1982) formal test for a 1st-order ARCH effect is reported. Under the null hypothesis of conditional homoscedasticity, the ARCH-statistic is distrib-

<sup>17</sup> It appears that the size of non-normality in West German daily stock returns is much more pronounced than that observed by Akgiray (1989) in the American market.  
<sup>18</sup> It can be questioned, however, whether the LB-statistics are indeed significant since heteroscedasticity can lead to the underestimation of the standard errors and therefore to the overestimation of the  $\chi^2$ -statistics.

uted as  $\chi^2(1)$ .<sup>19</sup> For both data sets, Table 1 yields clear evidence of an ARCH effect in the daily stock returns. Hence, there are strong reasons for believing that the distributional assumption of unconditional normality is inappropriate and therefore the application of the ARCH/GARCH model to the data seems motivated.

In order to provide an additional descriptive snapshot of volatility clustering, an empirical Markov-chain transition matrix for the daily stock market returns has been calculated. In the Markov-chain model the observations  $r_t$  are classified in an ascending order into  $J$  quantiles where the first quantile contains the largest declines in stock returns and the  $J$ -th quantile contains the largest increases. The quantiles are chosen such that all quantiles have the same number of observations. One can then count the number of times that an observation  $r_t$  falls into quantile  $J_i$  and  $r_{t+1}$  falls into quantile  $J_j$  and denote this number by  $n_{ij}$ . If the  $r_t$  are independent and identically distributed then the expected value is  $n_{ij}^* = (T - 1)/J^2$  for all  $i$  and  $j$ . The empirical transition matrix of  $n_{ij}$ 's is displayed in Figure II. It shows the daily stock returns classified into 10 quantiles. The height of the three-dimensional body gives the values of  $n_{ij}$ . For all entries, the expected number is  $n_{ij}^* = 10$ . Although the overall pattern of volatility is not very clear, there are several dominant peaks in and/or nearby the corners. These peaks can be interpreted in terms of periods of turbulence. The dominant peaks  $n_{11}$  and  $n_{21}$  are the number of cases where a strong decline of returns was followed by another strong decline while  $n_{96}, n_{98}$  and  $n_{1010}$  are the number of pairs of strong increases. Likewise,  $n_{110}$  is the number of cases where a strong increase followed a strong decline. Thus there is some informal support for the volatility clustering hypothesis.

A more rigorous test of the hypothesis is provided by estimating the autocorrelation function (ACF) of squared returns  $r_t^2$ .<sup>20</sup> The ACF for squared returns is significant at all lags up to 20 and the Box-Pierce  $Q$ -statistic is calculated as  $Q(24) = 1441.90$ . Thus there is again very strong rejection of the  $H_0$  of no serial dependence in variances for daily stock returns.

<sup>19</sup> The test appears to work well under conditional normality in finite samples. Moreover, as Weiss (1986) discussed, the LM test is also appropriate (subject to some moment conditions) for nonnormal distributions.

<sup>20</sup> According to Bollerslev (1988), the ACF and PACF for squared data can be used in the same way as in conventional ARIMA models to identify the order of the AR and MA component in models for variances.

