

Output, Prices and Interest Rates over the Business Cycle

How well does a Keynesian-Neoclassical Synthesis Model perform?*

By Werner Roeger**

1. Introduction

The correct interpretation of the relationship between real and nominal variables over the business cycle is one of the central macroeconomic challenges. For a long time there was nearly unanimous agreement on what had to be explained, namely a positive relationship between real output and prices at cyclical frequencies. The widespread acceptance of the Phillips curve as an empirical regularity signifies this common perception. In various papers Lucas (1972, 1976), for example, regarded this correlation as a central feature of modern business cycles. In a series of contributions, Kydland (1989), Cooley and Ohanian (1991), Kydland and Prescott (1991) and Backus and Kehoe (1992) challenged this view by providing empirical evidence that the correlation between prices and output over the business cycle is indeed strongly negative in industrial countries over the post-war period. Finally, Chada and Prasad (1994) reconciled both views. They find that detrended prices are strongly countercyclical, nevertheless there also exists a significant positive correlation between inflation and the cyclical output component. Cooley and Hansen (1994) added two additional stylised facts to this list, namely first the observation that prices are leading output and second a lead of GDP over inflation. Again, this is a surprisingly robust pattern in the data to be found for many industrialised countries, especially over the flexible exchange rate period. There exists another important but puzzling relationship between nominal and real variables, namely a strong negative correlation between short term nominal interest rates and the lagged growth rate of real GDP. Fuhrer and Moore (1995) give a recent

* Verantwortlicher Herausgeber / editor in charge: B. F.

** I would like to thank participants at the Annual Meeting of the Verein für Socialpolitik in Bern and two anonymous referees for helpful comments. The views expressed in this paper are those of the author and should not be attributed to the Commission of the European Communities.

account of this relationship for the US. This leading indicator property of the short term interest rate is also often exploited for forecasting GDP.

It is difficult to replicate these stylised facts with conventional equilibrium business cycle models. As recently shown by Cooley and Hansen (1998) neither the conventional real business cycle (RBC) model nor extensions of it, which allow for nominal wage contracting or limited information of economic agents, are able to generate a negative correlation of the price level and a positive correlation of inflation with real GDP. The paper therefore addresses the question, whether it is possible to account for these stylised facts by allowing for both monetary and technology shocks and by adding additional frictions on the adjustment of prices. Allowing for technology shocks is mainly motivated by the ability of the standard RBC model to generate negative price output correlations. Adding nominal price rigidities will strengthen the impact of monetary shocks on real variables in the model. Our assessment on the relative importance of these additional elements will be based on stochastic simulations of a model calibrated on West German data for the flexible exchange rate period and prior to German unification.

The model used here departs from the standard RBC model in at least three important dimensions. First, to motivate price rigidities we assume that firms are monopolistically competitive and face convex price adjustment costs. This is similar to Rotemberg (1982, 1996) and Hairault and Portier (1992) who also assume quadratic costs of adjusting prices as a source of nominal rigidity. Secondly, in order to better fit basic labour market facts at business cycle frequencies, we depart from the hypothesis of a fully neo-classical labour market and replace it by a search model along the lines introduced by Pissarides (1990) and implemented already into an RBC model for the US by Merz (1995) and Andolfatto (1996). Both authors show that by introducing trading frictions into the labour market a number of labour market stylized facts can be better replicated than with the standard neo-classical labour supply function. Among these stylised facts are the smaller cyclical variability of real wages relative to productivity and the fact that employment lags output. Such a departure from a the pure neoclassical specification of the labour market can also be motivated by the observation that only a very small segment of the labour market corresponds to the Walrasian notion of a spot market, where market participants meet to negotiate a new labour contract with a market clearing price every day. In addition, labour services offered and demanded in each segment of this market are certainly not homogeneous. Many highly differentiated skills and abilities are traded in the labour market. Neither for firms nor workers is the type of work required or the quality of work offered completely transparent. Given the information sets of both firms and workers it seems very difficult to

achieve an optimal match in very short periods of time. Thus trade in the labour market must be regarded as highly uncoordinated, time consuming and costly for both workers and firms. Our contribution, however, differs somewhat from existing models. Unlike in Merz and Andolfatto, intertemporal substitution of leisure does not play a role for labour supply decisions, instead we follow Pissarides and base wage behaviour on a concept of permanent income maximization, for given unemployment benefits as the reservation wage. Thus wages are set as a mark-up over benefits. This is a standard feature the search model shares with alternative labour market hypotheses, like union wage bargaining or efficiency wage models (see, for example, C. A. Pissarides (1998) for a recent exposition). We also extend that framework and introduce overlapping nominal wage contracts. Therefore the modification to the standard RBC model suggested in this paper not only implies different behaviour of labour market participants, but the labour market also exhibits nominal rigidities. Finally, we adopt an open economy framework. This is more appropriate, given the significant degree of openness of the German economy. The suggested framework may also prove suitable for data sets pertaining to other European countries.

These features make this model much less Walrasian than other models presented so far in the dynamic general equilibrium tradition. Indeed, with these assumptions our model can be regarded as representing a modern version of a “Keynesian-neoclassical synthesis”. By exposing this model to the RBC testing procedure, this analysis also helps to bridge the gap between dynamic equilibrium macro models and more traditional dynamic macroeconomic models like, for example, the IMF’s MULTIMOD or the European Commission’s QUEST II model.

The paper is organized as follows. In the next section the model is presented. Sections 3 and 4 discuss model solution and calibration. Section 5 presents the major stylised facts for Germany. In section 6 simulation results are presented and a sensitivity analysis is conducted in order to see more clearly the contribution of individual model features for an understanding of important business cycle facts. The paper ends with some concluding remarks.

2. A Dynamic Small Open Economy Model

The economy consists of a continuum of households indexed by z on the closed interval $[0,1]$, as well as N identical firms and a government. Firms are owned by domestic households and produce goods which are imperfect substitutes both within and across countries. There exists perfect capital mobility between the domestic economy and the rest of the world but zero

labour mobility. Households derive utility from consumption net of transaction cost $C_{t,z}^*$. The intertemporal utility function maximised by individual households is time separable and logarithmic and is given by

$$(1) \quad E_t \sum_{j=0}^{\infty} (1 + \theta)^{-j} \log(C_{t+j,z}^*) .$$

We follow Leeper and Sims (1994) and define gross private consumption $C_{t,z}$ as

$$(2) \quad (P_t^c/P_t)C_{t,z} = (P_t^c/P_t)C_{t,z}^* + \omega[Y_{t,z}/(M_{t,z}/P_t)]^v Y_{t,z} \quad \text{with } v > 0 .$$

Here P_t^c is the consumer price deflator which differs from P_t because households consume both domestic and foreign goods. It is assumed that transaction costs increase with the volume of transactions, here approximated by current factor income of the household ($Y_{t,z}$) and with velocity. According to this specification transaction costs approach zero as real money balances go to infinity. A non-monetary equilibrium does not exist in this economy, since transaction cost are unbounded as $M_{t,z}$ approaches zero.

Unlike in conventional neoclassical growth models, there is no leisure term in the utility function. Instead it is assumed that households negotiate wages with firms by maximising the surplus of both parties from a successful job match. Also, we assume that, because of important fixed costs associated with going to work, households will not choose hours of work but can only supply a fixed number of hours per period which we normalize to one¹. One very useful side effect of our specification in the context of open economy models will be the absence of a wealth effect in the labour supply decision of households. As pointed out by Devereux et al. (1992) and Correia et al. (1995)², such models of labour supply are better suited in an open economy context to capture typical cross country correlation patterns. Maximization is carried out subject to the following budget constraint in real terms

$$(3) \quad A_{t+1,z} = (1 + r_t)A_{t,z} + (Y_{t,z}^h - C_{t,z}P_t^c - T_{t,z} - i, M_{t,z})/P_t .$$

Each individual household can be in two states at each particular date. Either he is currently employed or unemployed. Current net labour income of the individual household ($Y_{t,z}^h$) is equal to W_t if the household is employed and equal to Z_t in the case of unemployment. The variable T_t represents a

¹ The analysis abstracts from any transition between in and out of the labour force.

² These authors arrive at a labour supply decision rule which neglects income effects by assuming a model with home production.

lump sum tax to the government. The last term in the budget constraint is equal to the interest foregone – it is the nominal short term interest rate – of holding money instead of capital. It measures the implicit consumption of money services. Households can transfer income across periods by holding real wealth (A_t) in the form of shares in domestic firms (V_t) yielding a real return equal to r_t government bonds (B_t) with a domestic nominal interest rate i_t it and net foreign assets ($e_t F_t$) denominated in domestic currency, where e_t is the nominal exchange rate defined as the price in D-Mark of one unit of foreign currency, yielding a foreign nominal interest rate of i_t^w as well as real money balances (M_t/P_t). Thus, real wealth is given by

$$(4) \quad A_{t,z} = V_{t,z} + B_{t,z} + e_t F_{t,z} + M_{t,z}/P_t .$$

Our formulation of the budget constraint assumes that households regard the three assets as perfect substitutes and capital is perfectly mobile internationally. Thus expected returns on government and foreign bonds are equalised period to period, according to the following interest arbitrage condition

$$(5) \quad i_t = i_t^w + E_t [\Delta e_{t+1}/e_t] .$$

Arbitrage between assets yielding a nominal return and equity requires that it is equal to r_t plus expected inflation between period $t+1$ and t

$$(6) \quad i_t = r_t + E_t [\Delta P_{t+1}/P_t] .$$

At each date the household decides about current consumption, the holding of wealth in the next period and real money balances, given his current and future expected wage and benefit income stream, lump sum taxes and interest rates and the current level of financial wealth. The optimal decision rules of the household can be obtained by maximising (1) subject to the constraints (2) and (3) with respect to consumption, wealth and real money balances, which gives the system of stochastic Euler equations (subject to the transversality condition)

$$(7a) \quad [C_{t+j,z}^* P_{t+j}^c P_{t+j}]^{-1} = [(1+r_t)/(1+\theta)] E_t \{ [C_{t+j+1,z}^* P_{t+j+1}^c P_{t+j+1}]^{-1} \}$$

$$(7b) \quad M_{t+j,z}/P_{t+j} = (\nu\omega)^{1/(v+1)} Y_{t+j,z} i_{t+j}^{-1/(v+1)} \quad \text{for } j = 0, 1, \dots$$

There is heterogeneity across individual households, since households can differ with respect to their initial endowment of real financial wealth, different un-/employment histories and different current occupational status.

