Chaos in the Dornbusch Model of the Exchange Rate*

By Paul De Grauwe and Hans Dewachter, Leuven**

I. Introduction

Ever since the empirical breakdown of (linear) structural exchange rate models, the predominant view on exchange rate dynamics has been based on the "news" model. In this model the only sources driving the exchange rate are random events.[1]

Recent research has revealed some problems with the "news"-model. First, there appears to be more structure in the time series of the exchange rate than the pure stochastic model can account for. This additional structure has been found in most exchange rates. See, for example, *Cutler, Poterba* and *Summers* (1990) who report significant autocorrelations in the exchange rates at different lags.

Second it appears that many, if not most, movements in the exchange rates cannot easily be accounted for by observable "news". In an analysis of high-frequency exchange rate data, *Goodhart* (1990) documents that very often the exchange rate does not respond to observable news, and that many exchange rate movements cannot be associated with news.

This recent empirical research suggests that in addition to random shocks, there are other driving forces in the exchange market that are important to understand its dynamics. In this paper we will focus on a (non-linear) speculative dynamics, in which the behavior of "chartists" and "fundamentalists" plays a prominent role. The analysis will be performed in the context of a structural model, the *Dornbusch* model, which has become the most popular textbook model of the exchange rate.[2] It will be shown that this

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** Hans Dewachter is Research Assistant at the National Fond for Scientific Research (Belgium).

[1] See *Frenkel* and *Mussa* (1985), *Levich* (1985), *Mussa* (1984).

[2] See *Dornbusch* (1976).

model together with a simple non-linear speculative dynamics is capable of generating a complex behavior of the exchange rate which is unpredictable, even in the absence of random shocks. Such behavior has been called "chaotic".[3] In addition, the model will be used to analyze the behavior of the exchange rate when random events ("news") occur with low frequency.

The remainder of the paper is organised as follows.

In section two we present the model. Section three reports the basic properties of the model. It will be shown that the model is able to generate chaotic motion. In section four we report the results of monetary policy shocks. Section five points out the importance of low-frequency stochastic shocks for some regions in the parameterspace of the model. Finally, section six briefly discusses the properties of the price level and the interest rate.

II. The Model

1. *The Dornbusch Model*

The version of the Dornbusch model that will be used in this paper consists of the following building blocks:

a) The Money Market Equilibrium Condition

(1) $$Ms_t = Y_t^{a} \cdot P_t \cdot (1 + r_t)^{-c}$$

where P_t is the domestic price level in period t, r_t is the domestic interest rate, Ms_t is the (exogenous) money supply, Y_t is the (exogenous) level of domestic output.

b) The Open Interest Parity Condition

(2) $$E_t\,(S_{t+1}) / S_t = (1 + r_t) / (1 + r_{ft})$$

where S_t is the exchange rate in period t (the price of the foreign currency in units of the domestic currency), $E_t\,(S_{t+1})$ is the forecast made in period t of the exchange rate in period $t + 1$, r_{ft} is the foreign interest rate.

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[3] In a previous paper one of the authors used a partial equilibrium model of the exchange rate. The chaotic results obtained there also depended on the existence of a J-curve effect, see *De Grauwe* and *Vansanten* (1990). Here we discard the assumption of a J-curve. This should make the results stronger.

c) Goods Market Equilibrium

The long run equilibrium condition is defined as a situation in which purchasing power parity (PPP) holds, i.e.:

$$S^*_t = P^*_t / P_f{}^*_t \tag{3}$$

where S_t^* is the equilibrium (PPP) exchange rate, $P_f{}^*_t$ the foreign and P^*_t the domestic steady state value for the price level in period t.

The short-term price dynamics is assumed to be determined as follows:

$$P_t / P_{t-1} = (S_t / S_t^*)^k \quad \text{where } k \geqslant 0 \tag{4}$$

That is, when the exchange rate exceeds its PPP-value, S_t^*, the domestic price level increases. Put differently, when the currency is undervalued this leads to excess demand in the goods market tending to increase the price level. The opposite occurs when the exchange rate is below its PPP-value (an overvalued domestic currency). Note that we assume full employment so that adjustment towards equilibrium is realized through price changes.

2. *The Speculative Dynamics*

We assume that there are two classes of speculators. One class is called "chartists", the other "fundamentalists". (See *Frankel* and *Froot* (1986) for a first attempt at formalizing this idea. A recent microeconomic foundation of this assumption is provided by *Cutler, Poterba,* and *Summers* (1990). Empirical evidence about the importance of these types of speculators is found in *Allen* and *Taylor* (1989) and *Frankel* (1990).)

The "chartists" use the past of the exchange rates to detect patterns which they extrapolate into the future. The "fundamentalists" compute the equilibrium value of the exchange rate. In this model this will be the (steady state) PPP-value of the exchange rate. If the market rate exceeds this equilibrium value they expect it to decline in the future (and vice versa). Another way to interpret this dual behavior is as follows. The "chartists" use the past movements of the exchange rates as indicators of market sentiments and extrapolate these into the future. Their behavior adds a "positive feedback" into the model.[4] As will become clear, this is a source of insta-

[4] Note that chartists themselves may believe that these movements are unrelated to the fundamentals. They consider these market movements to be important pieces of information reflecting other agents believes about market fundamentals.

bility. The fundamentalists have regressive expectations, i.e. when the exchange rate deviates from its equilibrium value they expect it to return to the equilibrium. The behavior of the fundamentalists adds a "negative feedback" into the model, and is a source of stability.

A second feature of the speculative dynamics assumed in the model is that the weights given to "chartists" and "fundamentalists" are made endogenous. More specifically, it will be assumed that when the exchange rate is close to the equilibrium (fundamental) rate, the weight given to the fundamentalists is at its lowest, whereas the chartists then have a maximal weight. When the market rate deviates from the equilibrium rate, the weight given to the fundamentalists increases with that deviation. That is, when the exchange rate continues to deviate from its fundamental value, fundamental analysis becomes increasingly important. There comes a point that it will overwhelm technical analysis in forecasting future exchange rates.

This assumption can be rationalized by introducing the idea that expectations made by fundamentalists are heterogenous, i.e. each fundamentalist makes a different calculation of the equilibrium rate (see also *Cutler, Poterba* and *Summers* (1990). If we assume that these calculations are normally distributed around the true equilibrium rate, we can conclude that when the market rate is equal to the true equilibrium rate, the high and low forecasts made by fundamentalists will offset each other (so that also their buy and sell orders will be offsetting). As a result, when the market rate and the fundamental rate coincide, the fundamentalists have a low weight in determining the movements of the exchange rate. These then will be dominated by the chartists. When, however, the market rate starts deviating from the fundamental rate, say it increases, those who have made a low forecast for the equilibrium rate will increasingly dominate the market. If the market rate has increased sufficiently, all fundamentalists will consider that market rate to be too high, and will expect it to go down in the future. Their weight in the formation of market expectations will be high, so that the weight given to the chartists becomes correspondingly small.

We now implement these two assumptions about the speculative dynamics as follows. We write the change in the expected future exchange rate as consisting of two components, a forecast made by the chartists and a forecast made by the fundamentalists:

$$E_t(S_{t+1})/S_t = (E_{ct}(S_{t+1})/S_t)^{mt}(E_{ft}(S_{t+1})/S_t)^{1-mt} \tag{5}$$

where $E_t(S_{t+1})$ is the market forecast made in period t of the exchange rate in period $t+1$; $E_{ct}(S_{t+1})$ and $E_{ft}(S_{t+1})$ are the forecasts made by the chartists and the fundamentalists, respectively; m_t is the weight given to the chartists and $1 - m_t$ is the weight given to the fundamentalists.

We assume that the chartists extrapolate recent observed exchange rate changes into the future, using a moving average procedure, i. e.

(6) $$E_{ct}(S_{t+1})/S_t = (S_t/S_{t-1})^d \cdot (S_{t-1}/S_{t-2})^e \cdot (S_{t-2}/S_{t-3})^f$$

where the coefficients d, e, and f are the weights of the moving average.

Admittedly this is a very crude assumption, and chartists typically use more sophisticated rules (in our further research we hope to study the implications of using more sophisticated chartists' forecasts). The use of simple rules, however, is not necessarily a disadvantage if we can show that very complex behavior of the exchange rate is possible even if chartists use these very simple forecasting rules.

The fundamentalists are assumed to calculate the equilibrium exchange rate (i. e. the exchange rate that leads to equilibrium in the model). In our model this is the PPP-rate. They will then expect the market rate to return to that fundamental rate (S^*_t) at the speed h during the next period, if they observe a deviation today, i. e.