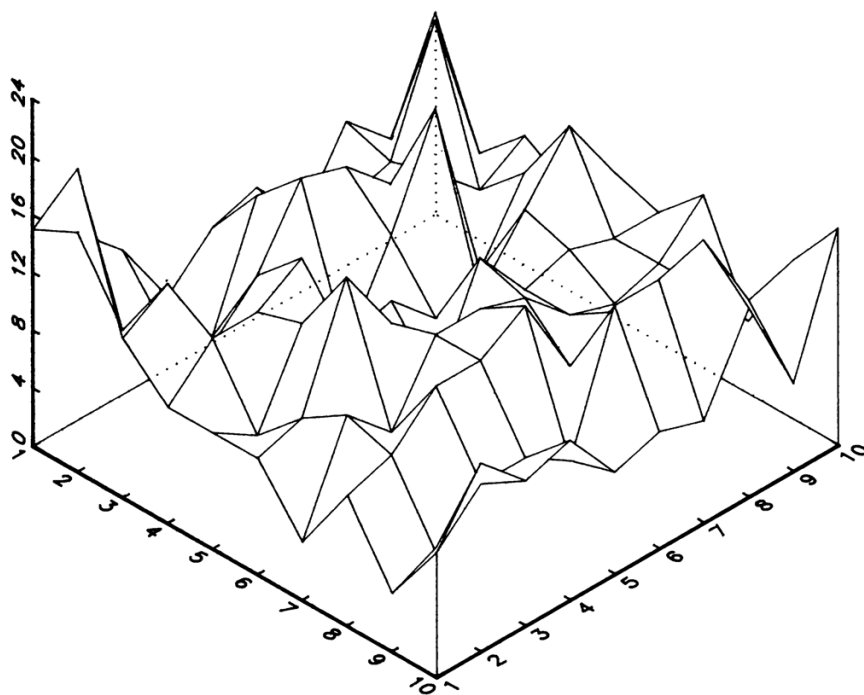


Figure II: Markov transition matrix: daily stock market returns

#### IV. GARCH-M (1,1) and SGARCH-M (1,1) Model Estimates

In this section of the paper GARCH-M and SGARCH-M models are estimated for  $r_t$ . Estimation of the parameters and hyperparameters is done by casting the model in state-space form and using the Kalman filter to construct the log likelihood via a prediction error decomposition.<sup>21</sup> Before estimation, the values of  $p$  and  $q$  in equation (2) and (3) have to be specified. Because estimation of the models is computationally burdensome and the GARCH (1,1) model has been widely accepted to be a good representation of stock returns only the results for this pair of  $p$  and  $q$  are presented in the next Table. The vector of fixed explanatory variables ( $X$ ) includes the intercept and a (0,1)-dummy taking the

<sup>21</sup> Hall (1990) provides the computational details of obtaining the maximum likelihood estimates. The approach is related to procedures in Watson and Engle (1983). No nonnegativity constraints have been imposed on the parameters in the numerical optimization.



value 1 for the data 89/10/16 and zero otherwise.<sup>22</sup> The estimation results are given in Table 2.

*Table 2*  
**Estimation of GARCH-M (1,1) and SGARCH-M (1,1) models for  $r_t$**

Parameters and Hyperparameters	GARCH-M (1,1)	SGARCH-M (1,1)
$\beta_0$	0.001 (2.2)	0.0007 (2.1)
$\beta_1$	-0.11 (7.1)	-0.11 (6.5)
$\delta$	0.0003 (1.5)	0.0007 (3.8)
$\alpha_0$	0.000001 (1.5)	0.00003 (5.2)
$\alpha_1$	0.76 (32.2)	0.75 (30.9)
$\phi_1$	0.02 (1.5)	0.23 (12.0)
Var( $\omega$ )	–	0.0004 (4.2)
<i>Diagnostics:</i>		
Log Likelihood:	-8870.30	-8778.40
Ljung-Box (8):	11.21	13.54
Ljung-Box (16):	18.66	20.67
Jarque-Bera test:	12.02	8.89

*Notes:* Asymptotic absolute values of the  $t$ -statistics are given in parentheses. The standard errors have been calculated using White's robust covariance matrix. The residuals and diagnostics have been extracted with a Kalman smoother [compare *Watson and Engle (1983)*].

The table includes the coefficient estimates and asymptotic  $t$ -statistics for the GARCH-M (1,1) and SGARCH-M (1,1) model. The restricted model constrains the variance in equation (3) to be zero, i.e.  $\text{Var}(\omega) = 0$ . The unrestricted model relaxes this assumption. The table also includes diagnostics to check whether the models are appropriate. Several

<sup>22</sup> An alternative procedure to estimate processes which are subject to identifiable discrete shifts has recently been provided in *Hamilton (1990)*.

