# Purchasing Power Parities for the DM

## A Cointegration Exercise*

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## I. Introduction

Purchasing Power Parity (PPP) is probably one of the oldest theoretical approaches to exchange rate determination. Notwithstanding, it has not lost its relevance as one of the major building blocks, i. e. equilibrium conditions of even very recent theoretical attempts to explain exchange rate behaviour. Most prominent among these is the monetary approach to exchange rate determination as presented, e. g. by *Bilson* (1976, 1978, 1979) in which the assumption is made that PPP even holds in the short run, or the *Dornbusch* model (Dornbusch 1976) which shows how deviations from PPP might occur and under what circumstances a movement back to the equilibrium should take place.

The term "equilibrium" here is very important because it makes the point clear that whenever deviations from PPP occur – however these deviations are measured for the moment – there are at least two variables that can bear the burden of adjustment towards equilibrium, namely the exchange rate and/or relative prices. This fact should be emphasized since it has become obvious to us that a great deal of public opinion, in e. g. the forex markets, sees PPP not as an equilibrium condition but as a theory determining the exchange rate per se.

This view of PPP is to some extent understandable and probably reflects a lot of earlier open economy thinking which indeed saw the exchange rate as the relative price of two national incomes. In this way of thinking, the exchange rate was mainly driven by trade flows, and whenever country A had a trade surplus with country B there was excess demand for A's currency on the foreign exchange markets and a subsequent revaluation. Since

* This paper has been presented at Seminars at the University of Basel and Swiss Bank Corporation and at the 6th International Meeting on Monetary Economics and Banking in Lyon, 1989.

trade flows are influenced by – among other things – relative prices (i.e., a trade surplus develops for goods in country A are cheaper than similar goods in country B), the causal flow in these models is clearly from price levels to the exchange rate. This is not to say that it was not understood that the exchange rate might have some influence on the internal price system, too. But this was generally not accounted for in those models. Since they are still relatively widespread, at least among practitioners, so it should not be too surprising that the exchange-rate-determination view of PPP is still prominent.

The more recent literature on exchange rate determination (overviews of the literature are provided by *Frankel* (1980), *Heri* (1982), *Kohlhagen* (1978) among others) regards the exchange rate as being the relative price of financial assets, whether money alone as in the monetary approach to the exchange rate (*Bilson* 1976, 1978, 1979) or other internationally tradable assets as in the portfolio balance approach (examples are *Branson* (1977), *Henderson* (1979) and *Kouri* (1976)). In those approaches, the major determinants of exchange rates are demand for and supply of domestic and foreign financial assets. Another important factor in these more recent models is expectations. Indeed, through the inclusion of expectations these theories result in quite a menu of different exchange rate determination approaches.

Here, PPP – if it features at all – is again used as an equilibrium condition and in a couple of models the above mentioned feedback from the exchange rate to the domestic price level is explicitly taken into account (one example in this tradition is *Bernholz / Gaertner / Heri* (1985)).

When it comes to empirical investigations of Purchasing Power Parity one gets the impression that the concrete approach ranks somewhat lower than the theory itself – at least over more recent experiences. *Frenkel* (1976, 1978, 1980) presents evidence broadly in favour of PPP for the 1920's, although his econometric methodology has been challenged by *Edison* (1985), who applies dynamic specification modeling techniques. Again *Taylor / McMahon*'s (1988) results are strongly supportive of PPP over the same sample period covered in *Frankel*'s paper. They explicitly test the equilibrium condition in a cointegration framework as suggested among others by *Engle / Granger* (1987). A similar exercise was performed by *Kirchgaessner* (1988) for the Seventies and Eighties with rather inconclusive results.

The plan of our paper is as follows: We first give a very brief introduction to the PPP concept (section II). In section III we introduce the cointegration framework which we use in section IV to get a clue as to whether PPP gives at least some information about possible long run co-movements of exchange rates and relative prices. Since PPP is indeed a very simplified concept for discussing the relationship between an asset price and its possible fundamental determinants, we show that a degree of pragmatism is required when tests are performed. Inclusion of dummy variables for periods of possible "bubble" behaviour of exchange rates or for estimation periods which do not always cover the overall period of the recent floating seem to be necessary in some cases. The last section is reserved for a summary and a couple of conclusions.

## II. The Theoretical Concept

The theoretical literature knows at least two different approaches to PPP theorizing – the absolute and the relative form of Purchasing Power Parity. The absolute form is often connected with the name of *Gustav Cassel* but the roots probably go back to *David Ricardo* or even to the school of Salamanca. In the absolute form an equilibrium is attained if, in two countries, goods or baskets of goods are purchased for the same price when denominated in the same currency, i. e.

$$e_{ij} = \frac{P_i}{P_j} \tag{1}$$

where $e_{ij}$ = exchange rate (domestic in terms of foreign currency)
$P_i$ = Price level in country $i$
$P_j$ = Price level in country $j$

As emphasized (1) is an equilibrium condition, and as such embodies no specific hypothesis concerning the direction of causality.