(7) $$E_{ft}(S_{t+1})/S_t = (S^*_t/S_t)^h$$

As indicated earlier, the weights given to chartists and fundamentalists are assumed to be endogenous and to depend on the deviation of the market rate from the fundamental rate.[5]

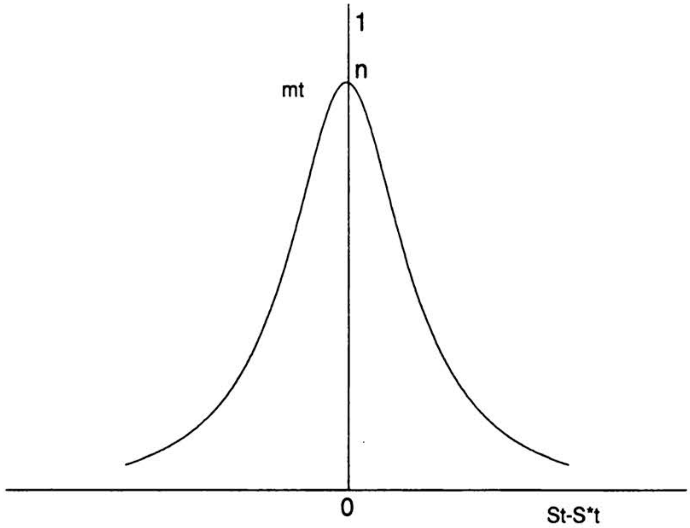
There are several ways one could implement this assumption. We will make m_t in equation (5) a negative function of the deviation of S_t from its equilibrium value S^*_t, using the following specification

(8) $$m_t = n/(1 + b(S_{t-1} - S^*_{t-1})^2)$$

where $0 < n < 1$ and $b > 0$.

Graphically we can represent this specification as follows:

[5] In *De Grauwe* and *Vansanten* (1990) these weights were assumed to be fixed.



From this figure it can be seen that when the market exchange rate is close to the fundamental rate the weight given to the chartists attains its maximum. This maximum is set at the value n (which is at most equal to 1). When the market rate deviates from the fundamental rate this weight tends to decline. For very large deviations it tends towards zero. The market expectations will then be dominated by the fundamentalists. Note also that the parameter b determines the speed with which the weight of the chartists declines. This coefficient can also be interpreted as measuring the sensitivity of the fundamentalists' expectations with respect to the deviations between the market and the fundamental exchange rate increases. With a high b the curve in figure 1 becomes steeper.

From the preceding it may appear that we have introduced a lot of "ad-hoc" assumptions in constructing the model. First, the specification of equation (8) determining the changing weights of chartists and fundamentalists may seem rather special. Other functional forms could be used. In appendix we present an alternative functional form, and we show that the main results of the paper remain intact. Second, and more fundamentally, we have not derived the specifications of the behavior of the speculators from an explicit optimizing framework. As a result, expectations cannot be called rational. Our defence here is to plead guilty, and to ask the reader to follow us so as to see how far such a non-linear specification of the speculative dynamics can go in explaining exchange rate movements. We want to show that the speculative dynamics which we assume here (and for which there is an increasing amount of empirical evidence) allows us to construct models that come closer towards understanding reality than the structural models that have been used up to now. In addition, we will show that in our model economic agents have no incentives to invest energy in trying to detect the dynamics of the underlying model. Thus, it becomes reasonable to assume that they use simple rules of thumb (like PPP) in computing the equilibrium rate.

We now proceed towards solving the model.

3. *Solution of the Model*

Substituting (6), (7) and (8) into (5), and (5) into (2) and using (3) we obtain an expression for $(1 + r)$, which can be substituted in (1). This yields the following equation:

$$P_t \cdot {P^*_t}^{-ch(1-mt)} \cdot \left({S_t}^{-h(1-mt)} \cdot E_{ct}\left(S_{t+1}/S_t\right)^{mt}\right)^{-c} = Z_1 \tag{9}$$

where $Z_1 = MsY^{-a}(1 + r_f)^c {P_f}^{-c(1-mt)}$ and brings together all the exogenous variables. Equation (9) describes the equilibrium in the money market together with interest parity.

Using (3) we rewrite equation (4) as follows:

$${S_t}^{-k} \cdot {P_t}^{(1+k)} \cdot {P_{t-1}}^{-1} = Z_2 \tag{10}$$

where $Z_2 = P_f$, which without loss of generality can be set equal to 1.

The system of equations (9) and (10) fully describes the dynamics of the model. We can solve it for the endogenous variables P_t and S_t. This yields:

$$P_t = Z_{1t} \cdot {P^*_t}^{ch(1-mt)} \cdot \left({S_t}^{-h(1-mt)} \cdot E_{ct}\left(S_{t+1}/S_t\right)^{mt}\right)^{c} \tag{11}$$

and

$$S_t = \left((G_2\, G_1)^{-1} \cdot {S_{t-1}}^{-f1}\, {S_{t-2}}^{-f2}\, {S_{t-3}}^{-f3}\, {S_{t-4}}^{-f4}\right)^{(1/f1)} \tag{12}$$

$$\begin{aligned}
\text{with}\quad f_0 &= (cdm_t - ch\,(1 - m_t))\,(1 + k) - k \\
f_1 &= (1 + k)\,cm_t\,(e - d) - (cdm_{t-1} - ch\,(1 - m_{t-1})) \\
f_2 &= (1 + k)\,cm_t\,(f - e) - c\,(e - d)\,m_{t-1} \\
f_3 &= -fcm_t\,(1 + k) - c\,(f - e)\,m_{t-1} \\
f_4 &= cfm_{t-1} \\
G_1 &= (Z1_t{}^{(1+k)}/Z1_{t-1}) \\
G_2 &= {P^*_t}^{(1-ch(1-mt))(1+k)}/{P^*_{t-1}}^{(1-ch(1-mt))} \\
m_i &= n/(1 + b\,(S_{t-i} - S^*_{t-i})^2)
\end{aligned}$$

The exchange rate is determined by its own past, the lagged prices, and the exogenous variables Z_1 and Z_2.

As can be seen from (11) and (12) the solution of the model is a complex system of non-linear difference equations. An analytical solution to this system cannot be derived. In the next two sections we will simulate the behaviour of this model. Our interest in the model is first to know whether it is capable of generating an unpredictable dynamics (section III). More specifically, we ask the question under what parameter values the system will exhibit "chaotic" behavior. Second, we are interested in the interaction between the deterministic dynamics of the model with the occurence of stochastic shocks (section V).

III. Existence of Chaos

In this section we turn to the question of the type of solutions the model is able to generate under different parameter values. We study under what configuration of parameters the model produces a chaotic movement of the exchange rate.

Let us first define chaotic motion. (We use the definition as provided by *Devaney* (1989)). A function like equation (12) is chaotic if:

(a) it has sensitive dependence on initial conditions
(b) it is topologically transitive
(c) periodic points are dense.

The intuition of this definition can be explained as follows. According to (a) a slight change in the initial conditions will (if sufficient time is allowed for) lead to a time path of the exchange rate which bears no resemblance whatsoever with the original time path. As will become clear, this has far-reaching implications for the predictability of the exchange rate. (b) implies that the consecutive exchange rates produced by iteration of equation (12) will eventually move from one arbitrarily small neighborhood to any other. Condition (c) introduces an element of regularity. It ensures that the exchange rate will remain within certain bounds around the steady state value (a strange attractor). Conditions b and c together also imply that the exchange rate has infinite periodicity, i.e. no cycles repeat themselves exactly.

Unfortunately there are no known methods to detect chaos, in an analytic way, in a difference equation of higher order like (12). One can use the characteristics (a) and (b), however, to detect chaos in an experimental way, i.e. through iterations of equation (12). In particular we will simulate the model and analyze whether the sensitivity to initial conditions holds (condition (a)). We analyze this by generating a minor difference in the initial condition of the system. If the system is not chaotic the solutions should asymptotically be equal. Second we analyze the periodicity of the solution, by checking whether cycles in the exchange rate repeat themselves.

This simulation approach has an obvious weakness. Since any simulation is finite in length, it is impossible to discriminate between chaos and solutions with a periodicity equal to the length of the simulation plus one.

A pragmatic solution to this problem is to consider simulation runs that are long enough. Here we chose to simulate the model over a period of 7000 observations. As a result, the difference between chaos and solutions that have a periodicity of 7000 or more becomes purely academic since agents will not be able to detect a periodicity higher than 7000. The *Dornbusch* model we use here typically has as a unit of time, a month (possibly a week).[6] Therefore, it would take at least 140 years (if the unit of time is a week) for a solution with periodicity 7000 to start a new cycle, and to be detectable. We consider such solutions to be equivalent to chaos.