Nevertheless, it is possible to derive an aggregate formulation of the consumption rule (see the Appendix) which is given by

$$(8) \quad C_t^* = \theta/(1 + \theta) [A_t + H_t] P_t / P_t^c$$

where H_t is the expected net present discounted income minus lumps sum taxes and transaction cost

$$(9) \quad H_t = E_t \left[\sum_{j=0}^{\infty} b_{tj} \left((L_{t+1} W_{t+j} + (1 - L_{t+j}) Z_{t+j} - T_{t+j} - (1 + v^{-1}) i_{t+j} M_{t-j}) / P_{t+j} \right) \right]$$

$$\text{with } b_{tj} = \prod_{k=0}^j (1 + r_{t+k})^{-1}.$$

Also simply summing over all households, aggregate money demand is given by

$$(10) \quad M_t / P_t = (\nu \omega)^{1/(\nu+1)} Y_t i_t^{-1/(\nu+1)}.$$

Total consumption of goods and services can further be divided up into domestic and imported brands. It is assumed that preferences over various brands can be expressed as a CES utility function

$$(11) \quad C_t = \left[\xi C_{dt}^p + (1 + \xi) C_{mt}^p \right]^{1/p} \quad \rho < 1, \sigma = 1/(1 - \rho).$$

The term σ defines the elasticity of substitution between domestic and foreign goods. Correspondingly, the price index for consumption goods is a weighted average of a price index for domestic goods P_t and imports P_t^m and is given by

$$(12) \quad P_t^c = \left[\xi P_t^{(1-\sigma)} + (1 - \xi) P_t^{m(1-\sigma)} \right]^{1/(1-\sigma)}.$$

There are $N(n = 1, \dots, N)$ firms in each region, supplying variants of domestic goods which are imperfect substitutes. Preferences of consumers over different brands of domestic and imported goods are given by the utility functions

$$(13) \quad C_{jt} = \left(\sum_{n=1}^N C_{dt,n}^y \right)^{1/y} \quad y < 1, \tau = 1/(1 - y), j = d, m.$$

Government:

The government pays unemployment benefits and purchases goods and services (G_t), where it is assumed that it allocates purchases over individual commodities by maximising the same CES subutility function as consumers do. It finances these expenditures by a lump sum tax and via money creation. Alternatively, it can issue debt. Thus the government budget constraint is given by

$$(14) \quad B_{t+1} = (1 + r_t)B_t + (1 - L_t)Z_t/P_t + G_tP_t^c/P_t - T_t/P_t - (M_t - M_{t-1})/P_t.$$

It is assumed in this analysis that Z_t is set by a fixed replacement ratio z_0 relative to the steady state wage level and G_t is set as a fixed ratio g_0 relative to steady state GDP. Since the discounted value of current and future tax revenues must equal the discounted value of government spending plus the initial value of outstanding debt, a debt rule must be imposed in order to make the evolution of the government budget sustainable. This requires that T_t is adjusted proportionally to the gap between the debt to GDP ratio and its target level b_0 according to

$$(15) \quad \Delta T_t = \varphi(B_t/Y_t - b_0).$$

Money supply follows an autoregressive process

$$(16) \quad (M_t/M_{t-1} - 1) = \chi_m(M_{t-1}/M_{t-2} - 1) + \varepsilon_t^m$$

where ε_t^m is a white noise shock with $E_{t-1}\varepsilon_t^m = 0$ and standard deviation σ^m .

Foreign Demand:

The level of total demand in the rest of the world D_t^w as well as the foreign price level P_t^w is exogenous. We further assume that foreign preferences imply identical elasticities of substitution between foreign and imported goods. Therefore export demand X_t is given by

$$(17) \quad X_t = \left(\frac{P_t^w}{(P_t^x/e_t)} \right)^\sigma D_t^w$$

where P_t^x is the German DM export price. German exports face a downward sloping demand function as long as $\sigma < \infty$.

Firms:

There are N firms indexed by n . Each firm produces a commodity which is an imperfect substitute of goods produced by the other firms. The technology of firm n is given by a constant returns to scale production function

$$(18) \quad Y_{t,n} = F(K_{t,n}, L_{t,n})\Gamma_t = K_{t,n}^{1-\alpha} L_{t,n}^{\alpha} \Gamma_t.$$

Γ_t is an economy wide exogenous shock to technology. It follows a stochastic process of the form

$$(19) \quad \log(\Gamma_t) = \chi_y \log(\Gamma_{t-1}) + \varepsilon_{t'}^y$$

where the $\varepsilon_{t'}^y$ are white noise with $E_{t-1}\varepsilon_t^y = 0$ and standard deviation σ^y . Generally, ε_t^y will be contemporaneously correlated across countries. Capital stock changes according to the rate of fixed capital formation J_t and the rate of geometric depreciation δ as

$$(20) \quad K_{t,n} = J_{t,n} + (1 - \delta)K_{t-1,n}.$$

Total investment expenditures are equal to investment purchases plus the cost of installation. The unit installation costs are assumed to be a linear function of the investment to capital ratio with a parameter ϕ . Total investment expenditure is therefore given by

$$(21) \quad I_{t,n} = J_{t,n}(1 + \phi/2(J_{t,n}/K_{t,n})).$$

In order to facilitate aggregation we interpret I_t as the physical requirement of a composite investment good by firm n . This composite good is produced from quantities $I_{nn'}$ of the different domestic and foreign goods which are combined using the same nested CES technology as households and the government. This implies that in terms of its investment demand, firm n will substitute between the investment goods in exactly the same way as consumers and the government. Workers are leaving firm n at rate s . In order to recruit new workers the firm has to open up job vacancies (O_t) and advertise actively. Recruitment costs for each vacancy are given by VC_t and it is assumed that they evolve proportional to wages at rate vc_0 . Each firm can fill existing vacancies within one period. Employment thus changes according to

$$(22) \quad L_t = O_t + (1 - s)L_{t-1}.$$

To allow for some sluggishness in prices it is also assumed that the firm faces quadratic price adjustment costs per unit of output

$$(23) \quad ADJ_{t,n}/Y_{t,n} = \gamma/2P_t(P_{t,n}/P_{t-1,n} - 1)^2.$$

These costs are proportional to the aggregate price level. To allow for a simple closing of the model it is assumed that real vacancy costs and real

adjustment costs are paid by the firm via the purchase of a CES basket of domestic and imported goods which is identical to that of private households and the government.

Preferences of domestic and foreign consumers and governments, and preferences of firms for investment goods imply imperfect substitutability between different brands, thus each firm has monopoly power in its market. We follow Dixit and Stiglitz (1977) and assume that firms are sufficiently small so as to take aggregate demand for domestic goods and the price level as given when setting prices for their own brand. Thus each firm faces the following demand function

$$(24) \quad Y_{t,n}^d = \left(\frac{P_t}{P_{t,n}} \right)^\tau \frac{Y_t^d}{N}$$

where $Y_{t,n}^d$ is the total demand addressed to firm n and Y_t^d is a total demand index defined as

$$(25) \quad Y_t^d = \frac{P_t^c}{P_t} (C_t + G_t + I_t) + VC_t O_t + ADJ_t + X_t .$$

Firm n sets prices such that

$$(26) \quad Y_{t,n} = Y_{t,n}^d$$

holds, i. e. output of brand n equals demand. Formally the decision problem of the firm can be characterised as follows. Each firm maximise the present value of its cash flow

$$(27) \quad V_{t,n} = E_t \left[\sum_{j=0}^{\infty} b_{tj} \left\{ P_{t+j,n} (Y_{t+j,n}) Y_{t+j,n} - W_{t+j} L_{t+j,n} - VC_{t+j} O_{t+j,n} - ADJ_{t+j,n} - \frac{P_{t+j}^c}{P_{t+j}} I_{t+j,n} \right\} / P_{t+j} \right]$$

subject to the demand constraint (26), as well as to the capital accumulation (20) and the employment adjustment constraint (22) and for given technology (18) and adjustment cost schedules for output prices (23) and capital (21). Define with λ_t^y , λ_t^k and λ_t^l the multipliers associated with these constraints. Differentiating the objective function with respect to $P_{t+j,n}$, $K_{t+j,n}$, $J_{t+j,n}$, $L_{t+j,n}$ and O_{t+j} ($j = 0, 1, \dots$), and imposing symmetry for domestic firms gives the following system of stochastic Euler equations for country aggregates (subject to the transversality condition)

$$(28) \quad \lambda_{t+j}^y = 1 - 1/\tau \left\{ 1 + \gamma/(1 + r_{t+j}) E_{t+j} [P_{t+j}/P_t - 1] - \gamma(P_t/P_{t-1} - 1) \right\}$$

$$(28c) \quad \lambda_{t+j}^y (1 - \alpha) Y_{t+j}/K_{t+j} = (r_{t+j} + \delta) \lambda_{t+j}^k - \phi/2 P_{t+j}^c / P_{t+j} (J_{t+j}/K_{t+j})^2 - E_{t+j} [\lambda_{t+j+1}^k - \lambda_{t+j}^k]$$

$$(28c) \quad (\phi J_{t+j}/K_{t+j} + 1) P_{t+j}^c / P_{t+j} = \lambda_{t+j}^k$$

$$(28d) \quad \lambda_{t+j}^y \alpha Y_{t+j}/L_{t+j} = (r_{t+j} + s) \lambda_{t+j}^l + W_{t+j}/P_{t+j} - E_{t+j} [\lambda_{t+j+1}^l - \lambda_{t+j}^l]$$

$$(28e) \quad \lambda_{t+j}^l = VC_{t+j}/P_{t+j}.$$

Equation (28a) gives the shadow value of output as a function of both the price elasticity of demand and the adjustment cost for prices³. Notice especially that in the absence of price adjustment costs, equation (28a) implies a constant mark up of prices over marginal cost, while for $\gamma > 0$ the mark up depends both on current and expected price changes. What does this imply for the cyclical behaviour of mark ups? Suppose current inflation is high relative to expected future inflation because of a cyclical peak in the current period. The optimal response of the individual firm, when confronted with this situation is to lower mark ups in order to avoid a costly increase in prices (relative to core inflation) that must be followed by a costly future decrease in prices. Price smoothing thus acts like a countercyclical mark up in this model. The Dixit Stiglitz model of monopolistic competition also implies that firms set the same domestic currency price in the domestic as well as the export market (see Dornbusch (1987) for a discussion of pricing behaviour under alternative market structures). Again, imposing symmetry we have

$$(29) \quad P_t^x = P_t$$

We also assume that foreign firms follow a similar pricing rule concerning their exports to Germany. This implies that German import prices are given by

$$(30) \quad P_t^m = e_t P_t^w.$$

Equation (28b) is the equation of motion of the marginal shadow value of capital λ_t^k . Equation (28c) is the first order condition for investment and it implies that the cost of a marginal unit of capital, including both its pur-

³ Second order terms are neglected in this formulation.

chase and adjustment costs, must equal the shadow value of capital λ_t^k . The cost of capital includes both the pure rental price and adjustment costs. Equations (28d) and (28e) define the law of motion of the shadow value of labour and show that labour demand is a positive function of output and negative function of total labour costs. Because it is costly for firms to fill existing vacancies, total labour costs are the sum of pure wage costs and vacancy costs. Notice also, λ_t^l can be interpreted as the present discounted value of expected profits from an occupied position.