Deviations from such an equilibrium, i. e. when the exchange rate between two countries does not reflect relative prices correctly, imply in principle some sort of arbitrage possibility on goods markets. In other words, since the basket of goods in country A is cheaper than in country B it is bought in the first and sold in the second. Those implied flows of goods result in flow demand and supply of foreign exchange which will, all things being equal, restore a new equilibrium. This is theory, of course, but more or less characterizes the traditional flow approach mentioned above, in which the exchange rate is defined as the relative price of different baskets of goods or different national outputs. In the more recent stock approaches, the exchange rate is viewed as the relative price of financial assets.

One of the more prominent among the stock approach theories is the monetary gambit, where the exchange rate is the relative price of two national monies. Seen through these monetarist glasses, a change in money supply in country i would lead to a proportional change in all nominal variables (such as exchange rates). According to this interpretation, therefore, the Purchasing Power Parity theory can alternatively be viewed as an implication of the monetaristic world of money neutrality.

The equilibrium as formulated in (1) is very restrictive at least if not a dogmatic monetarist view is pursued and if it is taken into account that a significant portion of goods and (especially) services are not even accessible to international trade or at least are not sufficiently comparable (such as all the specific national goods).

These facts mean that a perfect relationship of costs and prices between different countries does not hold good any longer.

Very often, therefore, equation (1) is augmented by a proportionality factor

$$(2) \qquad e_{ij} = \frac{P_i}{P_j} \cdot \alpha \qquad \alpha \neq 1$$

When $\alpha$ is assumed to be constant over time this leads to the relative form of Purchasing Power Parity.

This form postulates that in the sate of equilibrium, exchange rate movements proportionally correspond to the differences between changes in the price levels of the countries in question.

$$(3) \qquad \frac{e_{ij}, r+1}{e_{ij,t}} = \frac{P_{i,t}}{P_{j,t+1}} : \frac{P_{i,t}}{P_{j,t}}$$

Again, equations like (1) and (2) are equilibrium conditions. As such they do not have to hold at every point in time. Indeed foreign exchange market experience has proven that deviations from PPP not only exist but can be very large and long lasting.

The most important factors responsible for those deviations are (see *Heri / Wolf* (1985)).

(i) Transportation costs, taxes, tariffs and non-tariff barriers to trade (modern forms of protectionism).

(ii) Real factors such as differences in tastes, technology, and resource supplies.

(iii) Monetary factors such as shifts in monetary policy and reshuffling of international portfolios, and so on.

Therefore, whenever PPP is discussed, we should not forget that – being a purely bivariate concept – it may not only be the oldest but also the least sophisticated approach to the relationship between the exchange rate and possible fundamental variables. A great deal is disregarded, and only the empirical work can show whether this was wise or not.

But then, if we go into the empirical aspects of Purchasing Power Parity, there are even more problems to be mentioned.

*No. 1* is the choice of appropriate price indices.

This choice depends more or less on the fundamental interpretation of PPP. In the flow approach, in which commodity arbitrage plays a dominant role, there is a tendency to take price indices of traded goods (such as commodity prices, import and export prices).

In the monetary interpretation, on the other hand, a broader price index embracing a large basket of traded and non-traded goods is required. A consumer price index, unit labour costs, or – as a measure of inflationary expectations – an index based on long-term interest rates can be used for the calculations. The choice of the wholesale price index as done here is often used in this context as a sort of compromise.

*No. 2* is the choice of the appropriate base period in which the exchange rates in question are considered to be in balance. Like the index problems mentioned above, the selection of the base period can be a very arbitrary business too, because it implicitly requires that we can state precisely, for a selected period of time, when the exchange rate correctly reflects the relative competive position of two countries.

All these problems are not explicitly dealt with in this paper. Not that they are thought to be irrelevant, but we feel that the major results, i.e. the tendencies, should be recognisable even if some statistical problems cannot fully be resolved.

## III. The Cointegration Framework

Purchasing Power Parity belongs into the category of economic thinking for which *Granger* (1986, p. 213) writes: "At the least sophisticated level of economics lies the belief that certain pairs of economic variables should not diverge from each other by too great an extent – at least in the long run. Thus such variables may drift apart in the short run or according to seasonal factors, but if they continue to be too far apart in the long run then economic forces, such as market mechanisms or government intervention, will begin to bring them together again." Although such co-movement very often seems intuitively appealing, the correctness of long-term relatedness is an empirical matter after all. The idea underlying the cointegration framework allows specification and testing of models that deal with such long-run relationships at least for particular types of variables that are very often found to occur in macroeconomics.