The number of combinations of parameters is very large. Therefore, we restrict ourselves to an analysis of the parameters underlying the behavior of the speculators (the parameterfield (n, b)). The other parameters, in particular the income elasticity and the interest elasticity are fixed ($a = 0.5$, $c = 0.8$).[7]

By setting $Z_1 = Z_2 = 1$ we have a steady state for the five tupple (S_t, S_{t-1}, S_{t-2}, S_{t-3}, S_{t-4}) in the point (1, 1, 1, 1, 1). This steady state solution is independent of specific parameter positions in the parameterfield (n, b). Because of this independence we can evaluate the model characteristics in the neighbourhood of (1, 1, 1, 1, 1) for different parameter values.

We disturb the steady state for different values of (n, b) and analyse the behavior of the model from this moment on. The solution is either a stable one (the system returns to the pre shock position) or an unstable one (the system tends to a new position). In both cases the dynamics can be characterized by periodicity, by a limit cycle or by a chaotic motion.

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[6] The *Dornbusch* model has a goods market dynamics in which deviations from PPP are partially corrected during the next period. Therefore it implies a lot of time aggregation. Put differently, the model is not suited to describe, say, hourly or daily exchange rate movements.

[7] The simulation results presented in the paper were generated with the following weights for the chartists' moving average weights: $d = 0.6$, $e = 0.3$ and $f = 0.1$. As is shown in appendix the choice of these weights does not alter the main results of the paper. The other parameter values of the model are: $a = 0.5$, $c = 0.8$, $h = k = 0.45$.

The simulation results are presented in Table 1. We indicate the kind of solution we obtain for different combinations of the parameters n and b. The interpretation of the table can best be explained by considering an example: Take the second column. For values of $n \leqslant 0.5$ and $b = 10$ we obtain stable solutions. When n is increased to 0.55 the solution of the model exhibits a periodicity of 2 (i.e. each cycle repeats itself after two periods). When n is increased further the periodicity of the solution increases. With $n = 0.7$ we obtain an 4-period solution. Chaos is obtained by increasing n further (for example, $n = 0.74$ leads to chaos). There are values for n that will lead to explosive solutions. These solutions are obtained when $n = 0.8$ or higher. Thus, the model is capable of generating all the types of solutions possible.

Table 1: **Characteristics of the Model in the (*n*, *b*) Space**

n							
1.0	E	E	E	E	E	CH	CH
0.95	E	E	E	E	CH	CH	CH
0.90	E	E	E	E	CH	CH	CH
0.85	E	E	E	E	P16	P8	P8
0.80	E	E	E	E	CH	CH	CH
0.78	E	CH	CH	CH	CH	CH	CH
0.76	E	CH	CH	CH	CH	CH	CH
0.74	E	CH	CH	CH	CH	CH	CH
0.72	E	CH	P2	P2	P2	P2	P2
0.70	E	P4	P2	P2	P2	P2	P2
0.65	E	P2	P2	P2	P2	P2	P2
0.60	E	P2	P2	P2	P2	P2	P2
0.55	E	P2	P2	P2	P2	P2	P2
0.50	S	S	S	S	S	S	S
0.40	S	S	S	S	S	S	S
b (in thousands)	0	0.01	0.1	1	100	500	1000

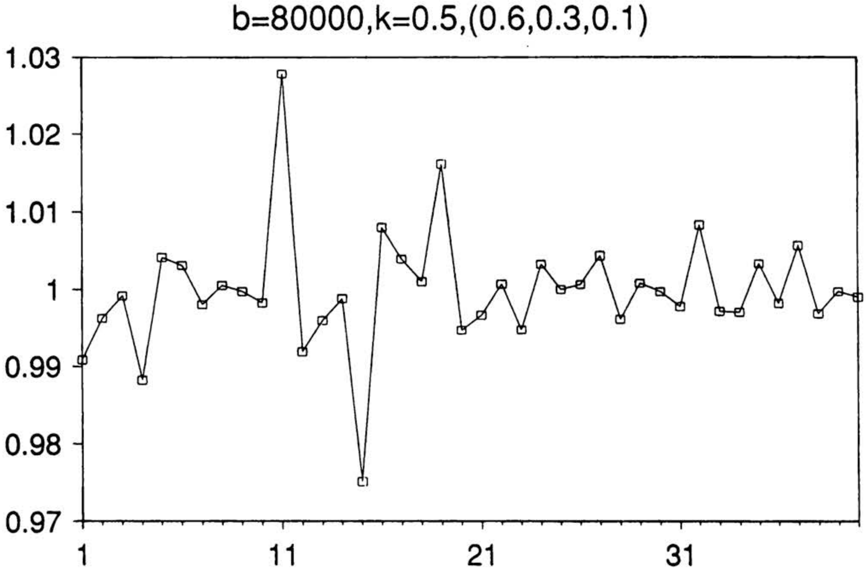
Where S is the stable solution, Pi displays periodicity i, CH is the chaotic solution, E. refers to explosiveness.

Table 1 also illustrates the role of chartists and fundamentalists in the dynamics of the solution. In general we find that an increase in the weight given to chartists (the parameter n) changes the solution from stable to cyclical and chaotic. Sufficiently high weights given to chartists can even make the system explosive. Consequently one can also conclude that an increasing weight given to the fundamentalists reduces the likelihood that chaos or instability is obtained.

The role of chartists and fundamentalists is also apparent in another way. As we move to the right in table 1, the sensitivity of the fundamentalists' expectations with respect to the deviations between the market and the fundamental exchange rate increases. We also observe that this movement leads to a reduction of the unstable region and an increase of the region of chaotic motion.

In figure 1 we present a few examples of chaotic motions for different configurations of the parameters. Note that the exchange rate fluctuates around a constant steady state value because we assume that the fundamentals are unchanged during the simulation, and the fundamentalists are aware of this. In a later section we analyze the model when fundamentals are allowed to change.

In order to show the sensitive dependence on initial conditions we simulated the model assuming first an initial shock in the exchange rate of 2.1 percent. We repeated the experiment with the initial shock equal to 2.2 percent. An example of the results of both experiments are shown in figure 2. (for the parameter configuration $n = 0.79$, $b = 2000$). We obtain similar results for all the chaotic solutions indicated in table 1.