Labour Market:

The basic incentive for search activities in the labour market by both workers and firms are the profit opportunities in present value terms which are associated with a successful job match for both parties. In the case of households, this is given by the difference between the present value of labour income a household can earn in the case of a successful current job match (H_t^l), versus the net present value of labour income in case of a failure (H_t^u). Both terms can most conveniently be expressed as arbitrage equations. The return from the human capital of an employed worker consists of three components: the current net wage rate, the expected capital loss from a job separation given by $s(H_t^l - H_t)$, where s is an exogenous separation rate, and the expected capital gain from an expected change in H_t^l .

$$(31) \quad r_t H_{t,z}^l = W_t/P_t + s(H_{t,z}^l - H_{t,z}^m) + E_t[\Delta H_{t+1,z}^l] .$$

Corresponding to this equation we can write an arbitrage equation for the human capital of an unemployed household as

$$(32) \quad r_t H_{t,z} = Z_t/P_t + p(\cdot)(H_{t,z}^u - H_{t,z}^l) + E_t[\Delta H_{t+1,z}^u] .$$

The return in this case consists of unemployment benefits, the expected capital gain from finding a job with probability $p(\cdot)$. Since the number of job matches is equal to the number of vacancies under the assumption that all vacancies can be filled within the current period (see eq. (22)), $p(\cdot)$ is given by $O_t/(1 - L_t)$. The last term is a capital gain from any expected change of H_t^u itself. As already discussed above, for the firm, the return from a successful job match is given by λ_t^l .

Following Pissarides (1990) we assume that each firm employs many workers and is large enough to eliminate all uncertainty about the flow of labour. Both parties also know about the profit opportunities of the other players. Wages are determined by an implicit bargain at the individual level, i. e. the firm engages in Nash bargains with each individual worker by tak-

ing the wage of all other employees as given. Thus wage contracts are set such as to maximise the product

$$(33) \quad \text{Max}(H_t^l - H_t^u)^\beta (\lambda_t^l)^{(1-\beta)}.$$

This agreement is based on the relative bargaining position of the two parties. The bargaining strength of workers is characterised by the parameter β ($0 \leq \beta \leq 1$). As will be seen below, this parameter determines the fraction of the total return from a successful job match going to workers. In order to introduce nominal rigidities into the wage setting process we assume that contracts last for 4 periods (quarters) and at each date, exactly one quarter of all workers signs a new contract with firms. This type of wage staggering has been suggested by Taylor (1980). In order to allow for the existence of different wage contracts at each date under symmetric technology, absence of skill differentials among workers and aggregate shocks to technology, it must be assumed that the labour market is spatially and/or occupationally segregated and there exists no possibility for workers to switch region and/or occupation. We assume that at each date t firms bargain with one quarter of the work force over a nominal wage contract W_t^c , which will remain fixed for one year. For the derivation of the wage contracting rule it is useful to rewrite the arbitrage conditions (31) and (32) for workers and for firms (eq. (28d)) such that they cover the whole contract length as follows

$$(34) \quad (1+r_t)H_{t,z}^l = \sum_{j=0}^3 E_t \left[b_{tj} \left(W_{t+j}^c / P_{t+j} + s(H_{t+j,z}^y - H_{t+j,z}^l) \right) \right] + E_t \left[(1+r_{t+4})^{-4} H_{t+4,z}^l \right]$$

$$(35) \quad (1+r_t)H_{t,z}^u = \sum_{j=0}^3 E_t \left[b_{tj} \left(Z_{t+j} / P_{t+j} + p(\cdot)(H_{t+j,z}^l - H_{t+j,z}^u) \right) \right] + E_t \left[(1+r_{t+4})^{-4} H_{t+4,z}^u \right]$$

$$(36) \quad (1+r_t)\lambda_{t,n}^l = \sum_{j=0}^3 E_t \left[b_{tj} \left(\lambda_{t+j,n}^y \alpha Y_{t+j,n} / L_{t+j,n} - W_{t+j}^c / P_{t+j} - s\lambda_{t+j,n}^l \right) \right] + E_t \left[(1+r_{t+4})^{-4} \lambda_{t+4,n}^l \right].$$

Workers who are currently engaged in wage negotiations care about their wage contract W_t^c and not the average wage rate W_t . Firms calculate the shadow value of the worker by also taking into account the wage contract of that worker. Maximising (33) with respect to W_t^c , under the restriction that

W_t^c remains fixed for the current and three consecutive periods, yields the familiar sharing rule for the division of the surplus where the fraction of the total surplus from a job match going to the worker depends on his bargaining strength

$$(37) \quad H_{t,z}^l - H_{t,z}^u = \beta(H_{t,z}^l - H_{t,z}^u + \lambda_{t,n}^l) .$$

Using this sharing rule and substituting (37), (38) and (39) gives the wage contract rule

$$(38) \quad W_t^c = E_t \left[\sum_{j=0}^3 b_{tj} (1 - \beta) Z_{t+j} / P_{t+j} + \beta (\lambda_{t+j}^y \alpha Y_{t+j} / L_{t+j} + VC_{t+j} / P_{t+j} p(\cdot)) \right] / E_t \left[\sum_{j=0}^3 b_{tj} P_{t+j}^{-1} \right] .$$

Wage contracts in the current period are indexed to an average of the current price level and expected price levels for three consecutive periods. They are further determined by labour productivity, unemployment benefits and labour market tightness in the current and three consecutive periods. The weights in which future expected variables enter the decision rule is given by b_{tj} . If wage contracts were signed for one period only we arrive at the wage equation as derived by Pissarides. This wage rule also exhibits the feature that the importance by which the marginal product of labour and labour market tightness influence the level of current wage contracts, depends positively on the bargaining power of workers. As the bargaining strength of workers diminishes, firms can tie wages more narrowly to the reservation wage. The average nominal wage rate in period t is thus given by the average value of all wage contracts signed in the current and the previous three periods

$$(39) \quad W_t = \frac{1}{4} \sum_{j=0}^3 W_{t-j}^c .$$

Equilibrium:

A monopolistically competitive equilibrium of this economy is given by a set of first order conditions of household z for consumption (7a) and real balances (7b) as well as an investment rule (28b, 28c), a labour demand rule (28d, 28e) and pricing rules for domestic (28a, 29) and foreign firms (30). P_t , P_t^x , P_t^m , i , r_t , e , simultaneously clear the markets for domestic and imported goods, the money, capital and the foreign exchange market. W_t is determined by the wage contracting rule (38, 39) and firms set employment optimally for given current and past wage contracts as well as their own

product price according to their FOC for labour. Thus the labour market equilibrium can coexist with involuntary unemployment in this model.

Prices, exchange rate and the interest rate also ensure that an intertemporal equilibrium condition between national saving and investment holds given a predetermined stock of net foreign assets. Using the budget constraint of private households and the government, together with the definition of the market value of the domestic corporate sector, yields the current account identity

$$(40) \quad e_{t+1}F_{t+1} = (1 + r_t)e_tF_t + Y_t - (C_t + I_t + G_t)P_t^c/P_t - ADJ_t - VC_tO_t$$

which relates the accumulation of net foreign assets to national savings minus investment. The equilibrium of the system ensures that a no Ponzi game condition is fulfilled for net foreign assets. Define national saving as

$$(41) \quad S_t = Y_t - (C_t + G_t)P_t^c/P_t - ADJ_t - VC_tO_t$$

then the no Ponzi game condition says that the present value of investment is constrained by the present value of national saving and the historically given level of net foreign assets

$$(42) \quad E_t \left[\sum_{j=0}^{\infty} (1 + r_{t+j})^{-j} I_{t+j} \right] = E_t \left[\sum_{j=0}^{\infty} (1 + r_{t+j})^{-j} S_{t+j} \right] + e_t F_t.$$

In contrast to a closed economy where savings and investment must be equalised period by period this condition shows that for an open economy with access to a world financial market savings and investment are only subject to an intertemporal solvency constraint.

