On Passive Money, Exchange Rates and Monetary Leadership*

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I.

The purpose of this paper is to consider the relation between the variability of the exchange rate and the possibility of national control over the domestic money supply, under the hypothesis that the international money stock is not a datum with respect to the world economy but is continually adjusted to the demand for international money (the "needs of trade criterion" or "demand standard").

This problem does not appear to have been examined before. Up to present the theory of passive or accomodating money has been stated in terms of a closed economic system (e. g. *Black* 1972, *Olivera* 1970 and 1971, *Sargent* and *Wallace* 1975, *Turnovsky* 1979) whereas the analytical literature on monetary interdependence among countries (extensively surveyed by *Mussa*, 1979) treats the world money supply as an exogenous datum.

II.

The argument will be developed on the basis of a general equilibrium model. We shall now enumerate its assumptions, part of which will be relaxed at a subsequent stage:

(1) The world economy comprises n countries and n products. Each country produces a single commodity. Labour is the sole scarce factor and enters into the production of every commodity. The supply of labour is a given quantity in each country. The input of labour required per unit of output is constant for each commodity. Wages are advanced at the beginning of the production period, which has equal length in all industries. Trade is unrestricted and there are no costs of transport.

(2) In each country the general means of exchange is a fiduciary paper-money issued by the respective government. Production is financed by the sale of riskless promissory notes, each of which obliges the issuer to pay one unit of money one period later. These bonds are tradeable everywhere but are expressed in the money of the debtor.

(3) Apart from the national currencies there exists an international circulating medium, which is universally accepted in exchange for products and services. The total quantity of this asset is regulated by the issuing agency to keep it in conformity with the market demand for it. Free convertibility is allowed between all monetary instruments, whether national or international.

(4) Each good has a uniform market-price, which is viewed by every consumer and producer as independent of his decisions. The budget constraint operates strictly upon each consumer. The excess-demand functions for commodities, real cash balances and real bonds are positively homogeneous of degree zero in nominal prices and assets. No distribution effects occur either within each country or between different countries. Future prices are expected with certainty to coincide with present prices.

As to the mathematical properties of the model, the excess-demand functions will be taken to be continuously differentiable. The matrix of first derivatives will be supposed to be non-singular at each equilibrium point, a traditional assumption which has been underpinned by contemporary findings about the density of the set of regular economies (see *Dierker*, 1974, chapter 10).

We shall use the following main symbols: q, money price; s, money wage; z, rate of interest; x, output; c, labour coefficient; r, exchange rate; N, supply of labour; M, quantity of national money; G, quantity of international money; E, B, L, H, excess demand for products, bonds, national money and international money, respectively. Subscripts will be employed, in a self-explanatory way, to identify countries or goods.

Under the hypotheses set forth we have the equilibrium equations:

$$q_i = (1 + z_i)\, s_i\, c_i\ ,$$
$$N_i = c_i\, x_i\ ,$$
$$E_i\,(X) = 0\ ,$$
$$B_i\,(X) = 0\ ,$$
$$L_i\,(X) = 0\ ,$$
$$H\,(X) = 0\ ,$$

where $i = 1, \ldots, n$ and $(X) = (r_1\,q_1, \ldots, r_n\,q_n, r_1\,s_1, \ldots, r_n\,s_n, z_1, \ldots, z_n, x_1 \ldots, x_n, r_1\,M_1, \ldots, r_n\,M_n, G)$. The rs are understood here as rates of conversion of national money into international money.

Because of the budget constraints, the equilibrium system includes just $5\,n$ linearly independent relations. Let us normalize the money variables by measuring them in wage units. We choose $r_h\,s_h$ as deflator, $h \in (1, \ldots, n)$. There result the $5\,n$ real variables:

$$r_i\,q_i/r_h\,s_h, r_j\,s_j/r_h\,s_h, z_i, x_i, r_i\,M_i/r_h\,s_h, G/r_h\,s_h \;,$$

with $i = 1, \ldots, n$ and $j = 1, \ldots, h-1, h+1, \ldots, n$. Clearly the real system is well determined.

What is usually called the monetary system — the equilibrium system in the $5\,n + 1$ undeflated unknowns — has exactly one degree of freedom. If country 1, for example, fixes both its national money supply and its exchange rate, thus making $r_1\,M_1$ a datum for the international economy, the monetary equilibrium becomes determinate. The values of the remaining $r_i\,M_i$ are then established, along with the rest of the monetary unknowns, by the equilibrium conditions.

It follows that one — but no more than one — of the n countries can manage at will its money supply and yet exercise control over the international value of its currency. The other $n-1$ countries must limit themselves to the regulation of either the exchange rate or the money supply.

The lack of international equality in this respect is due to the permissiveness of the world money stock. If G were autonomously prescribed, the single degree of freedom of the equilibrium system would thereby be eliminated without causing any asymmetry in the international distribution of monetary power.