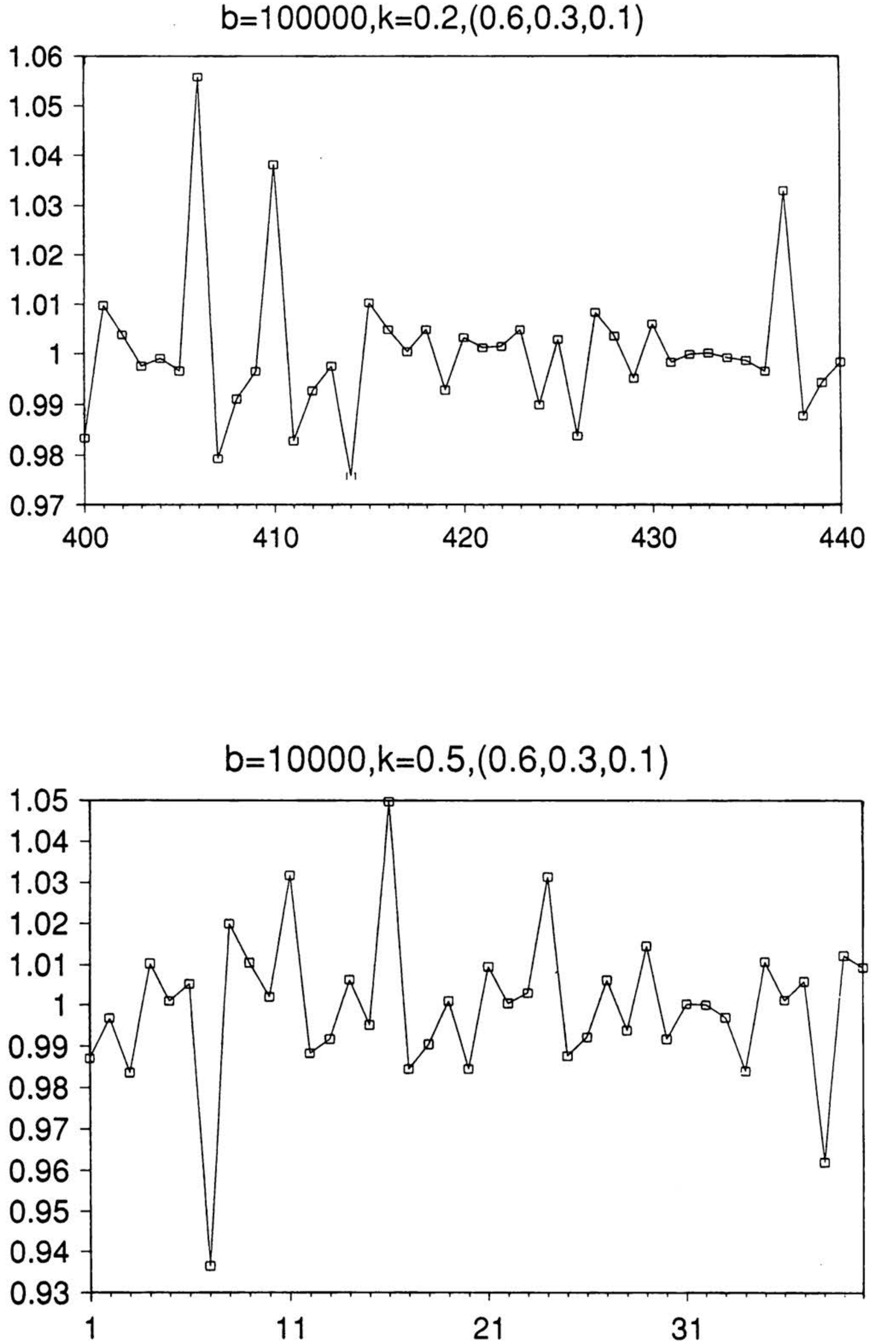


Figure 1: Chaotic Motion of the Exchange Rate.

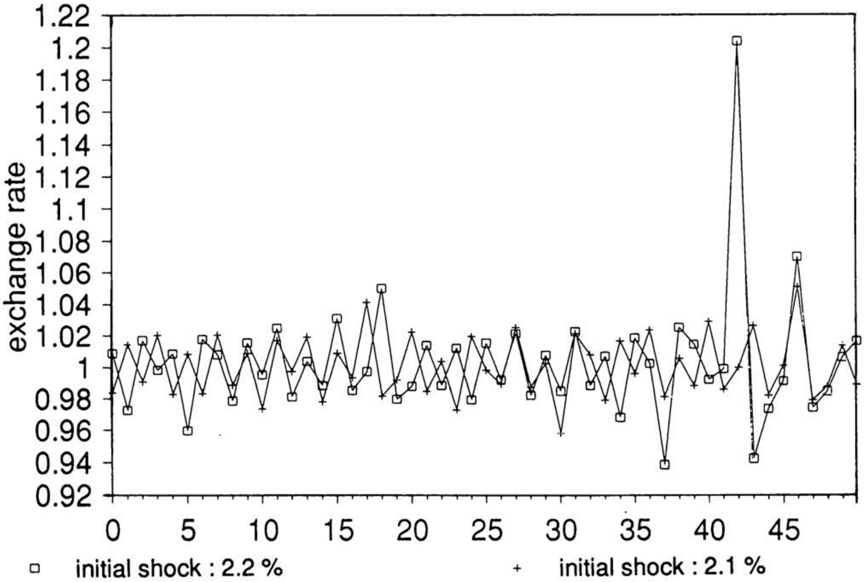


Figure 2: Dependence of Initial Conditions with Chaos.

Figure 3a shows, as an example, the phase diagram of the chaotic exchange rate series of figure 2 (assuming an initial disturbance in the exchange rate of 2.1 %). The horizontal axis shows S_t and the vertical axis S_{t+1}. The diagram presents the 6000 observations of one simulation experiment. Note that each observation falls in a different point, a characteristic of chaotic motion. No cycle repeats itself. And yet, there is a lot of structure in the phase diagram. This is illustrated further by a blow-up of the same diagram in figure 3b.

The examples of figures 1 to 3 illustrate the nature of chaotic motion. We ran simulations of 7000 periods during which time no cycle repeated itself. More importantly, a small difference in initial condition leads after a few periods to time paths of the exchange rate which are completely different. It is in this sense that exchange rate movements in this model are unpredictable. In order to forecast the exchange rate using this structural model we would need to know the initial condition with a degree of precision that is unattainable in social sciences.

In fact we need to know not only the initial conditions with extreme precision, but also the parameter values of the structural model with the same kind of precision in order to be able to predict the exchange rate. We illustrate this by presenting two simulations in figure 4. The first simulation is

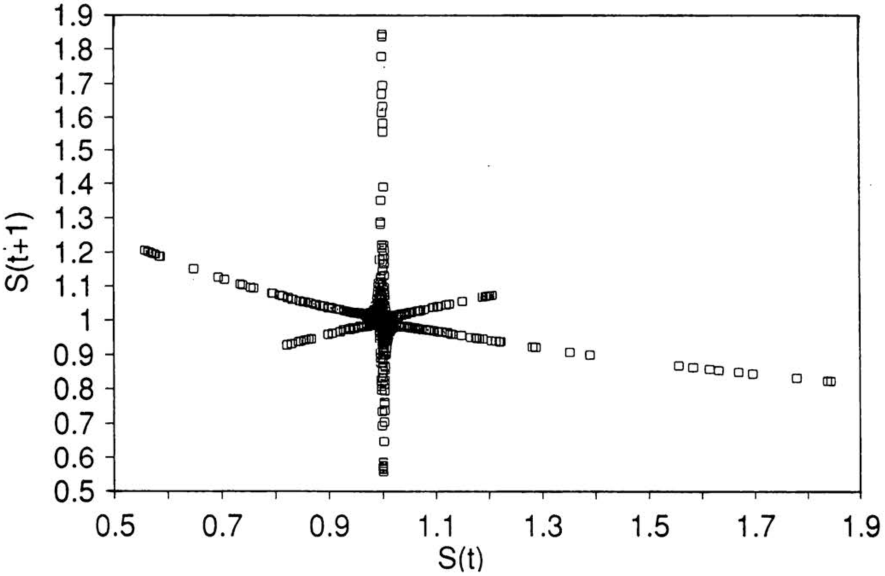


Figure 3a: Phase Diagram of Chaotic Motion.

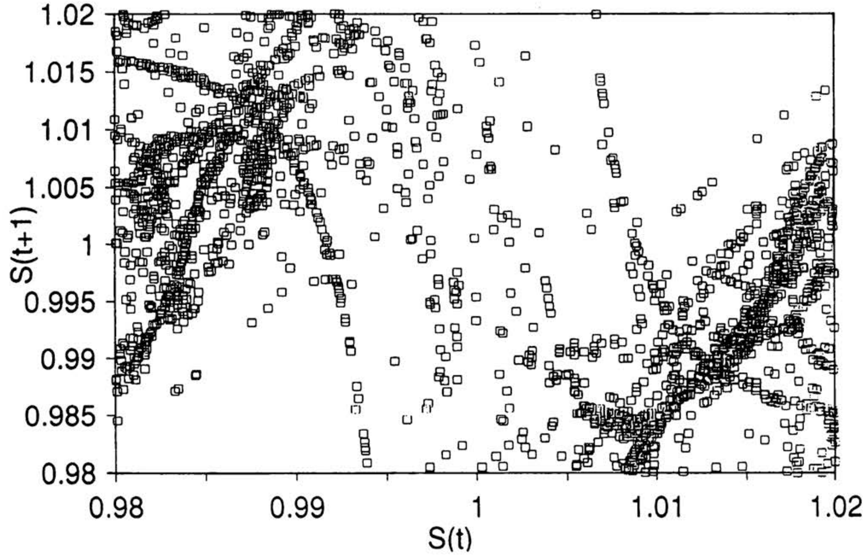


Figure 3b: Phase Diagram of Chaotic Motion (blow-up).

the same as in figure 2 (with initial condition: a shock of 2.2% in the exchange rate). In the second simulation we have decreased one parameter (n) by 0.5%. This second simulation can be interpreted as coming from a model in which a slight measurement error has been made compared to the true model underlying the first simulation. The results of figure 4 indicate that the slight measurement error produces a completely different time path of the exchange rate. This implies that in order to be able to use the model for predictive purpose we would have to know its underlying structural parameters with an extreme degree of precision. Under those conditions it is also unlikely that economic agents will have incentives to invest time and money in order to obtain information about the underlying structure of the model. The slightest error in the information processing will make it useless.

We will return to the problem of predictability of the exchange rate in a later section where we analyze the behavior of the model in an environment in which stochastic shocks occur "once in a while".

IV. Effects of Money Supply Changes

In this section we analyze how exogenous disturbances affect the solution of the model. We will focus here on permanent changes in the domestic money stock.