3. Solution Method

The Solution Algorithm:

To solve this nonlinear forward looking model, we depart from the usual procedure of linearising around the steady state and then solving the linear approximation. Instead we use a method developed by Laffarque (1990) and Boucekkine (1995) to solve the nonlinear model by Newton Raphson. For this solution method to work, the model economy must converge to a steady state growth path. It is well known (see for example Matsuyama (1987)) that the neoclassical open economy growth model with finitely lived consumers attains a steady state for a constant exogenous level of employment. Given

our assumptions on the labour market, it is easy to see that the steady state solution of our model can be obtained recursively, i. e. the level of employment in the steady state can be determined prior to solving for the remaining variables. Given our assumption on the indexation of vacancy costs and unemployment benefits to the steady state level of wages with rate z_0 and vc_0 respectively, a steady state unemployment rate can easily be obtained. Let X^* be the steady state value of variable X_t . Setting P^* equal to one, from (39) using the fact that $W^{c*} = W^*$ in the steady state and substituting the FOC for labour and vacancies (eq. (28d) and (28e)) into the wage contract and observing that $p(\cdot) = sL^*/(1 - L^*)$ in the steady state yields

$$(43) \quad 1 = (1 - \beta)z_0 + \beta \left[1 + (r^w + s)vc_0 + vc_0 \frac{sL^*}{(1 - L^*)} \right].$$

The steady state level of employment only depends on the exogenous world interest rate (r^w) and labour market parameters. This demonstrates that there will be a constant employment rate in the steady state and we can use the result in Matsuyama to show that there exists a stable steady state. Now let $y_t(n \times 1)$ and $x_t(k \times 1)$ be vectors of endogenous and exogenous variables respectively. The model can be expressed compactly as

$$(44) \quad f_t(y_{t-1}, y_t, E_t y_{t+1}, x_t) = 0$$

where f_t is a vector of nonlinear dynamic equations. The model is formulated in such a way that the variables contained in x are either constants (i. e. world interest rate, world demand, competitors prices) or white noise shocks⁴. After the realisation of the random shocks at t the model is solved for T periods, where T is chosen large enough for the model to attain the steady state. We set T equal to 200 periods which corresponds to 50 years. To solve the model with starting date t , the system is stacked for the T periods as

$$(45) \quad F(z, x; t) = \begin{bmatrix} f_t(z_t, x_t) \\ f_{t+j}(z_{t+j}, E_t x_{t+j}) \\ f_{t+T}(z_{t+T}, E_t x_{t+T}) \end{bmatrix} = 0.$$

where $z_{t+j} = (E_t(y_{t+j+1}), E_t(y_{t+j}), E_t(y_{t+j+1}))$. The system is then solved by Newton Raphson subject to the predetermined variable y_{t-1} and the terminal condition y_{t+T+1} , which is set equal to the steady state value y^* . The

⁴ We treat both TFP and money as endogenous variables by including their AR processes as model equations. The white noise error terms ε_t^y and ε_t^m are treated as exogenous variables.

steady state solution itself is calculated by setting the elements of the vector x equal to their unconditional expectation. The solution at date t yields the realisation of y_t under forward looking rational expectations. This solution is stored. New realisations of y_t and x_t are drawn for $t + 1$ and the solution procedure is repeated over the range $t + 1$ to $t + T + 1$, now with y_t as the predetermined variable. This process is repeated 100 times in order to generate a time series for y_t and x_t of 100 observations.

Impulse Responses to Technology and Monetary Shocks:

For an understanding of some basic properties of the model with respect to technology and monetary shocks it is useful to present impulse response functions. In order to highlight the effect of nominal rigidities for the adjustment process we present results both with and without nominal rigidity. To make the impact of both shocks comparable to each other, a positive one standard deviation shock was given to the technology and the money innovation respectively. Figure 1 shows the dynamic adjustment of TFP and money supply to a positive innovation in the first period.

The technology shock has a persistent effect on output. As shown in Figure 2a, the shock is magnified and propagated over time, leading to a hump shaped adjustment of GDP. The inability of the standard RBC model to generate this adjustment pattern has recently been criticised by Cogley and Nason (1995)⁵. In the context of our model the presence of nominal rigidities seems to be the main reason for this adjustment pattern, since it is absent under the assumption of flexible prices and wages (see Figure 2b) and resembles that of the standard RBC model. To understand more clearly the economic mechanism for the adjustment under nominal rigidities, it is interesting to notice that employment declines initially with a positive shock to technology and only recovers after some quarters (it lags output in that sense). This is due to the price response of firms. Because prices adjust sluggishly to the fall in cost, this restricts the expansion of aggregate demand and firms initially react to the technological improvement with a reduction in employment. Only as prices adjust to the new cost conditions will demand expand fully and firms hire new labour. Because the immediate price response under full price flexibility leads to a full demand expansion, firms start increasing employment immediately. Though even in this case the adjustment of employment is only completed after several quarters, the employment expansion is not strong enough to dominate the reversal of technology to its long run trend as dictated by the AR(1) process. Since prices

⁵ Burnside and Eichenbaum (1996) have recently shown that the hump shaped adjustment pattern can also be generated by an RBC model with labour hoarding and fluctuations in capacity utilisation.