A rather new jargon has been developed in the cointegration literature. A stationary series $X_t$, i.e. a series with time invariant mean and covariance function, is said to be integrated of order zero, or

$$X_t \sim I(0) \tag{4}$$

If the series needs to be differenced in order to achieve stationarity it is said to be integrated of order one, i.e.

$$X_t \sim I(1) \tag{5}$$

and more generally, if the series needs to be differenced $d$ times in order to become $I(0)$ it is called

$$X_t \sim I(d) \tag{6}$$

The most obvious example of $I(0)$ or $I(1)$ processes are a white noise $\varepsilon_t$ for which $\varrho_k = \mathrm{COR}(\varepsilon_t, \varepsilon_{t-k}) = 0$ for all $k = 0$, and a random walk for which $X_t = X_{t-1} + \varepsilon_t$, respectively.

For a pair of variables to be cointegrated a necessary condition is that they be integrated of the same order. If two variables $X_t$ and $Y_t$ are both $I(1)$ then it is generally true that a linear combination like

$$Z_t = X_t + a\,Y_t \tag{7}$$

will also be $I(1)$.

However, if a constant a exists such that

$$Z_t \sim I(0) \tag{8}$$

which means that although $X_t$ and $Y_t$ do both have infinite variances, the linear combination $Z_t$ is stationary,[1] then the two series are said to be coin-

[1] It is important to note here that a $I(0)$-process has a mean and a tendency to return to its mean, and autocorrelations that decline rapidly with lag increases.

An $I(1)$-process, on the other hand, will be relatively smooth, wander widely and only rarely return to an earlier value. Autocorrelations of an $I(1)$-process are all near one in magnitude even for a large $k$ since they can be shown to be

$$\varrho_k = 1 - |k|/t \tag{i}$$

tegrated – which means they don't drift apart from each other by too great an amount and they have similar tendencies.

Although the above discussion might seem somewhat remote from our main theme of Purchasing Power Parity, it is obvious that if economic theory suggests a long-run relationship between two variables like

$$X_t = a + bY_t + \varepsilon_t \tag{9}$$

then the above framework is of interest since unless $\varepsilon_t$ is $I(0)$ in case of $X_t$ and $Y_t$ being $I(1)$ – a process which is very typical for macroeconomic time series (*Granger* (1986), p. 214) – $Y_t$ and $X_t$ will tend to drift apart infinitely. Hence cointegration of a pair of variables is a necessary condition for the existence of a linear long-run relationship.

Empirical tests of the cointegration framework have to be performed in at least two steps.

The first step is a test of whether $X_t$ and $Y_t$ are $I(1)$. A test philosophy for this has been provided by *Dickey / Fuller* (1981). It is based on the following regression equation

$$\begin{aligned} X_t &= a + bX_{t-1} + \sum_{j=1}^{n} c_j \Delta X_{t-j} + \text{dTREND} + \varepsilon_t \\ Y_t &= a + bY_{t-1} + \sum_{j=1}^{n} c_j \Delta X_{t-j} + \text{dTREND} + \varepsilon_t \end{aligned} \tag{10}$$

where the *ns* are selected to be large enough to insure white noise residuals.

The relevant test statistic in this framework is the ratio of $b$ over its OLS standard error. The null hypothesis

$$H_0:\ X_t, Y_t \sim I(1)$$

and the null is rejected for (absolutely) large negative "$t$-ratios". The test statistic does not, however, have a $t$-distribution under the null because of the theoretically infinite variance of $X_t$ and $Y_t$. However, *Fuller* (1976) reports tables and critical values for those $t$-ratios.

The second step of the cointegration test consists of the so-called "cointegration regression",

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A pure random walk, for example, solves to

$$X_t = \varepsilon_t + \varepsilon_{t-1} + \varepsilon_{t-2} + \ldots \tag{ii}$$

with a variance of $\sigma_x^2 = t\sigma_\varepsilon^2$ with can obviously become indefinitely large as $t$ increases. See (e.g. *Pindyck / Rubinfeld* (1976) p. 431 ff.).

(11) $$X_t = a + b\,Y_t + \varepsilon_t$$

and the test of whether the residuals $\varepsilon_t$ appear to be $I(0)$ or not.

*Stock* (1984) has shown that when series are cointegrated OLS estimates of the cointegration equation (11) result in so-called super-efficient estimates of the cointegration factor a in equation (7). This means that if $X_t$ and $Y_t$ are cointegrated the OLS residuals from (11) will be consistent estimates of the $I(0)$ linear combination $Z_t$ in (7). One should emphasize here that this sort of empirical methodology is in stark contrast to the "philosophy of differencing" non-stationary time series before regressing them on each other, which was very fashionable in much of the empirical literature only a few years ago.