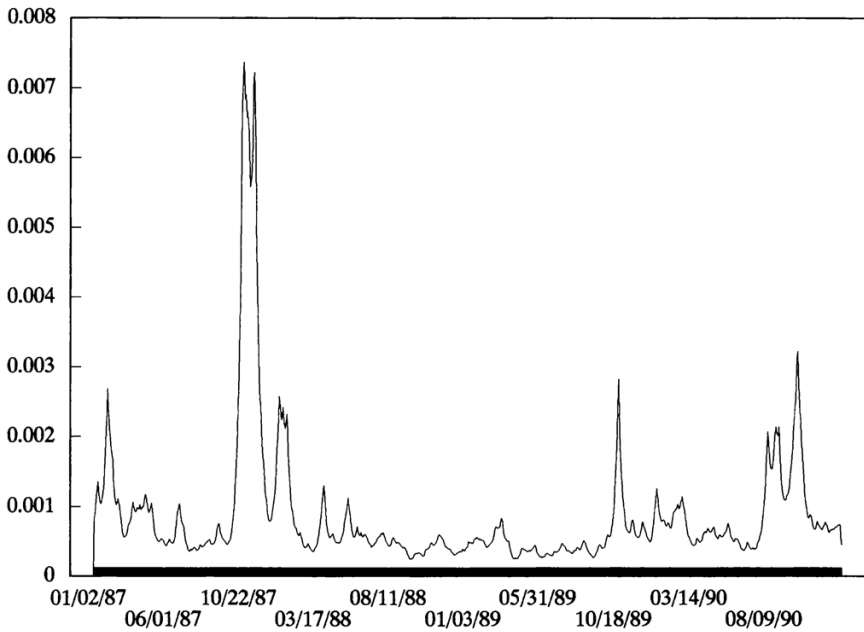
remarkable findings regarding the returns generating process can be drawn. First, under the assumption of conditional normality, all parameters of the SGARCH-M (1,1) model in the second column are highly significant with plausible parameters. Second, the SGARCH-M model reveals that there is not only a significant ARCH effect but also a positive and clearly significant time-varying risk premium present in the returns generating process.<sup>23</sup> Third, the SGARCH-M results find a significant variance for the GARCH equation, indicating that equation (3) is more appropriate than equation (2). Fourth, the sum of  $\alpha_1 + \phi_1$  is less than unity, though rather close to one, which indicates a very long persistence of shocks in volatility.<sup>24</sup> As a diagnostic check on the overall appropriateness of the SGARCH-M (1,1) process, various statistics are given for the estimated model. In particular,  $LB(k)$  is the Ljung-Box test statistics for an autoregressive process of order  $k$  in the residuals and the Jarque-Bera statistics tests the hypothesis of conditional normality. In no case can the hypothesis of uncorrelated returns be rejected. The Jarque-Bera normality test, however, is still not passed at the 5 percent level. Nevertheless, the strong convergence to normality provides supporting evidence of good fit to data. The results for the GARCH-M (1,1) model in the first column are less convincing. Both coefficients  $\delta$  and  $\phi_1$  turn into insignificance which indicates that an ARCH (1) model may be more appropriate. Additionally, the test on overall normality of the residuals points to more significant deviations for the deterministic ARCH/GARCH specification.

As mentioned in the outset, one motivation for this study was to improve upon the means by which stock price uncertainty is measured. It is therefore relevant to analyse the evolution of the time-varying conditional measure of risk over time. Figure III shows the conditional volatility computed from the estimates of the SGARCH-M (1,1) model specification using data over the whole sample period.

Focusing on the late 1980s, the plot indicates mean reversion in volatility and indicates no positive trend in volatility during “normal” market periods. Instead, the average conditional measure of risk over the 1987 - 1990 period has increased due to transitory periods of abnormally high volatility. Following these high-volatility episodes, the estimated condi-

<sup>23</sup> Although statistically significant, the positive effect of  $\delta$  on  $r$  is, however, quantitatively small.

<sup>24</sup> This indicates a near integrated GARCH process with persistent conditional variance. In fact, a conventional  $t$ -type test does not reject the null of a unit root in the conditional variance.



*Note:* The conditional volatility has been extracted with a Kalman smoother [compare *Watson and Engle (1983)*].