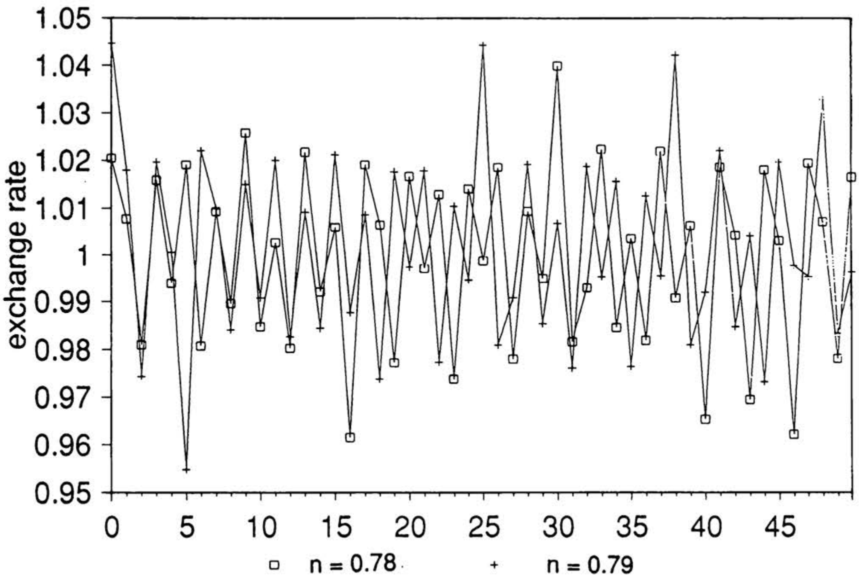


Figure 4: Sensitivity on Measurement Error Simulation 2450 to 2500.

An important characteristics of the *Dornbusch* model is that its steady state solution exhibits typical monetarist results. In particular, an increase of the domestic money stock by x% leads to an increase of the exchange rate and the domestic price level by the same x%. Thus, in the steady state PPP and the quantity theory holds, so that money is neutral in the long run.[8]

A second characteristics of the model is that the dynamics of the adjustment after the shock depends on the initial conditions. We illustrate this by applying a permanent increase in the domestic money stock of 5%. We do this in two simulations that have different initial conditions. The results are shown in figure 5. The shock in the money stock of 5% occurs in period 10 in both simulations. The only difference is that the initial conditions for the exchange rate differ (by 0.1%). We observe that the time path of the exchange rate looks quite different in both simulations. The steady state value of the exchange rate, however, is the same in the two simulations.

Note also that the exchange rate does exhibit the overshooting phenomenon. Following the news in the money stock, the new exchange rate overshoots its new steady state value. Thereafter the exchange rate moves around the new "strange attractor".

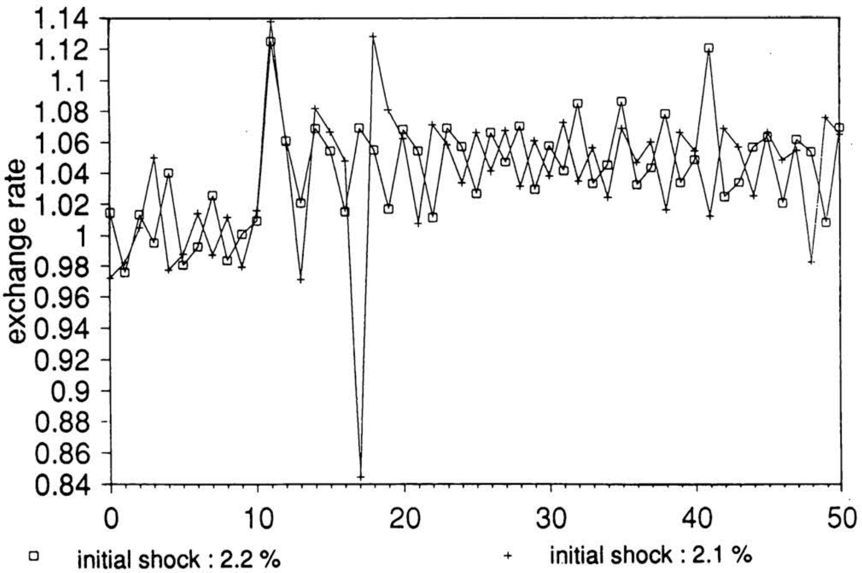


Figure 5: Permanent Shock in the Money Stock (5%) Simulation 95 to 130.

[8] It can be shown that in the steady state ($S^* = S_{t-i} = S_{t-j}$, for all j and i) the monetarist properties are satisfied, since $S^* = Ms^* = P^*$.

An interesting feature of figure 5 is the fact that there are several periods during which the exchange rate moves as much or even more than during the period when the "news" in the money stock occurs. As a result, for an outside observer of the time series of figure 5 it is not immediately obvious that in period 10 (when the shock in the money stock occurs) a change in a fundamental has occurred. It will take some time before this becomes clear. When the size of the disturbance is large relative to the inherent dynamics of the time series the fundamental change that occurs in period 10 can be inferred more quickly. We illustrate this phenomenon in figure 6 where we assume that the increase in the money stock in period 10 is 10 % (instead of 5 %).

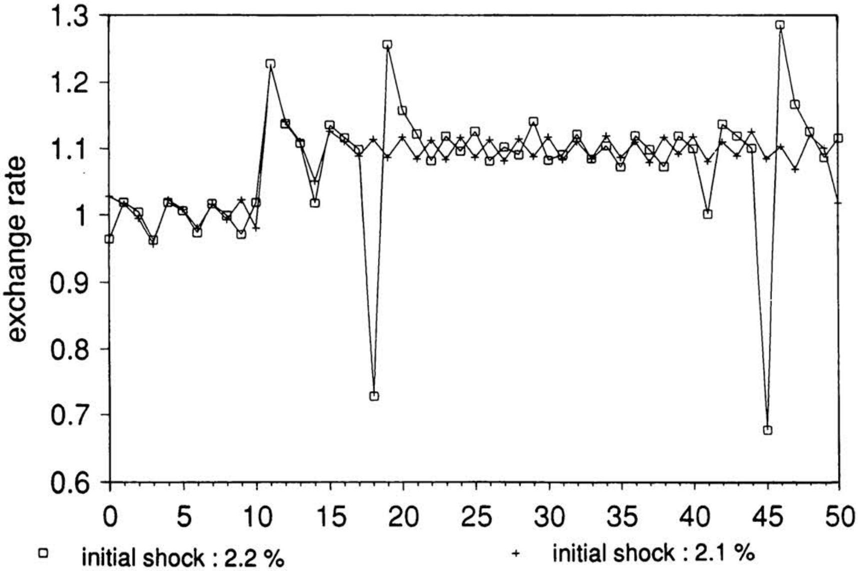


Figure 6: Permanent Shock in the Money Stock (10 %)
Simulation 95 to 130.

V. The Role of News in the Exchange Rate Dynamics

In the model analyzed so far we have established that for certain parameter values chaotic motion can occur. The characteristics of this dynamics is that it is unpredictable. We achieved this result without having recource to stochastic disturbances. In reality of course, "news" occurs and can be an important driving force of the exchange rate. In this section we therefore introduce news. We will take the view, however, that "news" does not occur every period. This seems to be more realistic than to assume that news occurs every period.

We will simulate the model assuming that random shocks occur in the money stock. These random shocks occur infrequently, i. e. most of the time there is no disturbance. Once in a while, there is a random shock in the money stock.

The simulations were constructed as follows. First we specify a random walk, i. e.

$$Ms_t = Ms_{t-1} + u_t \tag{17}$$

with u_t uniformly distributed with mean 0 and variance 0.1.

We perform this experiment using frequency ⅕ and 1⁄15, i. e. the change in the money stock occurs every 5 and 15 periods respectively. Between these periods no change in the money stock occurs.

We apply these shocks to the model that has as a solution an 8-period cycle. We do this to show that periodic solutions together with infrequent news are sufficient to generate complex dynamics. This then should hold a fortiori when we embed this infrequent news in the chaotic area. To show the latter we have also applied the money stock shocks to the model in the chaotic region. The resulting time series of these simulations are shown in figures 7 to 9.

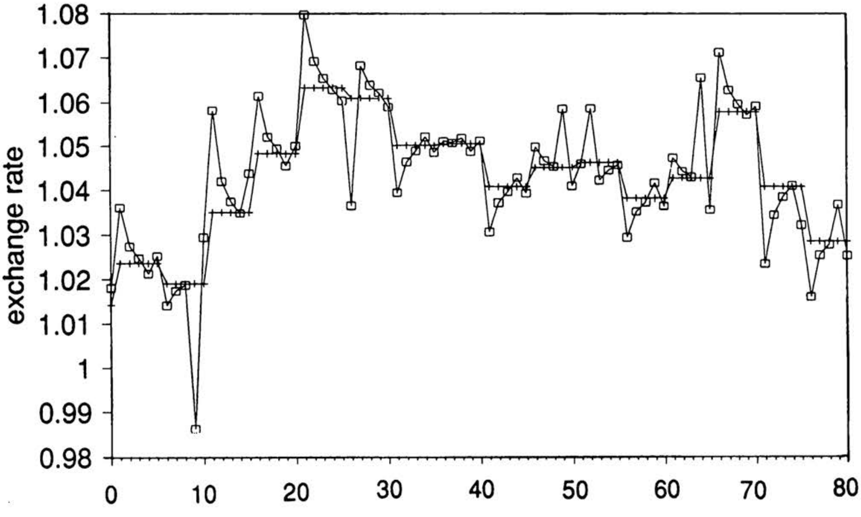


Figure 7: Simulation of the Exchange Rate with News Frequency ⅕ Embedded in Period 8 Solution.