A simple test of the null hypothesis

$$H_o:\ X_t,\, Y_t\ \textit{not}\ \text{cointegrated}$$

is based on the previously mentioned *Dickey / Fuller* test for $\varepsilon_t$. It turns out, however, that not only can we not use the conventional $t$-test, but in this case even the critical values tabulated by *Fuller* (1976) are not useful. *Engle / Granger* (1987), however, have tabulated critical values for this kind of exercise.

To come back now to the question of Purchasing Power Parity it should be noted that quite a lot of the exchange rate literature – at least the work dealing with the efficient markets hypothesis – draws the conclusion that exchange rates appear to follow a random walk.[2] Thus there seem to be good reasons to assume that exchange rates may be $I(1)$. If the nominal exchange rate and domestic-to-foreign price ratios are to be cointegrated, i. e. if there is a long-run (linear) relationship between exchange rates and prices which can be detected in a cointegration framework, then relative prices must be $I(1)$ also and the residual of the cointegration (OLS) regression in which prices are regressed on exchange rates or vice versa must be $I(0)$. Results of tests for the West German mark rates are discussed in the next section.

## IV. Empirical Results

The data we used for testing the degree of cointegration of exchange rates and relative prices are for West Germany. We used the respective DM-rates and the ratio of the West German price indices to those of the US, France,

[2] See *Giddy / Dufey* (1976), *Heri* (1986), *Levich* (1979) among others.

Italy, the Netherlands, Switzerland, Austria, the UK, Japan, Canada, Sweden, Finland, Norway, Denmark and Australia. The price series used are the respective wholesale prices or equivalents thereof. The use of quite a wide spectrum of different bilateral rates and prices allows us to apply the framework not only to freely floating rates (assuming they exist at all) but also, for example, to the EMS – where exchange rates are to some extent supposed to change with respect to changes in relative prices.

As mentioned above, the first tests have to deal with the question of whether exchange rates and relative prices are $I(1)$, i.e. non-stationary in levels.

*Table 1* gives results for an augmented *Dickey / Fuller* test over the period from 1974 to 1987 with monthly data. The tests were performed with and without inclusion of a trend variable and Table 1 shows the results for both of the test runs.

Given are $t$-statistics for the parameter of the respective lagged level variable in a regression of first differences on a constant, two own lagged values and the one-period lagged level variable. The first two columns give the results for the respective exchange rate. With one single exception all the parameters have the theoretically implied negative sign, and as expected most exchange rates series seem to be more or less $I(1)$. We mentioned above that this result is not very surprising given the conclusions of a lot of foreign exchange market efficiency studies in recent years.

The next two columns present the results for the international price ratio variables. The results are very similar to the ones above in that most $b$-parameters do not seem to be significant but with the exception of 3 out of 28 they do have the correct signs. Most price ratios therefore seem to be $I(1)$, too. This result had not necessarily to be expected on theoretical grounds although it was emphasized for example by *Granger* (1986) that many macroeconomic time series appear to be $I(1)$.[3]

We have argued above that this sort of time series behaviour was necessary in order to perform the straightforward cointegration tests that were discussed in the last section. We therefore continued by running the cointegrating regressions for all the bilateral rates and prices, normalising alternatively on the nominal exchange rate and relative prices.

The cointegrating regression is interesting not only because its error process is an unbiased estimate of $Z_t$, the linear combination of prices and exchange rates in equation (7), but also because it gives an unbiased esti-

[3] See also, for example, *Nelson / Plosser* (1982) on this point.

*Tabelle 1*
**Augmented Dickey / Fuller test statistics for exchange rates and relative prices**

| Relation | Exchange Rates | | Relative Prices | |
|---|---|---|---|---|
| | $d = 0$ | $d = 0$ | $d = 0$ | $d = 0$ |
| DM/US$ | − 0.84 | − 0.85 | − 1.95 | − 2.48 |
| DM/FF | − 0.44 | − 2.63 | 1.00 | − 2.91 |
| DM/LIRA | − 1.81 | − 2.63 | − 1.79 | − 1.45 |
| DM/HFL | − 1.19 | − 3.28* | − 2.59 | − 1.63 |
| DM/SFR | − 2.42 | − 2.83* | − 2.85* | − 0.17 |
| DM/AS | − 3.33* | − 2.00 | − 2.67 | − 3.18* |
| DM/POUND | − 1.61 | − 2.19 | − 3.30* | − 3.55** |
| DM/YEN | − 0.76 | − 2.90 | − 0.90 | − 2.36 |
| DM/C$ | − 0.68 | − 1.14 | − 1.21 | − 2.29 |
| DM/SKR | − 0.23 | − 2.89 | 0.17 | − 2.55 |
| DM/FMK | − 1.09 | − 1.73 | − 2.20 | − 3.71** |
| DM/NKR | 0.53 | − 1.52 | 0.67 | − 1.08 |
| DM/DRK | − 0.56 | − 1.37 | − 0.97 | − 1.17 |
| DM/A$ | − 1.36 | − 1.24 | − 0.72 | − 2.16 |