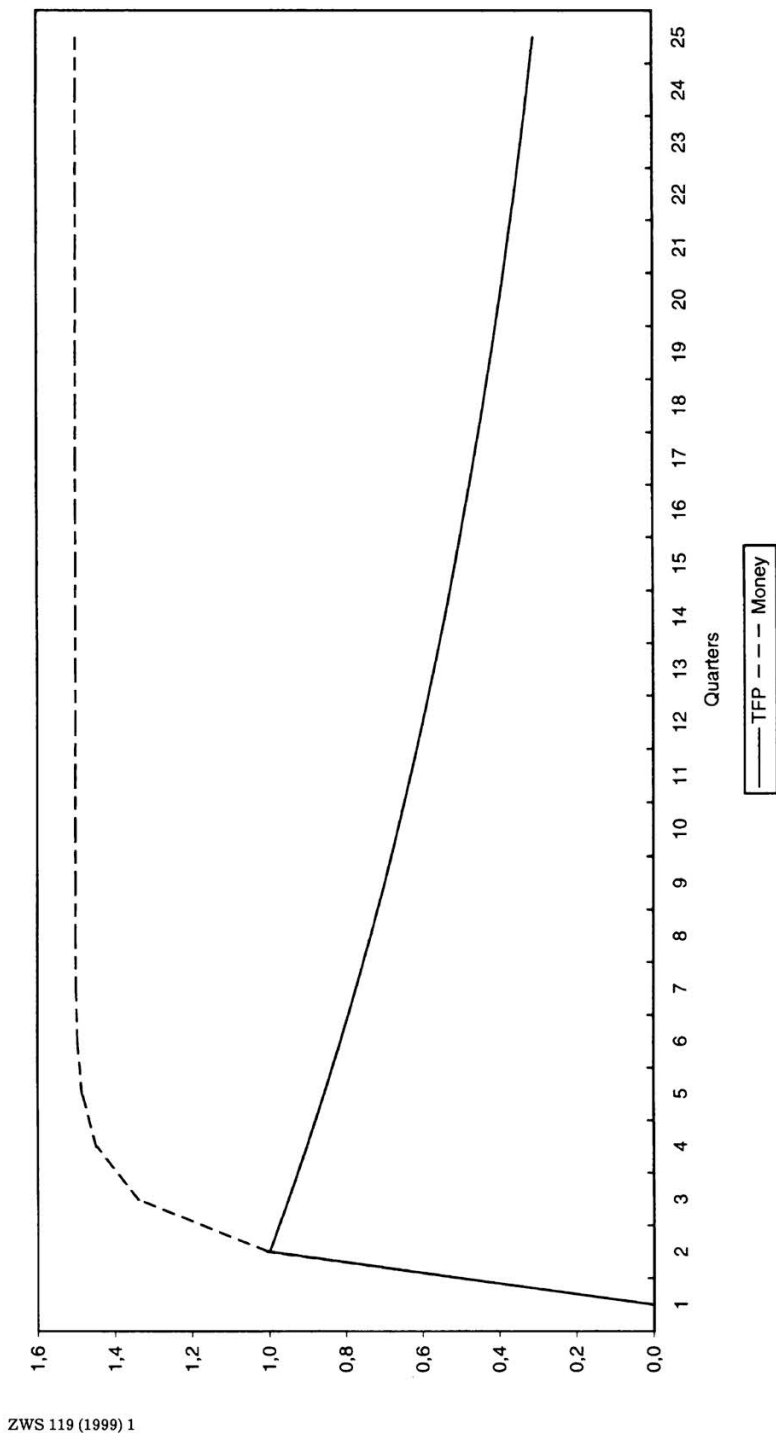


Figure 1: Technology and Money Shock

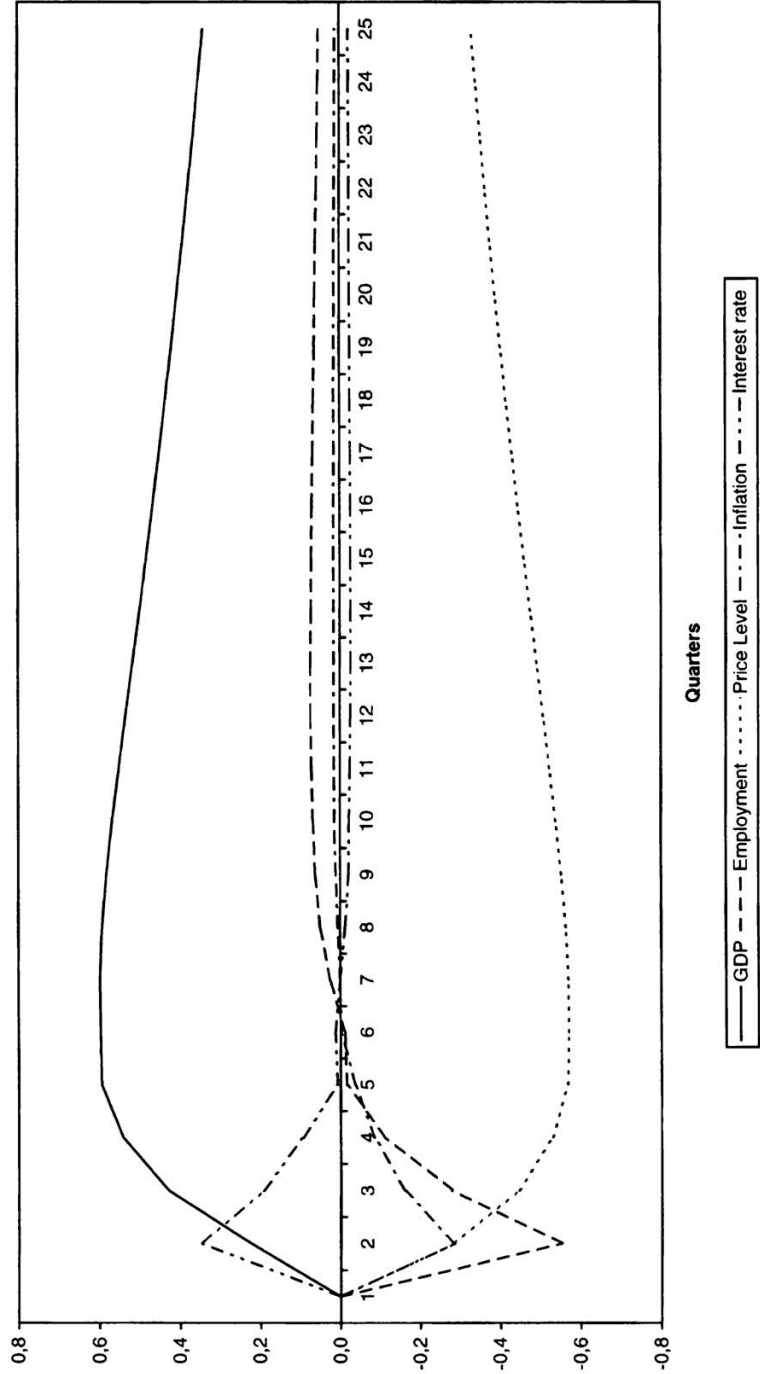


Figure 2a: Technology Shock, Nominal Rigidity

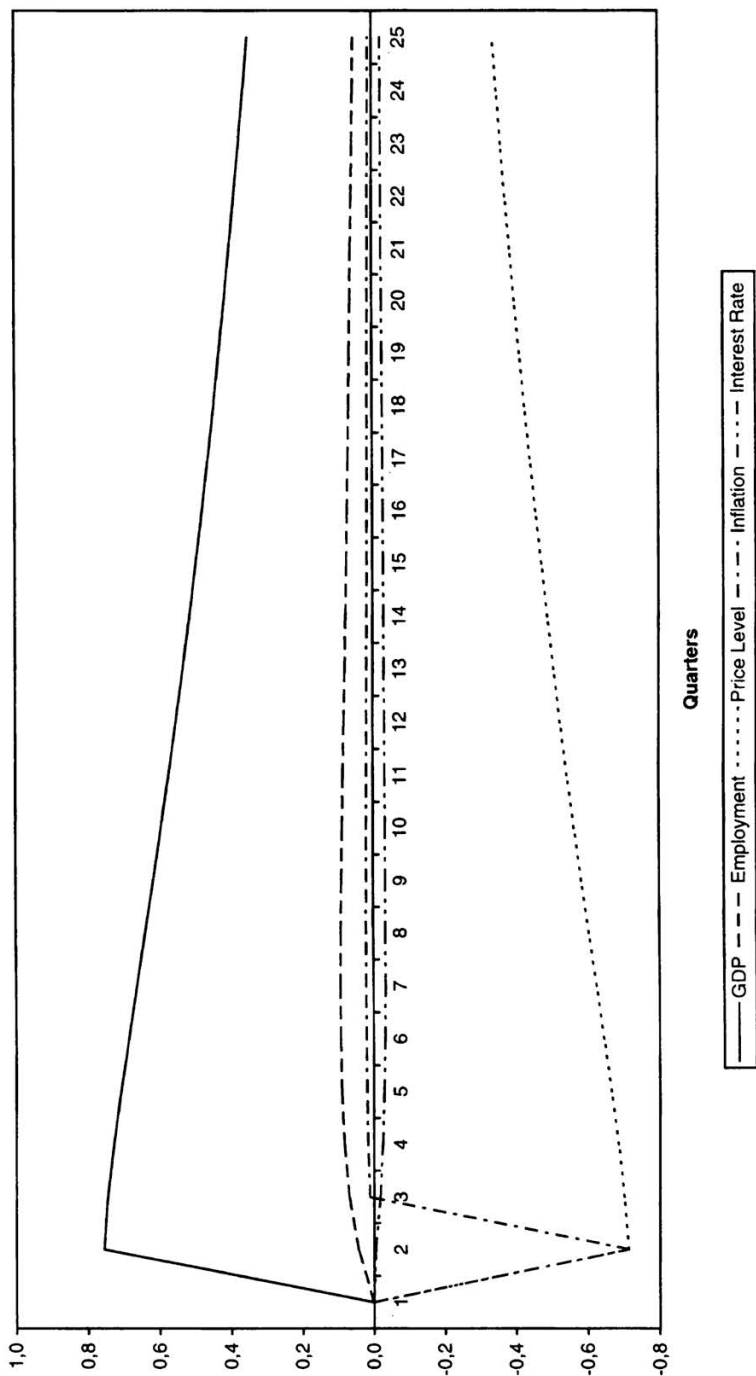


Figure 2b: Technology Shock, Flexible Prices and Wages

jump downwards immediately and return slowly to their long run equilibrium values. This price response allows for a strong initial adjustment of output and labour demand rises. In both cases, the positive employment effect in the medium run is due to constant unemployment benefits. The technological shock increases the surplus from a job match which is shared between workers and firms, this allows for a positive employment response.

A shock to money growth leads to a permanent increase in the price level under both assumptions on the degree of nominal rigidity (see figure 3a and 3b). With sluggish prices and wages the adjustment of the price level takes longer and associated with the money shock is a temporary increase in real output and employment. This effect is caused by a fall in real interest rates which is required to clear the money market under price rigidity. This leads to a temporary increase in investment and consumption demand. Because nominal wages are also slow to adjust employment also increases. The adjustment pattern of inflation and output to a monetary shock is very similar thus we should expect a high contemporaneous correlation between these two variables if money were the dominant shock to the economy.

4. Model Calibration

The empirical objective of the paper is to replicate important stylised facts of the German economy over the flexible exchange rate period. Our data set therefore begins in 1974:I. In order to avoid considering shocks related to German unification the data set ends in 1988:IV. All parameter estimates are as much as possible restricted to this sample period. To select parameter values we largely follow standard procedures, i. e. we base these values on evidence from growth observations and some microeconomic evidence. In cases where this is not possible parameters are chosen in the neighbourhood of existing studies. The parameter θ is set equal to .01 which implies a steady state annual rate of interest equal to 4%. The interest elasticity of money demand is based on a regression of real money balances on GDP and a short term nominal interest rate⁶. We obtain a value of $-.84$ for Germany. In the model experiments we set the income elasticity equal to one, i. e. we attribute deviations of the estimated elasticity from this value to an exogenous trend of velocity. The output elasticity of labour is set equal to the average wage share divided by one minus the Lerner index. Recent estimates of the Lerner index by the OECD (see Oliveira Martins, Scarpetta and Pilat (1997)) suggest values for τ of 6.66 for Germany. No information is

⁶ The data on money, GDP, the GDP deflator and the short term nominal interest rate for Germany is obtained from Deutsche Bundesbank (1974:I-1988:IV). As money stock variable M3 is used.