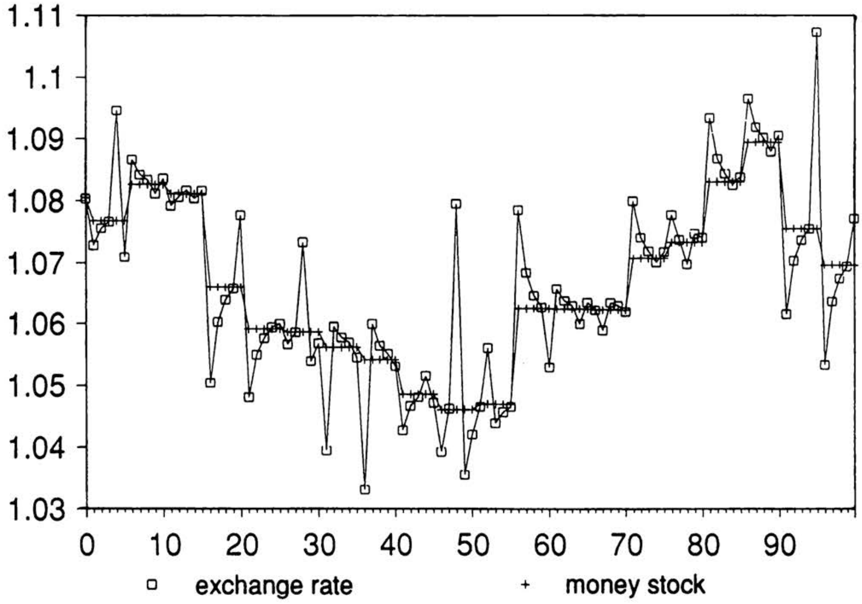


Figure 8: Simulation of the Exchange Rate with News Frequency 1/5 Embedded in Chaos Solution.

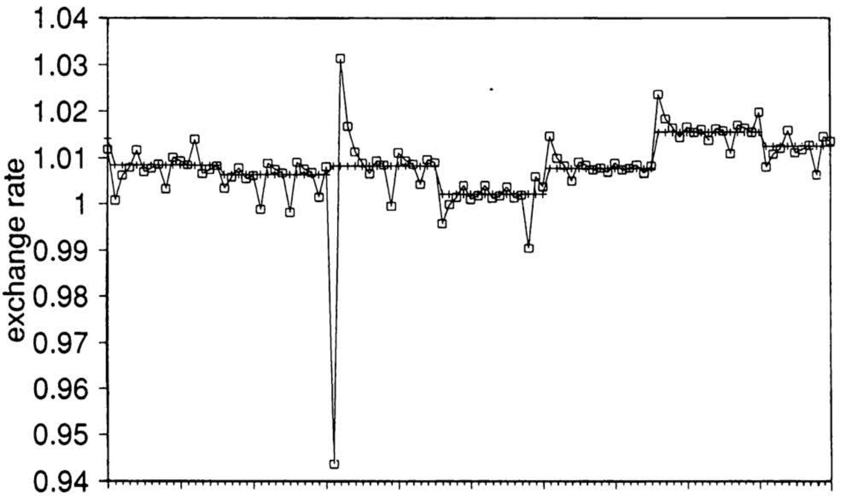


Figure 9: Simulation of the Exchange Rate with News Frequency 1/15 Embedded in Period 8 Solution.

The results of figures 7 and 8 confirm that infrequent occurrence of news (i. e. news every five periods) is sufficient to eliminate most the systematicity in the movements of the exchange rate. Note, however, that if the frequency with which news occurs is much lower than the periodicity of the cycle, the exchange rate often returns into a preditable periodicity. This is the case with a news-frequency of $1/15$ in a model that exhibits a 8-period cycle as is illustrated in figure 9.

The main reason why we do not need news to occur every time period to generate unpredictable exchange rate movements has to do with the non-linear nature of the model. The latter is able to generate autonomous dynamics. Even when the latest relevant news brought the model in a low periodic field, the exchange rate will have a nonperiodic solution before it displays its asymptotic characteristics and therefore it will be non predictable if the frequency of the news is such that it brings the model into another state before the asymptotic solution is displayed.

There is another noteworthy aspect to the results of figures 7 and 8. It appears that although the news in the money stock has quite often a substantial impact on the exchange rate, there are also many periods during which shocks in the money stock do not seem to affect the exchange rate very much. In addition, many large changes in the exchange rate occur at times when there is no news in the money stock. (The latter result was also found in the previous section which discussed the effects of permanent changes in the money stock.) These results are consistent with the recent empirical studies by *Charles Goodhart* (See Goodhart (1989) and (1990)) who showed that many exchange rate changes of the major currencies could not easily be traced back to observable news. Our model allows us to understand this empirical phenomenon.

VI. Some Preliminary Empirical Tests

Out of the large empirical literature concerning the exchange rate one dominant result emerges, i. e. unit root tests applied to exchange rates cannot easily be rejected.

A first test of our model therefore consists in applying unit root tests to the simulated exchange rates. Failure to reject the unit root hypothesis can then be interpreted as evidence (albeit preliminary) that the model's prediction of exchange rate patterns is not in contradiction with observable exchange rate behavior.

In order to test for unit roots we applied the augmented *Dickey-Fuller* test to the simulations of the model with infrequent news (1/5) using both the model under the 8-period cycle and under chaos.

We first estimated an equation of the form:

$$\Delta S_t = a_1 S_{t-1} + a_2 \Delta S_{t-1} + a_3 \Delta S_{t-2} + a_4 \Delta S_{t-3} \tag{18}$$

We tested the null hypothesis that $a_1 = 0$. The regressions of equation (18) were performed on simulation samples of different sizes. The results are presented in table 2. In none of the different cases can we reject the null hypothesis of a unit root, i.e. the t-values are well below their critical *Dickey-Fuller* values.

Table 2: **Test of $H_0 = 0$ in Equation (18)**

model with 8-period cycle

sample size	a_1	t-value	R2	DW	Q(20)
500	-0.000032	-0.07	0.4	2.1	22.7
1000	-0.000086	-0.32	0.3	2.1	72.2
3500	-0.000016	-0.11	0.3	2.1	80.3

model with chaos

sample size	a_1	t-value	R2	DW	Q(20)
500	-0.0004	-0.2	0.5	2.1	35
1000	-0.0004	-0.3	0.5	2.1	80
3500	-0.0001	-0.2	0.4	2.1	100

Despite the fact that we cannot reject the unit root hypothesis, some structure in the time series of the exchange rates exists. This can be seen from the estimated coefficients a_2, a_3 and a_4 (and their standard errors) of equation (18) as presented in table 3. These results suggest that there is autocorrelation in the exchange rate series, and that the random walk may not be the appropriate model to characterize exchange rate movements. A number of recent empirical studies have also tended to reject the random walk hypothesis to describe the observed exchange rate movements of the major currencies.[9]

[9] See *Cutler, Poterba* and *Summers* (1990).

Table 3: **Coefficients a_2, a_3 and a_4 in Equation (18)**

model with 8-period cycle			
sample size	a_2	a_3	a_4
500	-0.73 (0.04)	-0.48 (0.05)	-0.27 (0.04)
1000	-0.69 (0.04)	-0.45 (0.04)	-0.24 (0.03)
3500	-0.64 (0.02)	-0.40 (0.02)	-0.22 (0.02)
model with chaos			
sample size	a_2	a_3	a_4
500	-0.86 (0.03)	-0.60 (0.04)	-0.30 (0.03)
1000	-0.86 (0.04)	-0.61 (0.04)	-0.29 (0.03)
3500	-0.81 (0.02)	-0.56 (0.02)	-0.28 (0.02)

Note : the numbers in brackets are standard errors

VII. Chaos in the Price Level and in the Interest Rate

In this section we analyze the dynamics of the interest rate and the price level. Our main result is that a chaotic motion of the exchange rate implies that the interest rate and the price level also exhibit chaotic motion. We illustrate this feature for the interest rate in figure 10. This is the simulated interest rate under the same conditions as those prevailing for the exchange rate in figure 1. Note that the vertical axis shows the difference between the domestic and the foreign interest rate. Since the latter is assumed to be constant, the numbers can be interpreted as the changes in the domestic interest rate. Figure 11 presents the simulated price level.