The table is based on the following regression equation:

$$X_t = a + bX_{t-1} + \sum_{j=1}^{n} c_j \, \Delta X_{t-j} \, (+ \text{ dTREND}) + \varepsilon_t$$

in which the role of $x(t)$ is taken either by the respective bilateral exchange rates or the respective relative prices. The statistical significance of the $b$'s is tested by use of the critical values from *Fuller* (1976). The null hypothesis is that the series in question is $I(1)$.

The rejection region is $\{t \varepsilon R | t < c\}$ with $c = -3.15$, $-3.45$ or $-4.04$ at a significance level of 10%, 5% or 1% respectively.

*, **, *** indicate significance at the 10%, 5% and 1% levels, respectively.

mate of parameter $b$ (see equation (9)), for which theory has an explanation, too.

If PPP is indeed a relevant equilibrium condition, i.e. if the real exchange rate is a constant in the long run, then $b$ should not be too far away from unity. Unfortunately, we do again not have correct estimates of the standard errors of $b$ under the null of $b = 1$. However, *Kirchgaessner* (1988) provides estimates of standard errors under the null of $b = 0$. Using the critical values given there, we can test whether prices do at least have anything to do with exchange rates or vice versa at all. The results are in Table 2.

The cointegrating regressions have in most cases again been estimated for the period from 1974 to 1987. In some cases, though, the estimation period

*Tabelle 2*
**Cointegrating Regressions**

| Relation | Estimation period | Regression | $t$ statistic for coefficient $b$ |
|---|---|---|---|
| DM/US$ | 74 - 87 | $e(t) = \ \ 0.75 + 0.42\, p(t)$<br>$p(t) = -\,0.00 + 0.22\, e(t)$ | 4.1 |
| DM/FF | 74 - 87 | $e(t) = \ \ 4.85 + 1.04\, p(t)^{***}$<br>$p(t) = -\,4.32 + 0.87\, e(t)^{***}$ | 41.0 |
| DM/LIRA | 74 - 87 | $e(t) = -\,1.85 + 0.89\, p(t)^{***}$<br>$p(t) = \ \ 1.69 + 1.09\, e(t)^{***}$ | 92.8 |
| DM/HFL | 74 - 87 | $e(t) = \ \ 4.50 + 0.54\, p(t)^{***}$<br>$p(t) = -\,6.22 + 1.38\, e(t)^{***}$ | 22.4 |
| DM/SFR | 74 - 87 | $e(t) = \ \ 5.30 + 1.25\, p(t)^{***}$<br>$p(t) = -\,3.42 + 0.62\, e(t)^{***}$ | 24.2 |
| DM/SFR | 74 - 85 | $e(t) = \ \ 5.28 + 1.23\, p(t)^{***}$<br>$p(t) = -\,3.74 + 0.69\, e(t)^{***}$ | 28.8 |
| DM/AS | 74 - 87 | $e(t) = \ \ 2.66 + 0.16\, p(t)$<br>$p(t) = -\,1.72 + 0.61\, e(t)$ | − 4.2 |
| DM/POUND | 74 - 87 | $e(t) = \ \ 1.38 + 0.62\, p(t)^{***}$<br>$p(t) = -\,1.76 + 1.28\, e(t)^{***}$ | 25.4 |
| DM/POUND | 80 - 87 | $e(t) = \ \ 1.48 + 1.07\, p(t)^{***}$<br>$p(t) = -\,1.22 + 0.81\, e(t)^{***}$ | 25.0 |
| DM/YEN | 74 - 87 | $e(t) = -\,0.29 + 2.77\, p(t)^{***}$<br>$p(t) = \ \ 0.10 + 0.27\, e(t)^{***}$ | 22.2 |
| DM/C$ | 74 - 87 | $e(t) = \ \ 0.56 + 0.81\, p(t)^{*}$<br>$p(t) = -\,0.27 + 0.61\, e(t)^{*}$ | 12.7 |
| DM/SKR | 74 - 87 | $e(t) = \ \ 0.21 + 0.99\, p(t)^{***}$<br>$p(t) = -\,0.25 + 0.96\, e(t)^{***}$ | 59.8 |
| DM/FMK | 74 - 87 | $e(t) = \ \ 0.97 + 0.74\, p(t)^{***}$<br>$p(t) = -\,1.49 + 1.09\, e(t)^{***}$ | 26.3 |
| DM/NKR | 74 - 87 | $e(t) = -\,0.60 + 0.88\, p(t)^{***}$<br>$p(t) = \ \ 1.09 + 1.02\, e(t)^{***}$ | 39.4 |
| DM/DKR | 74 - 87 | $e(t) = -\,1.10 + 0.94\, p(t)^{***}$<br>$p(t) = \ \ 1.14 + 1.04\, e(t)^{***}$ | 82.4 |
| DM/A$ | 74 - 87 | $e(t) = \ \ 0.03 - 0.75\, p(t)$<br>$p(t) = -\,0.61 - 0.54\, e(t)$ | − 10.6 |