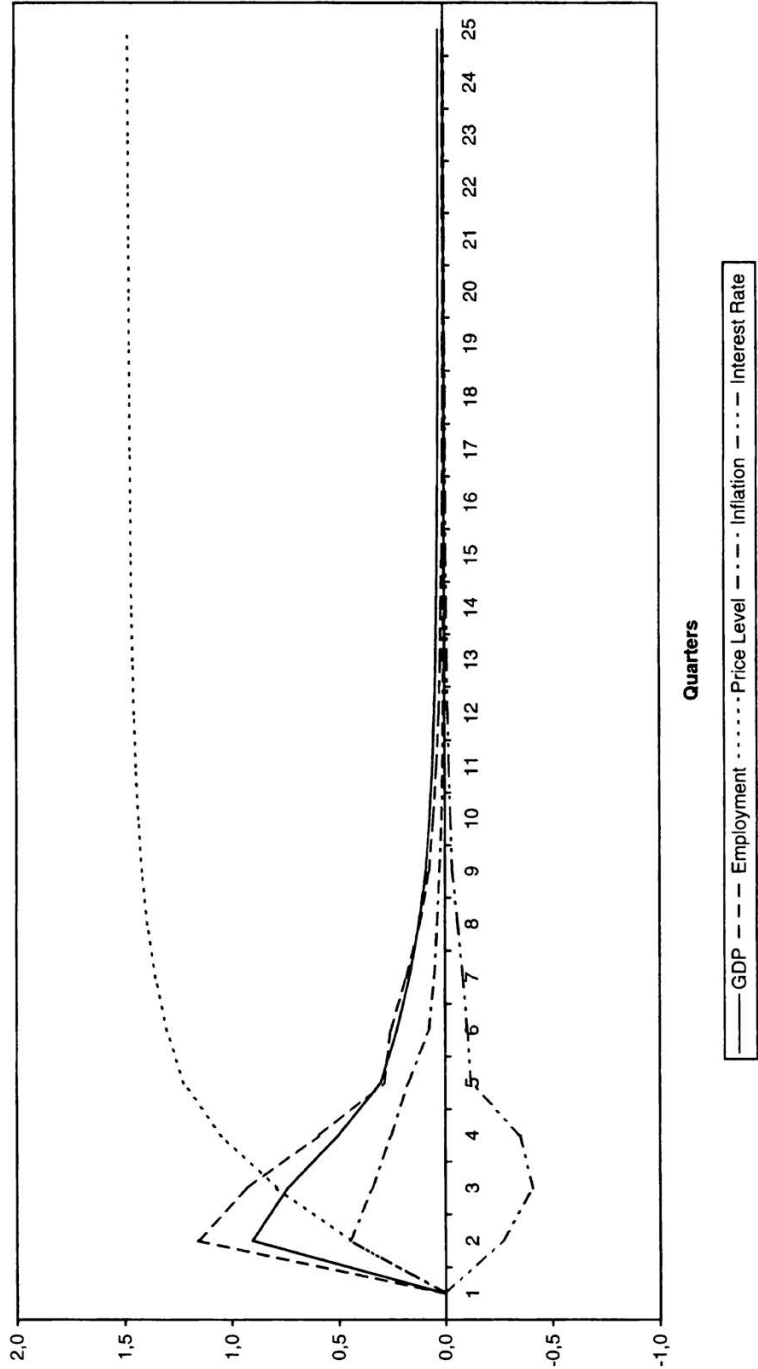


Figure 3a: Monetary Shock, Nominal Rigidity

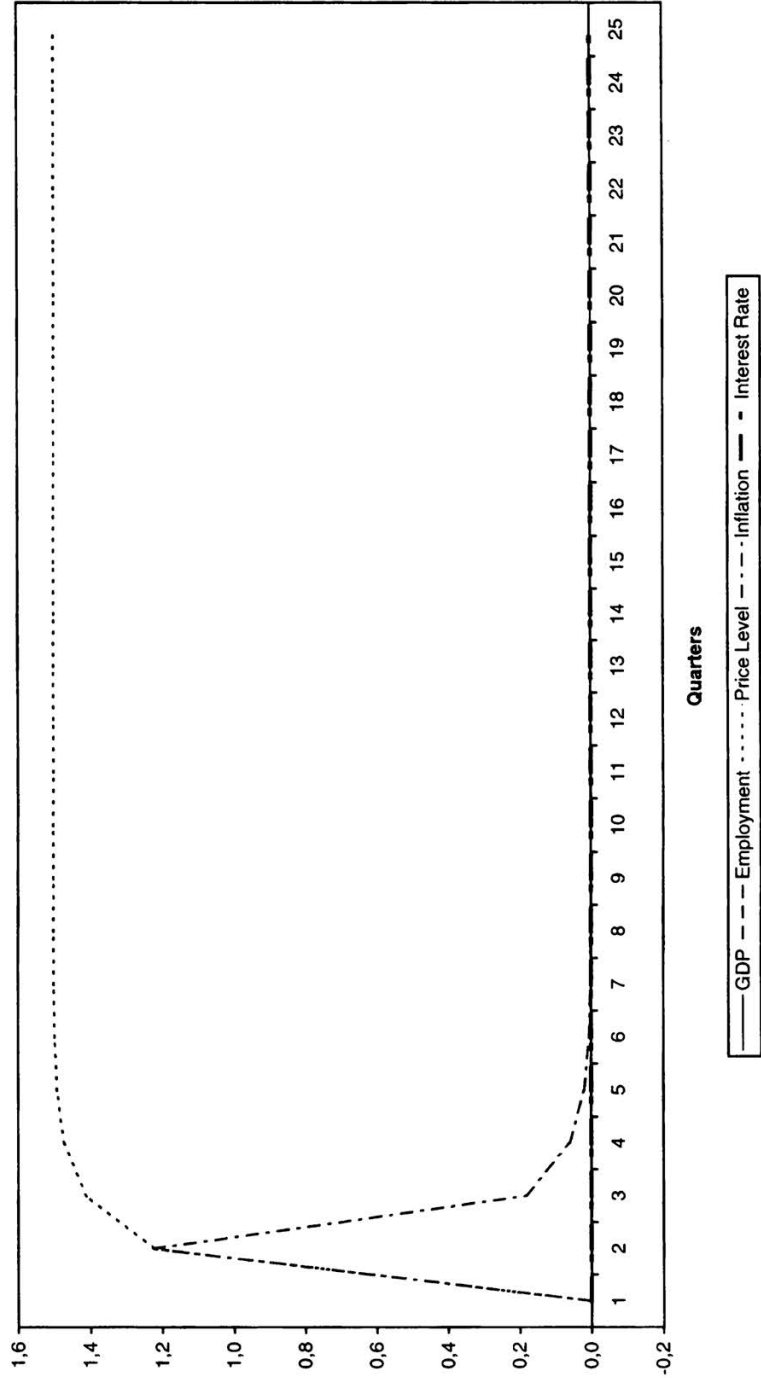


Figure 3b: Monetary Shocks, Flexible Prices and Wages

available on the size of price adjustment costs. These costs should, however, be small. We have selected a value of 10 for γ , i. e. a change of prices by 1% cause costs which amount to 0.1% of total output. The mark-up estimate together with the average wage share yield a value for the output elasticity of labour of .66⁷ for Germany.

Table 1
Parameter Values

Utility Function		
θ	0.01	Rate of time preference
σ	1.00	Price elasticity of imports
ξ	0.69	Share of domestic goods
ν	0.19	Transaction cost parameter
Fiscal Policy and Money Supply		
ψ	0.01	Parameter of debt rule
b_0	2.40	Debt target
χ_m	0.26	Parameter of money supply rule
σ_m	0.0063	Standard deviation of money shock
Technology		
α	0.65	Output elasticity of labour
δ	0.02	Depreciation rate
ϕ	8.00	Adjustment cost parameter (investment)
γ	10.00	Adjustment cost parameter (prices)
τ	6.66	Average mark-up
χ_y	0.96	Technology parameter
σ_y	0.0117	Standard deviation of technology shock
Labour Market		
s	0.015	Separation rate
η	0.60	Elasticity of job matches w. r. to U
β	0.50	Bargaining strength of workers
z_0	0.40	Replacement ratio
vc_0	0.13	Vacancy cost parameter

The depreciation rate is set to 2% per quarter which corresponds to the mean rate in our data set over the sample period. The adjustment cost parameter is more difficult to pin down on the basis of information on first

⁷ From equation (28d) it is obvious that the wage share is not an exact measure of α in the presence of search costs. However, the bias implied by the presence of search costs is negligible.

moments only. It has, however, been noticed before (see, for example, Men-doza (1991)) that the parameter θ has a crucial effect on the volatility of investment. It is therefore set in such a way as to make investment about three times as volatile as GDP in Germany.

With respect to the separation rate s we draw on information provided by Layard et al. (1990) from data on gross labour market flows. According to their figures the inflow rate into unemployment is about two per cent for Germany per quarter. Burda and Wyplosz (1994) have recently estimated an elasticity of matches with respect to unemployment (η) of about .6 for Germany. We assume in this analysis that firms can fill vacancies within one quarter. Though information on vacancies are notoriously unreliable, some studies indicate that this is a plausible simplification. A study by Ours and Ridder (1992), for example, reports average vacancy durations of 45 days for the Dutch economy. Similar estimates can be found for Germany (see Erdmann (1990)). The parameter β referred to as bargaining strength of workers is set to .5. This implies that all relevant differences between the two parties are captured by the two terms $(H_t^l - H_t^u)$ and λ_t^l (see Binmore, Rubinstein and Wolinsky (1986) for a discussion of the symmetry axiom).

The level of unemployment compensation determines the reservation wage. To capture both benefit duration and coverage, Layard et al. (1990) have calculated expenditures on benefits per unemployed person as a per cent of output per worker for major OECD countries for the year 1987. According to these figures the ratio for Germany is slightly below 20%. We therefore assume that unemployment benefits amount to roughly 40% of gross wages. Given the fact that all other parameters have been chosen, the parameter vc_0 can be selected such that the model replicates the steady state unemployment rate. Finally we set the price elasticity of imports in both regions equal to one. A value in this neighbourhood can often be found in empirical studies on import and export equations. The share of imports in total GDP is set to .31, which is the mean value over the period 1975 to 1998 for Germany. Both the domestic and the foreign price level as well as the nominal exchange rate is set equal to one in the steady state.

Within an open economy framework it would be possible to subject the model to various international shocks such as fluctuations of competitors prices, world demand and foreign interest rates. To keep as close as possible to the existing literature we neglect the international shocks and concentrate entirely on stochastic shocks to technology and money. An analysis of the impact of foreign disturbances will be left for future research. The parameters of the technology process and of the money supply rule are determined by estimating equation (19) and (16) for Germany with OLS over the period 1974:I to 1988:IV. For the Solow residual it was assumed that it fol-

lows a stationary process around a deterministic trend. In the construction of the series we followed Hall (1988) and corrected the raw TFP series by the mark-up component. M3 was used as monetary variable for Germany. A debt target of 60% is assumed and the parameters of the debt rule are taken from the Commission's QUEST II model.

5. The Stylised Facts of Business Cycles

This section presents the salient features of the German business cycle. Since in models with sticky prices the exchange rate and monetary policy regime may have important consequences for macroeconomic aggregates, we restrict ourselves to the period after the break-down of the Bretton Woods system. Table 2 gives information on the relative volatility, measured by relative standard deviations, persistence, measured by first order autocorrelations, and comovements, measured by cross correlations of major macroeconomic time series. Concerning volatility, it can be noted that private consumption is less volatile⁸ and private investment is substantially more volatile than GDP. Consumption, investment and GDP are also highly persistent and the two GDP components are strongly positively correlated with GDP. The trade balance on the other hand behaves countercyclical. Danthine and Donaldson (1993) and Bakus and Kehoe (1994) find similar relationships in more comprehensive cross sections of OECD countries.

The standard deviation of employment⁹ is about 68% of GDP. This is similar in other European countries. Bakus et al. report a value of .85% for total Europe. In contrast to the US, the volatility of productivity exceeds the volatility of employment in Germany. The real wage rate on the other hand fluctuates substantially less than productivity. This is also true for the US. Employment lags output. A noteworthy additional feature is the negative correlation between productivity and employment in Germany which is also shared by many European countries. For the US this correlation is slightly positive.

The main focus of this paper will be about the interactions between real variables and prices. We will concentrate our discussion on the following stylised facts: There is a negative correlation between output and prices.

⁸ It is important to notice that the consumption series reported here includes durables and non durables, while we only model non durable consumption. Kydland et al. (1994) point out that the volatility of non durable consumption is substantially smaller than that of total consumption. For the US they report a relative volatility of .75 for total consumption, compared to .52 for non durables.

⁹ We restrict ourselves to employment instead of hours. As noted by Burdett and Wright (1989), fluctuations in total hours worked are largely the result of fluctuations in employment.

Table 2
Some Stylized Facts of the German Economy (1974:I-1988:IV)

	SDEV	AR1	-4	-3	-2	-1	0	1	2	3	4
						Correlation of $X(i)$ with GDP					
GDP	1.24	0.57	0.28	0.37	0.46	0.54	1.00	0.54	0.48	0.40	0.35
Consumption	1.08	0.80	0.30	0.44	0.52	0.60	0.66	0.48	0.50	0.45	0.30
Investment	2.67	0.80	0.21	0.38	0.52	0.59	0.76	0.60	0.50	0.44	0.24
Trade/GDP	7.54	0.79	0.02	-0.09	-0.26	-0.36	-0.38	-0.36	-0.39	-0.29	-0.23
Employment	0.84	0.94	-0.27	-0.13	0.04	0.21	0.40	0.54	0.65	0.70	0.72
Productivity	1.21	0.56	0.49	0.48	0.45	0.41	0.75	0.19	0.05	-0.07	-0.15
Real Wage	0.71	0.26	-0.04	0.13	0.23	0.10	0.43	0.21	0.21	0.20	0.22
Money	1.00	0.79	0.20	0.24	0.20	0.11	-0.02	-0.16	-0.19	-0.13	-0.10
Price Level	0.56	0.84	-0.54	-0.61	-0.67	-0.58	-0.46	-0.24	-0.10	0.04	0.14
Inflation	0.33	-0.13	-0.04	-0.10	-0.10	0.12	0.16	0.32	0.18	0.19	0.09
Interest Rate*	0.66	0.94	-0.33	-0.38	-0.39	-0.37	-0.31	-0.24	-0.13	-0.11	-0.02
Correlation			Productivity-Employment								
			-0.15								

Unless otherwise stated, data are filtered with the HP-Filter.