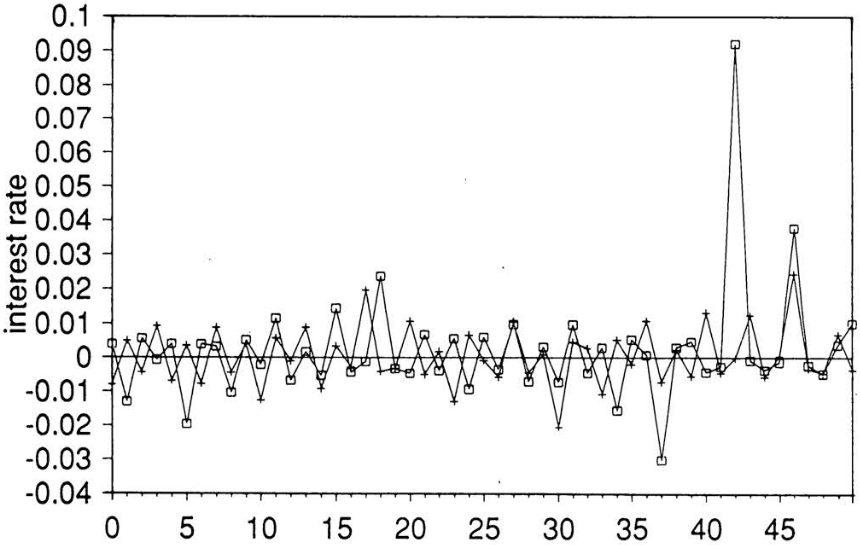


Figure 10: Chaotic Motion of the Interest Rate.

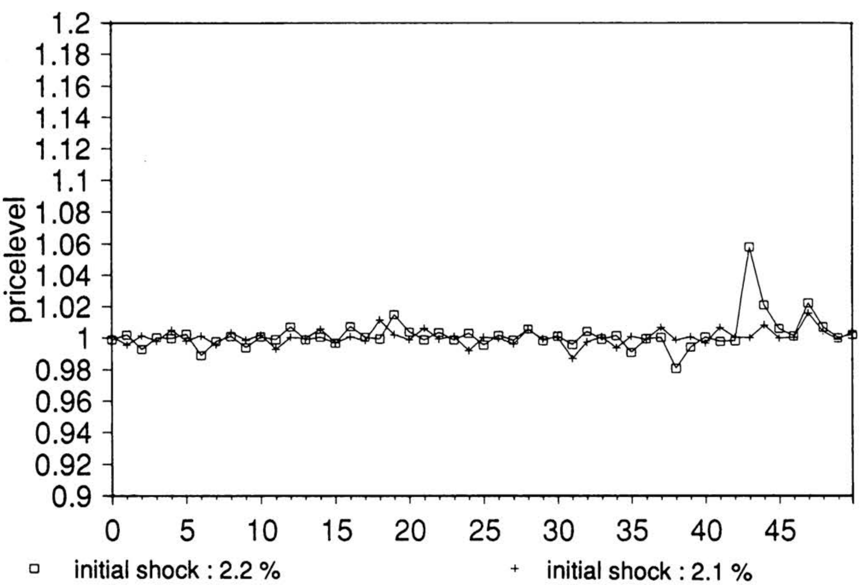


Figure 11: Chaotic Motion of the Pricelevel.

Figures 10 and figure 11 show the movements of the domestic interest rate and the domestic price level assuming the two different initial conditions as in figure 1. We obtain the same qualitative result, i. e. a small disturbance in the initial conditions leads to a completely different time path of the interest rate and price level, making these variables difficult to predict.

Finally, note that although the qualitative results are the same, both the interest rate and the price level display a smaller volatility than the exchange rate.

VIII. Conclusion

In this paper we have constructed a monetary model of the exchange rate based on the celebrated *Dornbusch* model. We have added a speculative dynamics in which "chartists" and "fundamentalists" interact and in which the weight given to these two classes of speculators changes depending on the market circumstances. The forecasting rules we have assumed for these two groups of speculators are extremely simple, if not crude. The "chartists" are assumed to extrapolate recent changes in the exchange rate using a simple moving average procedure, whereas the fundamentalists base their expectations on simple PPP calculations. We show that these simple rules implemented in the Dornbusch model are sufficient to produce very complex exchange rate behavior (chaos). These exchange rate changes are essentially unpredictable, despite the fact that the underlying model is deterministic.

The model does not generate chaos for all parameter values. For some plausible parameter values we found a cyclical behavior of the exchange rate, in other words a predictable behavior. We did find, however, that when "news" is infrequent, i. e. its frequency is not much higher than the periodicity of the exchange rate, this is sufficient to make the time series of the exchange rate unpredictable. In this connection, we found that although "news" in the money stock most often has strong effects on the exchange rate, at other times it fails to have much perceptible effect on the exchange rate. In addition, quite often turbulence in the exchange market occurs without any "news".

The results of our model allow to develop a more sophisticated view of the role of news in the foreign exchange market. The "news-paradigm" that has dominated thinking about the foreign exchange market, requires "news" to occur whenever the exchange rate changes. This has led to the situation in which market observers search for news whenever the exchange rate moves. As a rule, these observers will detect some random event which can be made responsible for the "inexplicable" movement in the exchange rate.

The results of our model lead to a different view. The speculative dynamics produced by the interaction of speculators using different pieces of information is capable of generating a complex dynamics which we do not fully understand. Although "news" remains important, we do not need to invoke it to explain *every* observed movement of the exchange rate. Many such movements are unrelated to the occurence of news, but follow an (as yet) not fully understood dynamics. Our model therefore can be considered to provide a synthesis view of the "news" model that up to recently dominated academic thinking, and the more popular view that exchange rate movements are driven by a speculative dynamics. The latter view has acquired some academic respectability recently by the work (among others) *Shiller* (1984), *Delong* et alii (1990).

Another implication of our model is that initial conditions matter. This is of particular importance to evaluate the effects of monetary disturbances. We found that the same monetary shock has quite different effects on the dynamics of the exchange rate depending on the circumstances (initial conditions) in which it is applied.

A final implication of the results of our model relates to the rational expectations assumption. We have found that very small measurement errors in the estimation of the underlying structural model completely change the exchange rate dynamics predicted by the model. This feature destroys the usefulness of structural models for predictive purposes. It may also explain why the out-of-sample forecasts made by structural models have most often been worse than "random-walk" forecasts (*Meese* and *Rogoff* (1983).

The rational expectations assumption has been based on the idea that economic agents use all relevant information, including the knowledge concerning the underlying structure of the model in which these agents operate. In our model, however, there is no incentive for economic agents to invest time and effort to gain knowledge about the underlying structural model. In order for this knowledge to be useful for predictive purposes, it would need to have a degree of precision which (today) is impossible to attain in the social sciences. It is therefore likely that economic agents use easy "rules of thumb" to compute the fundamental rate, and that they do not bother to use sophisticated structural models for predictive purposes.

Appendix

In this appendix we present some results of our model using a different specification of equation (8). We maintain the same basic assumption, i.e.

that as the exchange rate moves away from its fundamentals, the weight given to the fundamentalists tends to increase, so that the weight of the chartists declines.

Here we selected a log-linear functional form as follows:

$$(8') \qquad m_t = n - b\,(|\log(S_{t-1}) - \log(S^*_{t-1})|) \quad \text{for all } m_t > 0$$

$$m_t = 0 \text{ for } m_t < 0$$

where n is the maximum value given to the weight of the chartists. This weight tends to decline as the "misalignment" between the market and the fundamental exchange rate increases. For some critical value it becomes zero.

We simulated this version under the same parameter values as the original model. As in the case of the model discussed in the main text we foud regions of parameters for which a chaotic solution obtains. As an example we produce such a solution (for $n = 0.82$ and $b = 100$).

As can be seen from figure A1 the qualitative feature of the chaotic results in the paper can be found as well in this version of the model. The sensitivity on initial conditions is maintained in this version of the model.