The cointegrating regressions are

$$e_t = a + bp_t + \varepsilon_t$$

$$p_t = a + be_t + \varepsilon_t \qquad \text{while } p = \frac{p_i}{p_j}$$

The rejection regions for the null of $b = 0$ is $\left\{ t \varepsilon R | t < c \right\}$ with $c = 12.40$, 14.87 and 19.65 for 10%, 5% and 1% significance level, respectively. (*Kirchgaessner* 1988)) has furthermore been adapted to take account of possible policy changes or other arguments which will be discussed below.

The first impression one gets after a look at Table 2 is in fact that in most cases prices (exchange rates) are highly significant – judged by the critical values as given by *Kirchgaessner* (1988) – in an exchange rate (price) equation. The only exceptions are the DM/US$ case, the DM/A$ case and the DM/Austrian schilling case. In the last of these however, a caveat applies, since both relative prices and the exchange rate did not seem to be $I(1)$ in the first place (see Table 1) in which case the cointegration regression is misspecified.

A second look at the table reveals that the estimates of $b$, in many cases, are indeed not too far removed from unity and the results are largely invariant to the choice of the normalising variable.

The major reason, of course, for estimating the regressions in Table 2 is the fact that cointegration between $e$ and $p$ imposes the $I(0)$-restriction on the error terms $_t$ estimated in Table 2. In order to test this proposition we have performed augmented *Dickey / Fuller* tests on the cointegrating regression residuals.[4] The results are in Table 3.

As mentioned above, if $e$ and $p$ are $I(1)$ and cointegrated, we would expect $I(0)$-error terms in a cointegrating regression. In this case we would expect significant $b$'s in equations like the ones given in the footnote to Table 1.

Table 3 shows that in 7 out of 14 cases the $I(1)$ null hypothesis can be refuted which means essentially that in 50% of the cases studied exchange rates and relative prices seem to be cointegrated and that in these cases some sort of long-run relationship between exchange rates and relative prices seems to exist.

Some comments on the choice of estimation period as given in Tables 2 and 3 seem called for here. The general estimation period is from 1974 – roughly one year after the "breakdown" of the Bretton Woods System – to the end of 1987. It is clear that this period has been loaded with exogenous

[4] In Table 2 we do not use the cointegrating regression *Durbin / Watson* (CRDW) statistic in order to test directly whether $p$ and $e$ are cointegrated – a procedure which has often been used in similar exercises (e.g., *Taylor / McMahon* (1988)). The reason is that since it has been shown by *Engle / Granger* (1987) that this test is highly dependent on the dynamics of the errors in the cointegrating equation and that the power of an augmented *Dickey / Fuller* test is higher than the power of a CRDW-test if the error processes are of a higher order than 1. Such error processes are indeed to be expected in a PPP framework since it is well known that adjustment towards a PPP equilibrium – if it occurs at all – is very sluggish. Augmented *Dickey / Fuller* is indeed the test procedure recommended by *Engle / Granger* under general circumstances.

*Tabelle 3*
**Augmented Dickey/Fuller test statistics for residuals from cointegrating regressions**

| Relation | Estimation period | Normalized on $e$ | Normalized on $p$ | |
|---|---|---|---|---|
| DM/US$ | 74 - 87 | − 1.05 | − 2.08 | |
| DM/FF | 74 - 87 | − 2.16 | − 1.75 | |
| DM/LIT | 74 - 87 | − 3.01* | − 2.87* | a) |
| DM/HFL | 74 - 87 | − 3.56** | − 4.35*** | |
| DM/SFR | 74 - 87 | − 2.49 | − 2.11 | |
| | 74 - 85 | − 3.60** | − 3.17** | |
| DM/AS | 74 - 87 | − 3.20* | − 2.47 | |
| DM/POUND | 74 - 87 | − 2.21 | − 2.08 | |
| | 80 - 87 | − 3.75** | − 3.50** | |
| DM/YEN | 74 - 87 | − 2.65 | − 2.64 | |
| DM/C$ | 74 - 87 | − 1.22 | − 1.35 | |
| DM/SKR | 74 - 87 | − 2.98* | − 2.89* | b) |
| DM/FMK | 74 - 87 | − 1.77 | − 1.80 | |
| DM/NKR | 74 - 87 | − 2.39 | − 2.53 | |
| DM/DKR | 74 - 87 | − 2.97* | − 3.05* | |
| DM/A$ | 74 - 87 | − 1.25 | − 0.46 | |

The table is based on a similar regression as in Table 1. But here $x(t)$ is the residual of the respective cointegrating regression and $d$ is defined as zero. $n$ is usually taken to be 3. Exceptions are given in the footnotes below.