* Nominal short term interest rate, not filtered. Columns 3 to 11 give correlation between unfiltered interest rates and unfiltered growth rate of GDP.

Also, there is a pronounced phase shift in the sense that future output is more highly correlated with current prices than current output. The price level is about half as volatile as GDP and highly persistent, while inflation is about half as volatile as prices and much less persistent. Also inflation is procyclical and it lags behind output in the sense that current inflation is more strongly correlated with past output than with current output. As documented already in other studies, no strong conclusions can be drawn about the correlation between money and GDP. We also do not observe a strong correlation between money (M3) and output for Germany. In contrast to this, another widely observed correlation is that between short term nominal interest rates and GDP growth. Here we note a strong negative correlation and it is also the case that short term nominal interest rates lead output growth.

6. Simulation Results

In this section we will ask the question how this model, whose parameters have been chosen to fit trend information and some microeconomic evidence, is able to replicate the second moments of the German data described in the previous section, i. e. the volatility and comovement of macroeconomic aggregates at business cycle frequencies. In assessing the following results it is important to keep in mind that the model parameters have not been selected with an eye towards optimising the fit of these second moments. The only exception is the selection of the adjustment cost parameter for capital. This is the standard RBC model evaluation procedure (see Kydland and Prescott (1996) for a recent methodological exposition of this approach.).

For that purpose stochastic simulations are generated and the reported results are average values of 100 simulations over 80 periods¹⁰. We first look at the model results, using the benchmark parameters contained in Table 1. With respect to GDP and its components the model replicates the basic stylized facts. The relative variability of private consumption is smaller than that of output, while private investment is about three times as volatile as GDP. Both components are strongly positively correlated with GDP and exceed the observed correlation. Notice, however, by allowing for foreign price disturbances this correlation would be reduced. The trade balance is strongly countercyclical, as also found in the data. Again, allowing for foreign shocks would reduce this correlation in absolute value. The model also replicates some important labour market facts, like the positive correlation between employment and GDP over the business cycle, a standard deviation

¹⁰ We do not use the first 20 generated observations.

of productivity exceeding that of real wages as well as a lead of output over employment. It also generates the observed negative correlation between employment and productivity. The standard deviation of employment, however, seems too large compared to GDP. I suspect that allowing for more convex labour adjustment costs could improve the fit of the model along this dimension.

Regarding the interactions between nominal and real variables, there is a pronounced negative correlation between detrended output and the price level. Prices are leading output, and the lead is about as pronounced in the model economy as in the data. Consistent with the observed data, the model economy generates a positive contemporaneous correlation between the rate of inflation and detrended GDP and inflation is lagging GDP. The model is also successful in replicating the stylised facts between the short term nominal interest rate and the growth rate of GDP. Like in our data set, the money output correlation is weak. However, there are also dimensions in which the model is less successful. The relative standard deviation of prices as well as of inflation exceeds the observed volatility substantially.

In attempting to interpret the contribution of crucial model hypotheses for the reported simulation results, it is useful to perform a sensitivity analysis. In particular, we want to explore the importance of our assumptions on wage and price setting behaviour and money demand and the role played by technology and monetary shocks respectively. To address these questions we have performed various simulation experiments by slightly perturbing the benchmark model. In the following five experiments we change exactly one assumption in each case. In our discussion we will concentrate entirely on the interactions between nominal and real variables.

a) Only Technology Shocks

As shown in Table 4, without stochastic shocks to money supply, various stylised facts cannot adequately be captured by the model economy. There is negative correlation between prices and output exceeding the observed correlation. The model fails to generate a price level which is leading output in the sense defined above. In addition, the correlation between inflation and output is now negative. Interest rates do no longer lead output. Though the contemporaneous correlation between short term rates and output has the correct sign, interest rates do not lead output.

Table 4
Model with Real Shocks only

	SDEV	AR(1)	-4	-3	-2	-1	0	1	2	3	4
					Correlation: $X(i) - \text{GDP}$						
Price	0.645 (0.119)	0.872 (0.035)	-0.215 (0.141)	-0.456 (0.112)	-0.713 (0.069)	-0.927 (0.021)	-0.992 (0.002)	-0.839 (0.043)	-0.599 (0.090)	-0.349 (0.126)	-0.128 (0.149)
Infl.	0.313 (0.030)	0.431 (0.095)	-0.392 (0.117)	-0.484 (0.095)	-0.514 (0.065)	-0.425 (0.047)	-0.131 (0.025)	0.304 (0.040)	0.479 (0.051)	0.503 (0.073)	0.448 (0.101)
IS*	0.366 (0.034)	0.528 (0.083)	-0.056 (0.122)	-0.180 (0.120)	-0.392 (0.105)	-0.729 (0.057)	-0.917 (0.018)	-0.403 (0.088)	-0.112 (0.109)	0.024 (0.105)	0.078 (0.106)
Standard Deviation:											
GDP		Empl		Prod		WR					
0.663 (0.126)		0.600 (0.055)		0.966 (0.132)		0.636 (0.125)					

* Nominal short term interest rate, not filtered. Columns 3 to 11 give correlation between unfiltered interest rates and unfiltered growth rate of GDP.

b) Only Money Shocks

Results for this experiment are reported in Table 5. Without technology shocks the model fails to reproduce the negative correlation between the price level and output altogether. Although there is now a positive correlation between inflation and GDP, this correlation is too strong. In addition, the money output correlation is too strong. Finally, the contemporaneous correlation between the short term nominal interest rate and output is positive.

c) Flexible Wages and Prices:

With this set of assumptions, the model comes close to the RBC model. Under full price and wage flexibility, we can see from the results in Table 6 that the negative correlation between prices and output is too high. Prices do not lead output under this set of assumptions. The correlation between inflation and output has the wrong sign and the correlation of money and nominal interest rates with output becomes insignificant.

aa) Inflexible Prices and Flexible Wages

In order to assess the role of price and wage rigidity for fitting the cross correlation pattern between real and nominal variables we also performed an experiment with flexible wages and inflexible prices and allow for both real and monetary shocks. Comparing Table 6.1 to Table 3 one can see that wage rigidity is not crucial for generating the cross correlations we are interested in. However, the absence of wage rigidity would make the model fail along another dimension, because it leads to an implausibly large standard deviation for real wages and not enough employment volatility.

d) No Interest Elastic Money Demand

The results reported in Table 7 are close to those of the benchmark model. Nevertheless, it is interesting to notice that now the contemporaneous correlation between prices and output is stronger than that between lagged prices and output, i. e. the model fails to replicate the characteristic correlation pattern between prices and output at various leads and lags. The correlations between inflation and output are again closer to the benchmark model. Nominal interest rates still lead output.

Table 5
Model with Money Shocks only

	SDEV	AR(1)	Correlation: $X(i) - GDP$								
			-4	-3	-2	-1	0	1	2	3	4
M	0.835 (0.138)	0.812 (0.060)	-0.453 (0.090)	-0.422 (0.052)	-0.260 (0.062)	0.092 (0.055)	0.611 (0.042)	0.663 (0.058)	0.574 (0.114)	0.454 (0.144)	0.341 (0.144)
Price	0.648 (0.126)	0.907 (0.025)	-0.504 (0.110)	-0.516 (0.073)	-0.431 (0.045)	-0.190 (0.037)	0.229 (0.028)	0.474 (0.046)	0.576 (0.074)	0.565 (0.107)	0.483 (0.131)
Infl.	0.268 (0.031)	0.586 (0.100)	-0.103 (0.103)	-0.013 (0.128)	0.214 (0.143)	0.581 (0.102)	0.994 (0.001)	0.591 (0.100)	0.254 (0.126)	-0.010 (0.119)	-0.184 (0.098)
IS*	0.662 (0.062)	0.538 (0.100)	-0.111 (0.099)	-0.244 (0.077)	-0.351 (0.062)	-0.337 (0.055)	0.590 (0.052)	0.334 (0.075)	0.187 (0.092)	0.086 (0.104)	0.057 (0.081)
Standard Deviation:											
GDP		EMPL		PROD		WR					
0.534 (0.061)		0.666 (0.071)		0.140 (0.012)		0.324 (0.048)					

* Nominal short term interest rate, not filtered. Columns 3 to 11 give correlation between unfiltered interest rates and unfiltered growth rate of GDP.

Table 6
Model without Nominal Rigidities

		Correlation: $X(i) - \text{GDP}$										
		SDEV	AR(1)	-4	-3	-2	-1	0	1	2	3	4
M		0.833	0.822	-0.018	-0.025	-0.020	-0.020	-0.014	-0.008	-0.006	-0.006	-0.008
		0.124	0.040	0.196	0.194	0.189	0.189	0.189	0.185	0.181	0.184	0.187
	Price	1.279	0.728	-0.057	-0.175	-0.318	-0.506	-0.725	-0.500	-0.313	-0.163	-0.052
		0.188	0.072	0.152	0.156	0.155	0.137	0.092	0.136	0.160	0.161	0.153
Infl.		0.917	-0.013	-0.144	-0.169	-0.200	-0.261	-0.297	0.306	0.258	0.210	0.160
		0.061	0.100	0.110	0.119	0.084	0.066	0.075	0.079	0.075	0.092	0.107
IS*		0.428	0.433	-0.079	-0.102	-0.066	-0.091	0.063	0.074	0.066	0.067	0.061
		0.037	0.099	0.097	0.093	0.082	0.080	0.075	0.094	0.074	0.083	0.078
Standard Deviation:												
GDP			EMPL		PROD		WR					
0.973			0.022		0.954		0.959					
(0.138)			(0.071)		(0.134)		(0.135)					

* Nominal short term interest rate, not filtered. Columns 3 to 11 give correlation between unfiltered interest rates and unfiltered growth rate of GDP.