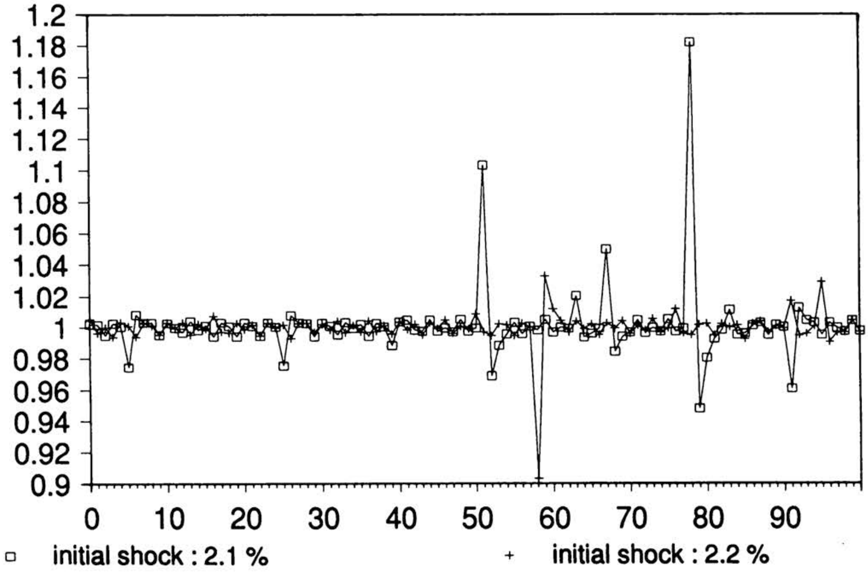


Figure A1: Sensitive Dependence on Initial Conditions Simulation 900 to 1000.

We also analyzed the extent to which the results are sensitive to the assumption that "chartists" use a 3-period moving average. Experimentation with different lags leads to the conclusion that our results are not sensitive to the choice of the lag. As an example we show the time series of the exchange rate in a model where chartists use a one period moving average (i.e. $d = 1$, $e = f = 0$). See figure A2 (we have set $n = 0.85$). The results show the same sensitivity on initial conditions.

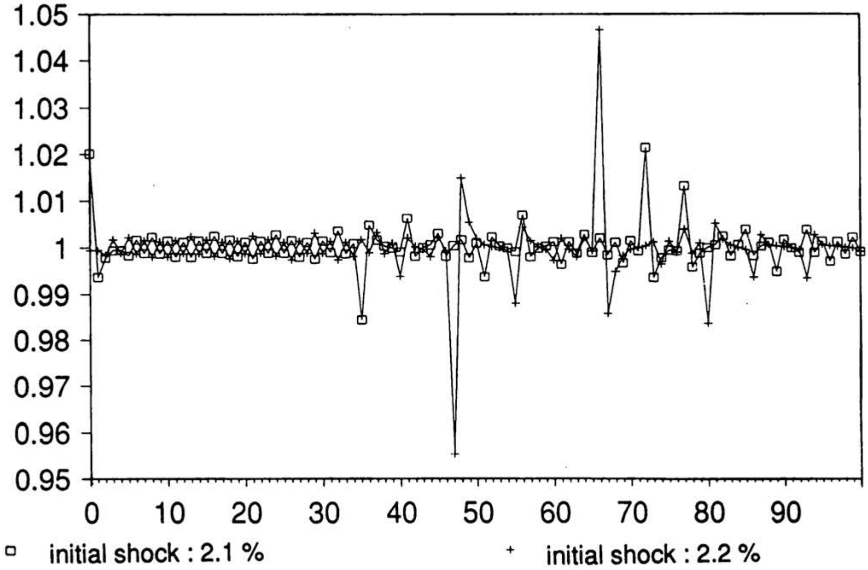


Figure A2: Sensitive Dependence on Initial Conditions Simulation 900 to 1000.

References

Allen, H. and *Taylor,* M.: Charts, Noise and Fundamentals: A Study of the London Foreign Exchange market, CEPR Discussion Paper, no. 341, Sept. 1989. – *Cutler,* D., *Poterba,* J. and *Summers,* L.: Speculative Dynamics, NBER Working Paper, No. 3242, January 1990. – *De Grauwe,* P., and *Vansanten,* K.: Deterministic Chaos in the Foreign Exchange Market, CEPR Discussion Paper, No. 370, January 1990. – *Delong,* B., *Shleifer,* A., *Summers,* L., *Waldmann:* Noise Trader Risk in Financial Markets, Journal of Finance, 1990. – *Devaney,* R.: An Introduction to Chaotic Dynamical Systems, Addison Wesley Company, Second Edition, Menlo Park, 1989, pp. 60 - 68. – *Dornbusch,* R.: Expectations and Exchange rate Dynamics, Journal of Political Economy 84, pp. 1161 - 1176, 1976. – *Frankel,* J. and *Froot:* The Dollar as a Speculative Bubble: A Tale of Chartists and Fundamentalists, NBER Working Paper, No. 1854, March 1986. – *Frankel,* J.: Chartists, Fundamentalists and Trading in the Foreign

Exchange Market, NBER Reporter, winter 1989/90, 9 - 12. – *Frenkel,* J. and *Mussa,* M.: Asset Markets, Exchange Rates and the Balance of Payments, in Jones, Ronald W. and Kenen, Peter B., eds., Handbook of International Economics Vol. II, Amsterdam, North-Holland, 1985. – *Goodhart,* C.: News and the Foreign Exchange Market, LSE Financial Market Group, Discussion Paper No. 71, January 1990. – *Levich,* R.: Empirical Studies of Exchange Rates: Price Behaviour, Rate Determination and Market Efficiency, Handbook of International Economics, vol II, Jones R. and Kenen P., Elsevier Science Publishers, 1985. – *Meese,* R. and *Rogoff,* K.: Empirical Exchange Rate Models of the Seventies: Do They Fit Out-of-Sample?, Journal of International Economics, 1983, 3 - 24. – *Mussa,* M.: The Theory of Exchange Rate Determination, in *Bilson,* John F. O. and *Marston,* Richard C., eds., Exchange Rate Theory and Practice, Chicago: University of Chicago Press, 1984, pp. 13 - 78. – *Scheinkman,* J. and *Lebaron,* B.: Nonlinear Dynamics and Stock Returns, Journal of Business vol. 62 no. 3 (1989). – *Shiller,* R.: Stock Prices and Social Dynamics, Brooking Papers on Economic Activity, 1984, No. 2, 457 - 98.

Summary

Chaos in the Dornbusch Model of the Exchange Rate

In this paper a model of the exchange rate is presented. The model incorporates interactions between different classes of agents. We consider two classes: fundamentalists who use PPP to forecast future exchange rates and chartists, using simple moving average schemes.

Moreover we allow the relative strength of these two classes to change over time. Imposing a specific functional form for the relative strength we obtain a closed form solution for the model.

The main conclusion of the paper is that these interactions can lead to chaotic paths of the exchange rates. This source of instability is independent of any arrival of news and can explain the recent findings of exchange rate movements that cannot be attributed to new information.

Zusammenfassung

Chaos im Dornbusch-Modell für den Wechselkurs

Dieser Beitrag beschreibt ein Modell für den Wechselkurs. Er berücksichtigt Interaktionen zwischen unterschiedlichen Kategorien von Akteuren. Wir befassen uns mit zwei dieser Kategorien, nämlich mit den Fundamental-Analysten zum einen, die ihren Prognosen für den jeweiligen künftigen Wechselkurs die Kaufkraftparität zugrundelegen, und den Chart-Analisten zum anderen, die dazu einfache Tabellen für den gleitenden Mittelwert verwenden.

Ferner unterstellen wir, daß sich die relative Stärke dieser beiden Kategorien im Zeitverlauf ändert. Gibt man für die relative Stärke eine spezifische funktionelle Gliederung vor, erhält man ein Modell in geschlossener Form.

Die Hauptschlußfolgerung dieses Beitrags besteht darin, daß diese Interaktionen zu chaotischen Entwicklungen der Wechselkurse führen. Diese Labilitätsquelle ist unabhängig von neuen Erkenntnissen und kann für die kürzlich festgestellten Wechselkursbewegungen verantwortlich zeichnen, die sich nicht auf neue Erkenntnisse zurückführen lassen.

Résumé

Chaos dans le modèle du cours du change de Dornbusch

Cet article présente un modèle du cours du change. Le modèle incorpore des interactions entre différentes classes d'agents. Nous considerons deux classes: celle des fondamentalistes qui utilisent la «PPP» pour prévoir les cours du change futurs, et celle des charistes, qui utilisent des simples mouvements moyens.

En plus, nous permettons à ces deux classes de modifier au cours du temps leur force relative. En imposant une forme fonctionelle spécifique pour la force relative, nous obtenons une solution pour le modèle en soi.

La conclusion principale de cet article est la suivante: ces interactions peuvent mener les cours de change dans des directions chaotiques. La source d'instabilité est indépendante de toute arrivée d'informations et peut expliquer les conclusions récentes sur les mouvements du cours de change, qui ne peuvent pas être attribués à nouvelles informations.