The null hypothesis is that the series in question is $I(1)$. The rejection region is $\left\{ t \varepsilon R | t < c \right\}$ with $c = -2.84$, $-3.17$ and $-3.77$ at a significance level of 10%, 5% or 1% respectively (*Engle / Granger* (1987)).

a) with $n = 2$, b) with $n = 6$

real shocks which induced dramatic changes in real exchange rates at least over shorter periods of time. In general, we have not tried to deal with these shocks, although there may be some sort of "bubble-dummy" or similar devices to take account of foreign exchange market anomalies which might have been justified in our study. The only two facts we have tried to account for are the extreme behaviour of the British pound in the early and mid-70's and the lack of depreciation of the Swiss franc against the DM in the last two years notwithstanding the better inflation performance of West Germany against Switzerland. If we adapt the estimation period in those two cases, Table 3 reveals that we get cointegration of exchange rates and prices in both instances.

Another interesting result in Table 3 is the fact that in the DM/FF case the series do not seem to be cointegrated notwithstanding the fact that Germany and France are major players in the EMS. Here one gets the impression that although currencies are adapted from time to time to changes in price competitiveness, what is done is usually too little and too late. Indeed, if you look at the inflation differences accumulated over the last four years between France and West Germany, you get the impression that a DM-revaluation is overdue, notwithstanding the transitory strength of the franc in mid-1988.

## V. Summary and Conclusions

Some sort of purchasing power parity (PPP) is probably lurking in the mind of every economist who's talking about equilibrium exchange rates. Indeed, the idea that the rate of exchange of two currencies should somehow reflect their internal purchasing power is almost as old as economics itself.

Accordingly, PPP has been empirically tested in every period in which it was a bone of contention, including the current period of (quasi) flexible exchange rates which began in the mid 70's. Furthermore, due to its simplicity, PPP was always a popular guinea pig for new statistical applications whenever they were developed.

This paper is in the same tradition. We think the newly developed concept of cointegration is especially well suited to deal with the issue of Purchasing Power Parity because the cointegration framework allows a sort of direct test of long-run equilibrium relationships. Such exercises were rather difficult to perform until recently, since the (near) unit-root-characteristics of most economic (level) time series made it hard to apply conventional statistical methods to those variables.

In section III of our paper we have given a very brief introduction to the cointegration framework which we have applied in the following sections to exchange rates and prices for West Germany and 14 of its major trading partners (US, France, Italy, Netherlands, Switzerland, Austria, UK, Japan, Canada, Sweden, Finland, Norway, Denmark and Australia). The estimation period we use covers, with two exceptions, the whole of the current floating period, i.e. from 1974 to 1987.

Briefly summarized, the results indicate that

(i) in most cases analysed, not only the exchange rate but also relative (wholesale) prices seem to be $I(1)$, i.e. non-stationary in level.

(ii) In 50% of the cases relative prices and exchange rates seem to be cointegrated, i.e. a long-run equilibrium relationship seems to exist between relative prices and the exchange rate.

(iii) In most cases, the parameter connecting the two time series does not seem to be too far away from unity, although this hypothesis could not be tested explicitly in our paper since we did not want to deal here with the problem that we don't have correct standard errors of the parameters in the so-called cointegration regression, at least not under the null that the parameters are unity.

Part of our tests have been based on the time series behaviour of the error term in the so-called cointegration regression. PPP had to be rejected in every instance in which the error process seemed to be $I(1)$ (for details the reader is referred to section III). It should be noted however that in most cases in which the hypothesis that the error processes are $I(1)$ could not be rejected, the autocorrelation function fell off very rapidly, which might be an indication that the critical values used are somewhat to restrictive.

Overall, the results are more in favour of Purchasing Power Parity for the DM rates than a couple of papers which use more traditional econometric methods seem to suggest. Furthermore, they seem to imply that although PPP is definitely not a forecasting device for exchange rates, it can help in designing risk-management procedures or at least in giving an idea about long-run tendencies in bilateral exchange rates.