Further insight into the described hegemony situation is provided by the displacements of equilibrium which result from variations of monetary data. The comparative static phenomena that interest us are summarized in the following table, where λ denotes a positive scale factor and $j = 2, \ldots, n$:

$$\lambda\,M_1 \to \lambda\,(s_1, q_1, r_2\,s_2, r_2\,q_2, r_2\,M_2, \ldots, r_n\,s_n, r_n\,q_n, r_n\,M_n, G) \;,$$
$$\lambda\,r_1 \;\to \lambda\,(r_2\,s_2, r_2\,q_2, r_2\,M_2, \ldots, r_n\,s_n, r_n\,q_n, r_n\,M_n, G) \;,$$
$$\lambda\,M_j \to (1/\lambda)\,r_j, \lambda\,(s_j, q_j) \;,$$
$$\lambda\,r_j \;\to (1/\lambda)\,(s_j, q_j, M_j) \;,$$

the omitted variables being unaltered.

It is evident from the table that, in terms of the "policy assignment problem", country 1 can employ M_1 to rule its international monetary balance and r_1 to dictate the international price level. The other members of the system can only pursue domestic monetary goals.

The preceding observations do not depend closely on the model to which they refer. Any general equilibrium model of international trade and payments would yield similar results. Indeed, apart from the monetary specifications, we have used no more than the determinateness of the real system and the corresponding dimensional property of the monetary equilibrium.

III.

We now pass on to characterize the international passive-money economy from a dynamical point of view. In the equations system of the foregoing section let us take $r_1, \ldots, r_n$ and M_1 as exogenous data. The equilibrating process may then be formulated by the differential equations:

$$
\begin{aligned}
dx_i/dt &= q_i - (1 + z_i)\, s_i\, c_i\,, \\
ds_i/dt &= c_i\, x_i - N_i\,, \\
dq_i/dt &= E_i\,(X)\,, \\
dz_i/dt &= -\,B_i\,(X)\,, \\
dM_j/dt &= L_j\,(X)\,, \\
dG/dt &= H\,(X)\,, \\
(i = 1, &\ldots, n;\quad j = 2, \ldots, n)
\end{aligned}
$$

where t signifies time. In accord with standard practice, the reaction constants have been equated to unity.

Alternately, if $M_1, \ldots, M_n$ and r_1 are given, we put

$$
dr_j/dt = L_j\,(X)
$$

in lieu of the expression for dM_j/dt, with the same convention as regards the speed of adjustment. The rest of the dynamical equations carry over without change.

In both cases there exist integral solutions over any compact set of initial values (see *Abraham* and *Robbin*, 1967, page 57, Parameterized Flow Theorem).

The resulting dynamical systems may be linked up with models of balance-of-payments adjustment, if we agree to identify the internal

excess supply of money with the foreign deficit of each national economy. In terms of this construction, the adjustment rule laid down for M_j implies that the latter varies proportionately to country j's balance-of-payments surplus. A similar meaning attaches to the equation for r_j, when M_j is a datum.

Some key facts about the stability of equilibrium are perceived by focusing attention on the strictly monetary part of the adjustment processes. For this purpose we assume that the level of employment and the real wage are constant in each country, and hence that the volume of output, the rate of interest and the relative price of each product remain stationary everywhere. To take advantage of the "dynamics on the real plane" techniques we set $n = 2$, and adopt $r_1 M_1$ instead of $r_h s_h$ as general deflator. The equilibrium conditions for the money markets then reduce to

$$L_2 \, (r_2 \, M_2, G) = 0 \; ,$$
$$H \, (r_2 \, M_2, G) = 0 \; .$$

The associated matrix of signs depends crucially on the substitutability or complementarity between the two assets. The decisive consideration in this respect is that the international medium can be used to make payments in every country, whilst the national currency has in principle no more than local circulation. The sign pattern is, therefore,

$$\partial L_2 / \partial \, (r_2 \, M_2) < 0 \; , \qquad \partial L_2 / \partial \, G < 0 \; ,$$
$$\partial H / \partial \, (r_2 \, M_2) > 0 \; , \qquad \partial H / \partial \, G < 0 \; .$$

Now, if r_2 is fixed, the money-market dynamics are given by

$$dM_2/dt = L_2 \, (r_2 \, M_2, G) \; ,$$
$$dG \, /dt = H \, (r_2 \, M_2, G) \; ,$$

a system which, as may readily be verified, possesses global asymptotic stability. The corresponding equilibrium is either a stable node or a stable focus.

On the other hand, if M_2 is fixed, the adjustment equations are

$$dr_2/dt = L_2 \, (r_2 \, M_2, G) \; ,$$
$$dG/dt = H \, (r_2 \, M_2, G) \; ,$$

and also here the equilibrium turns out to be globally stable. The approach to equilibrium may be monotone or oscillatory, just as in the previous system. A certain degree of "overshooting" may therefore happen, but only in the course of damped fluctuations.