*Figure III: The conditional volatility in the West German stock market 1987 - 1990*

tional volatility quickly reverts back to much lower “normal” levels. Using a rather crude classification, four periods can be observed. The first covers the October 1987 worldwide stock market crash when the conditional measure of risk shows extremely high and unstable values. Other than this crash period, the risk premium is low and comparatively stable over the August 1988 - September 1989 period, suggesting that the traditional constant variance assumption is probably realistic for this short sub-period. The stock market crash of October 1989 marks the beginning of the third period. This period is again characterized by a noticeable upturn in the conditional variance although the increase was not of such spectacular magnitude as that in October 1987. The last period of rising risk premiums began in September 1990. The most obvious explanation for this late 1990 experience are concerns about the economic consequences of German unification and related concerns about strong monetary growth and the consequential dangers of a pick up in inflation.

## V. Conclusion

This paper has examined the econometric evidence for the relationship between stock returns and stock returns volatility. The key result is that the *stochastic* GARCH-in-mean (1,1) model with a conditional normal density provides a reasonable description of daily West German stock returns in the entire 1987 - 1990 period. Consequently, the German stock markets show volatility clustering, i.e. large returns (of either sign) are more frequently observed in more volatile periods. The implied time-varying risk premiums have far-reaching economic consequences. One example is that the well-known variance bound tests suggested by *Shiller* (1981) are probably misspecified.<sup>25</sup> Finally, despite the fact that the SGARCH-M (1,1) model seems to be supported by the data, it seems obvious that future work in this area is called for. One extension would be to integrate other variables that could affect expected risk premiums.<sup>26</sup>

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<sup>25</sup> By assuming a *constant* risk factor, *Shiller* (1981) has derived a variance bound test and concludes that the stock price variation due to dividends is much too small to be consistent with the huge variance of stock prices.

<sup>26</sup> *Fama* and *Schwert* (1977), for example, show that the nominal interest rate can be used to predict stock returns. *Keim* and *Stambaugh* (1986) use (1) the yield spread between long-term corporate bonds and short-term Treasury bills, (2) the level of the S&P composite index in relation to its average level over the previous 45 years, and (3) the average share price of the firms in the smallest quintile of NYSE firms to predict stock returns while *Baillie* and *De Gennaro* (1989) investigate the impact of institutional features like settlement and check clearing procedures upon daily stock return data.

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### Summary

#### **The Message in Daily West German Stock Prices: Empirical Evidence Using GARCH**

Stock returns have long been recognized to be heteroscedastic as well as leptokurtic. One model that captures both characteristics is the GARCH process. This article is concerned with modelling the dynamic and distributional properties of fine frequency West German stock market data from January 1, 1987 to December 31, 1990. The stylized results are that the conditional heteroscedasticity in daily stock returns is well represented by a stochastic GARCH (1,1) process with near unit roots.

### Zusammenfassung

#### **Die empirische Modellierung von Tageskursen westdeutscher Aktienbörsen mittels GARCH-Modellen**

Zeitreihen von Aktienrenditen weisen in der Regel (links)-schiefe Verteilungen und Kurtosis-Werte, die signifikant größer als drei sind, auf. Aufgrund dieser Eigenschaft werden in dem Papier GARCH ( $p, q$ )-Modelle ("generalized autoregressive conditional heteroscedasticity models") für die täglichen Renditen am deutschen Aktienmarkt vom 1. Januar 1987 bis zum 31. Dezember 1990 spezifiziert und geschätzt. Die Ergebnisse zeigen, daß die täglichen Renditen durch GARCH (1,1)-Modelle beschrieben werden können.

### Résumé

#### **Le message des cours de bourse quotidiens ouest-allemands: évidence empirique avec des modèles de Garch**

Les séries chronologiques de rendements d'actions montrent en général des structures inclinées (vers la gauche) et des valeurs d'aplatissement qui sont nettement supérieures à trois. Sur base de cette propriété, des modèles de GARCH ( $p, q$ ) («generalized autoregressive conditional heteroscedasticity models») sont spécifiés et évalués dans cet article pour les rendements quotidiens sur le marché allemand des actions du 1er janvier 1987 au 31 décembre 1990. Les résultats montrent que les rendements quotidiens peuvent être décrit par des modèles de GARCH (1, 1).