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## Zusammenfassung

### Die Kaufkraftparitätentheorie für die Deutsche Mark

Die Kaufkraftparitätentheorie bildet einen der wohl ältesten theoretischen Bausteine zur Bestimmung langfristiger Wechselkursgleichgewichte. Auch in den neueren Ansätzen der Wechselkursdeterminierung hat sie als wichtige Gleichgewichtsbedingung Eingang gefunden und so über die Zeit kaum an Relevanz verloren. Im vorliegen-

den Papier wird dieser Gleichgewichtsansatz anhand der Theorie kointegrierter Prozesse auf seine empirische Evidenz hin untersucht. In einem einführenden Teil (Abschnitt I und II) wird kurz das wechselkurstheoretische Credo der letzten Jahrzehnte beleuchtet und das theoretische Konzept der Kaufkraftparitätentheorie in den wesentlichen Zügen vorgestellt. Ferner wird auf einige Probleme einer solchen empirischen Modellierung bzw. Überprüfung eingegangen.

Abschnitt III enthält eine Einführung in die unter anderem von *Engle* und *Granger* – zur Überprüfung langfristiger Gleichgewichtsbeziehungen – entwickelte Theorie kointegrierter Zeitreihen. Unter Abschnitt IV werden die empirischen Ergebnisse für 14 bilaterale Wechselkurse zwischen der DM und einer Reihe wichtiger Industrieländer für die Periode 1974 - 1987 präsentiert und diskutiert. Die Ergebnisse der Kointegrationsschätzung sprechen in der Hälfte aller Fälle für die Gültigkeit der Kaufkraftparitätentheorie. Interessant scheint dabei, daß neben den wirklich frei floatenden Wechselkursen DM/US-$ und DM/Yen, auch für den DM/FF-Fall – mit Frankreich als wichtigsten EWS Partner Deutschlands – die Hypothese einer lagfristigen Gleichgewichtsbeziehung im Sinne der Kaufkraftparitätentheorie verworfen werden mußte.

## Summary

### The Purchasing Power Theory with Relation to the D-Mark

The purchasing power theory represents one of the probably oldest theoretical components in determining long-term exchange-rate equilibrium. It forms part also of the most recent approaches to the determination of exchange rates as an important equilibrium condition and has thus hardly lost in relevance over time. The present paper analyzes this approach to equilibrium on the basis of the theory of co-integrated processes with regard to its empirical evidence. The introductory (part I and II) throws some light on the exchange rate credo of the last few decades and briefly presents the main characteristics of the theoretical concept underlying the purchasing power parity theory. Moreover, the paper deals with a number of problems involved in such empirical model-building and/or revision.

Part III contains an introductory to the theory of co-integrated time series as developed by *Engle* and *Granger*, among others, for reviewing long-term equilibrium relations. Part IV presents and discussed the empirical results of 14 DM-related bilateral exchange rates with a number of important industrialized countries in the period from 1974 to 1987. The results of the co-integration assessment speak in favour of the validity of the purchasing power parity theory in 50% of the cases. It may be interesting in this context that the hypothesis of long-term equilibrium relations within the meaning of the purchasing power parity theory had to be rejected also for the DM/FF rate – with France being Germany's most important partner in the EMS – besides the DM/US-$ and the DM/yen rates enjoying real freedom of float.

## Résumé

### La théorie de la parité du pouvoir d'achat pour le Deutsche Mark

La théorie de la parité du pouvoir d'achat forme un des plus anciens éléments théoriques pour déterminer les équilibres à long terme des cours de change. Dans les nouvelles théories servant à déterminer les cours de change, elle représente aussi une condition d'équilibre importante et a donc toujours conservé au fil du temps sa signification. Dans ce travail, l'évidence empirique de cette notion d'équilibre est analysée sur base de la théorie de processus cointégrés.

Dans l'introduction (chapitres I et II), les auteurs expliquent brièvement le crédo de la théorie des cours de change des dernières décennies et présentent les principales caractéristiques de la théorie de la parité du pouvoir d'achat. Ensuite, ils approfondissent certains problèmes d'un tel point de vue empirique et les analysent.

Le chapitre III comprend une introduction à la théorie des séries chronologiques cointégrées, développée entre autres par *Engle* and *Granger* – pour contrôler les relations d'equilibre à long terme. Dans le chapitre IV, les résultats empiriques pour 14 cours de change bilatéraux entre le DM et une série de pays industrialisés importants sont présentés et discutés pour la période allant de 1974 à 1987. Dans la moitié de tous les cas, les résultats de l'estimation de cointégration confirment la validité de la théorie de la parité du pouvoir d'achat. Il est intéressant de constater qu'à côté des cours de change vraiment flottants du DM et du dollar américain ainsi que du DM et du Yen, également pour le cas du DM et du FF – la France étant le partenaire le plus important du FME de l'Allemagne –, l'hypothèse d'une relation d'équilibre à long terme dans le sens de la théorie de la parité du pouvoir d'achat a dû être rejettée.