It is illustrative to compare these results with those that obtain under the monetarist prescription of keeping G constant (the stationary-economy form of the constant money growth rule: cf. *Mayer* 1975). The adjustment process with pegged exchange rates is then described by the equations

$$dM_1/dt = L_1 (r_1 M_1, r_2 M_2) ,$$
$$dM_2/dt = L_2 (r_1 M_1, r_2 M_2) ,$$

with partial derivatives

$$\partial L_1/\partial (r_1 M_1) < 0 , \qquad \partial L_1/\partial (r_2 M_2) > 0 ,$$
$$\partial L_2/\partial (r_1 M_1) > 0 , \qquad \partial L_2/\partial (r_2 M_2) < 0 .$$

The system just defined lacks qualitative stability: it can be stable or unstable depending on the parameter values. The case of instability is of the saddle-point type. Hence, in order to guarantee convergence, it is necessary to imagine that somehow the economy "jumps" to the stable arm of the saddle if it is initially off it.

Quite the same picture emerges from the adjustment process with flexible exchange rates, as represented by the equations

$$dr_1/dt = L_1 (r_1 M_1, r_2 M_2) ,$$
$$dr_2/dt = L_2 (r_1 M_1, r_2 M_2) ,$$

where the Ms are invariant through time.

Although we have assumed static expectations, the signs matrices utilized in the preceding argument are compatible with a variety of expectations hypotheses. In particular, the effects of learning processes upon the agents' expectations functions (see *Fuchs*, 1979) can be allowed for without changing the qualitative framework.

IV.

To recapitulate, we have seen that if the supply of international money is governed by the demand for it (the "needs of trade criterion" or "demand standard") then one and only one country may determine both its national money supply and the exchange rate between its home currency and the international circulating medium. We have found, furthermore, that the related mechanisms of international monetary adjustment are qualitatively asymptotically stable. And we have observed that neither this fact nor the indicated asymmetry in the distribution of monetary control can occur when the international money supply is exogenous.

The two parts of the conclusions arrived at differ in degree of generality. The hegemonic element presupposes that the international money supply is an endogenous variable, but the stability result depends moreover on the specific adjustment principle embodied in the needs of trade criterion. The equilibrium of the world money markets would be completely unstable if the international monetary authority followed a "foreign exchange standard" type of adjustment rule, expanding the international money stock in proportion to the excess supply of the hegemonic currency.

References

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Zusammenfassung

Über passives Geld, Wechselkurse und geldpolitische Führerschaft

Der Beitrag untersucht das Verhältnis zwischen der Veränderlichkeit des Wechselkurses und der staatlichen Steuerung der inländischen Geldmenge, unter Zugrundelegung der Hypothese, daß der internationale Geldbestand keine feststehende Größe darstellt, sondern laufend der Marktnachfrage angepaßt wird („Handelsbedarfskriterium" oder „Nachfragekriterium"). Bei Anwendung dieser Hypothese zeigt es sich, daß nur ein einziges Land seine eigene Geldmenge sowie auch seinen in internationalen Währungseinheiten ausgedrückten Wechselkurs bestimmen kann, während alle übrigen Länder keine andere Wahl haben, als entweder ihre Geldmengen oder aber ihre Wechselkurse festzulegen. Ferner wird festgestellt, daß die mit dem resul-

tierenden internationalen Gleichgewicht verbundenen Geldmechanismen qualitativ stabil sind, was bei sonst identischen Annahmen dann nicht zutrifft, wenn der internationale Geldbestand unveränderlich ist.

Summary

On Passive Money, Exchange Rates and Monetary Leadership

In this paper the relation is examined between the variability of the exchange rate and the national control over the domestic money supply under the hypothesis that the international money stock is not a datum but is continually adjusted to market demand (the "needs of trade criterion" or "demand standard"). It is shown that, under this hypothesis, one and only one country may determine both its national money supply and its exchange rate in international monetary units, whereas all the other countries must choose between fixing their money supplies and pegging their exchange rates. It is verified, moreover, that the monetary mechanisms associated with the resulting world equilibrium are qualitatively stable, a property which, under otherwise identical assumptions, does not obtain if the international money supply is rigid.

Résumé

De la monnaie passive, des taux de change et de la fonction de guide de la politique monétaire

L'étude examine le rapport entre la variabilité du taux de change et la maîtrise publique de la masse monétaire domestique dans l'hypothèse que les disponibilités monétaires internationales ne constituent pas une grandeur invariable, mais sont constamment ajustées à la demande du marché (critère des besoins commerciaux ou de la demande). Dans cette hypothèse, il se vérifie qu'un pays seulement est en mesure de déterminer tant sa propre masse monétaire que son taux de change exprimé en unités monétaires internationales, tandis que tous les autres pays n'ont pas d'autre choix que de fixer soit leurs masses monétaires, soit leurs taux de change. L'on constate en outre que les mécanismes monétaires liés à l'équilibre international qui en résulte sont qualitativement stables, ce qui ne se vérifie pas dans des hypothèses identiques lorsque les disponibilités monétaires internationales sont invariables.