Table 6.1
Model with Price Rigidity and Flexible Wages

		Correlation: $X(i) - \text{GDP}$										
		SDEV	AR(1)	-4	-3	-2	-1	0	1	2	3	4
M		0.833 (0.124)	0.822 (0.040)	-0.162 (0.199)	-0.156 (0.194)	-0.115 (0.194)	-0.021 (0.197)	0.163 (0.202)	0.191 (0.198)	0.167 (0.197)	0.132 (0.201)	0.099 (0.201)
Price		1.063 (0.182)	0.841 (0.044)	-0.221 (0.173)	-0.356 (0.158)	-0.487 (0.150)	-0.585 (0.141)	-0.560 (0.147)	-0.404 (0.172)	-0.245 (0.188)	-0.107 (0.184)	0.004 (0.176)
Infl.		0.578 (0.043)	0.258 (0.103)	-0.225 (0.129)	-0.246 (0.128)	-0.236 (0.108)	-0.178 (0.096)	0.045 (0.086)	0.281 (0.095)	0.288 (0.099)	0.257 (0.116)	0.209 (0.130)
IS*		0.712 (0.052)	0.402 (0.093)	-0.086 (0.108)	-0.134 (0.100)	-0.190 (0.091)	-0.364 (0.080)	-0.095 (0.100)	0.008 (0.104)	0.016 (0.091)	0.037 (0.096)	0.046 (0.080)
Standard Deviation:												
GDP			Empl	Prod		WR						
0.808			0.242	0.892		1.862						
(0.130)			(0.031)	(0.140)		(0.135)						

* Nominal short term interest rate, not filtered. Columns 3 to 11 give correlation between unfiltered interest rates and unfiltered growth rate of GDP.

Table 7
Model without Interest Elastic Money Demand

	SDEV	AR(1)	-4	-3	Correlation: $X(i) - \text{GDP}$					1	2	3	4
					-2	-1	0						
M	0.833	0.822	-0.229	-0.200	-0.101	0.073	0.278		0.292	0.249	0.195	0.144	
Price	0.124	0.040	0.192	0.188	0.189	0.193	0.197		0.192	0.191	0.194	0.195	
	0.960	0.891	-0.278	-0.413	-0.526	-0.541	-0.550		-0.406	-0.243	-0.094	0.025	
Infl.	0.180	0.030	0.179	0.159	0.148	0.146	0.155		0.180	0.195	0.192	0.183	
	0.429	0.442	-0.278	-0.295	-0.244	-0.122	0.066		0.313	0.357	0.332	0.268	
IS *	0.040	0.101	0.142	0.144	0.137	0.124	0.108		0.105	0.107	0.129	0.150	
	0.938	0.532	-0.117	-0.222	-0.292	-0.296	-0.080		-0.021	0.005	0.037	0.045	
	0.081	0.083	0.114	0.099	0.092	0.091	0.111		0.112	0.103	0.101	0.087	
Standard Deviation:													
GDP		EMPL		PROD		WR							
0.789		0.569		0.944		0.684							
(0.127)		(0.059)		(0.135)		(0.125)							

* Nominal short term interest rate, not filtered. Columns 3 to 11 give correlation between unfiltered interest rates and unfiltered growth rate of GDP.

These results allow us to draw the following tentative conclusions: The presence of money and technology shocks together with nominal rigidities seem to be essential for replicating major stylised facts between prices and output over the cycle. Ignoring one of these individual elements leads to a deterioration in the ability of the model to replicate the stylised facts along various dimensions. In addition, interest elastic money demand contributes towards an improvement of some dynamic cross correlation patterns between prices and output. Based on these simulation outcomes we can now attempt a more systematic interpretation of the role played by individual hypotheses.

The negative correlation between prices and output can best be understood by writing the money market equilibrium condition as a price level equation

$$(46) \quad \ln(P_t) = -\ln(Y_t) + \ln(M_t) + 1/(\nu + 1)\ln(r_t + E_t[P_{t+1}/P_t]) .$$

The different effects of money and technology shocks can be seen clearly from this formulation. A positive supply shock (see Table 4 and Figures 1a and 1b) increases income and therefore the demand for real balances. If money is not strongly accommodating the income expansion, the price level tends to fall. Nominal interest rates also tend to fall in response to a supply increase. This gives an additional positive impulse on the demand for money. Thus the negative correlation between prices and output can be replicated under the assumption that supply shocks are important and monetary policy is non-accommodating. A monetary shock (see Table 5 and Figures 2a and 2b), on the other hand, leads to an increase in both GDP and the price level. The positive effect on the price level is reinforced by the fact that the highly persistent nature of the estimated money supply process makes nominal interest rates rise, i. e. the expected inflation effect on short term rates dominates the reduction in real rates. Thus it is difficult to generate the observed correlation between prices and output with money shocks only. A comparison of Table 4 and Table 5 shows, however, that the presence of monetary shocks, helps to reduce the extreme negative correlation between prices and output which emerges when only real shocks are present.

The simultaneous presence of monetary and real shocks is also crucial for replicating the characteristic lead of the price level vis-à-vis output. Monetary shocks, in the presence of nominal rigidities, lead to a relatively short lived output effect accompanied and followed by a permanent increase in the price level. Supply shocks on the other hand induce a more immediate and only temporary price response combined with a more permanent change in output. Combining the correlations between prices and output as exhibited in Table 4 and 5 yields the characteristic cross correlation pattern

that we observe in the benchmark model. In this regard it is also interesting to observe the role played by the interest elasticity of money demand which seems to be a necessary ingredient for prices to lead output. It can be envisaged that with interest elastic money demand prices become more forward looking. Thus the presence of both nominal and real shocks, nominal rigidity and interest elastic money demand are needed to produce this pattern in the model economy.

In contrast to this result, the phenomenon that output leads inflation can be generated under various hypotheses. In particular it can be observed in cases without nominal rigidity, thus a straightforward Keynesian interpretation of this result would not be sufficient. Let us therefore look more closely at how this correlation pattern emerges in the case without nominal rigidity. Consider a positive supply shock, then the economy adjusts in the following way. Output increases in the short run but there will be an early tendency for output to revert back to trend. The price level is a close mirror image of this development it first falls and then starts to increase. Thus the supply shock gives rise to a positive correlation between inflation and lagged output, but a negative contemporaneous correlation. This pattern can also clearly be seen in Figures 1a and 1b. To match the correlation pattern more closely one therefore must allow for money shocks.

In interpreting the correlation between short term nominal interest rates and inflation from the above experiments the results suggest that especially the assumption of nominal rigidities is crucial. Only in the absence of price sluggishness we do not observe a significant negative correlation between the two variables. The negative correlation is especially pronounced in the case of supply shocks.

7. Conclusions

The model presented in this paper seems able to account for a sizeable number of stylized facts. The results indicate that real as well as monetary shocks, together with some degree of nominal rigidity, are important elements for an understanding of the interaction between nominal and real variables over the business cycle. It is found that by allowing for technology and monetary shocks, the model can replicate the negative contemporaneous correlation between prices and output as well as the positive contemporaneous correlation between inflation and output. The results generated by the model economy are also consistent with characteristic lead and lag relationships, such as the lead of prices and nominal short term interest rates relative to output and the lagged response of inflation to movements of GDP. The present analysis does, however, also suffer from important shortcomings. The model could especially be improved along at least two dimen-

sions. First, the volatility of nominal variables is too high and employment is too volatile. It could be envisaged that some alternative money supply rules, e. g. by allowing for some targeting of inflation could improve the fit of the model. With respect to the labour market it seems likely that adding more friction to labour demand (e. g. in form of quadratic adjustment costs) could be helpful.

Appendix

Deriving the Aggregate Consumption Rule

Using the first order conditions (7a), solving the budget constraint (3) forward and imposing the transversality condition gives the following decision rule for consumption of household z

$$(A.1) \quad C_{t,z}^* = \theta / (1 + \theta) [A_{t,z} + H_{t,z}] P_t / P_t^c .$$

The variable $H_{t,z}$ can be decomposed into the present value of current and future net income from employment or unemployment $H_{t,z}^y$ and the present value of lump sum taxes, transaction costs and foregone interest income from holding money $H_{t,z}^x$

$$(A.2) \quad H_t = H_{t,z}^y + H_{t,z}^x$$

with

$$(A.3) \quad H_{t,z}^y = E_t \left[\sum_{j=0}^{\infty} b_{tj} \left(Y_{t+j,z}^h - T_{t+j,z} - (1 + v^{i1}) i_{t+j} M_{t+j,z} \right) / P_{t+j} \right]$$

$$(A.4) \quad H_{t,z}^x = E_t \left[\sum_{j=0}^{\infty} b_{tj} \left(-T_{t+j,z} - (1 + v^{-1}) i_{t+j} M_{t+j,z} \right) / P_{t+j} \right] .$$

For the derivation of an aggregate consumption rule we observe that the human capital of employed and unemployed households which we denote with H_t^l and H_t^u respectively is given by the two arbitrage equations (31) and (32). Employment and unemployment (U_t) dynamics are given by (22) and by

$$(A.5) \quad \Delta U_t = sL_t - p(\cdot)U_{t-1}, \quad \text{where } U_t = 1 - L_t .$$

Since we make the assumption that each employed worker earns W_t and each unemployed worker receives Z_t , we can define aggregate permanent income from wages and benefits as

$$(A.6) \quad H_t^y = H_{t,z}^y L_t + H_{t,z}^u U_t .$$

Taking first differences of this expression and substituting (31), (32), (22) and (A.5) yields aggregate permanent income from employment and unemployment of the household sector

$$(A.7) \quad H_t^y = E_t \left[\sum_{j=0}^{\infty} b_{tj} \left(L_{t+j} W_{t+j} + (1 - L_{t+j}) Z_{t+j} \right) / P_{t+j} \right] .$$

Using (A.2) and simply summing over $A_{t,z}$ gives equation (8) and (9) in the text.

References

- Andolfatto, D. (1996), Business Cycles and Labor Market Search. *American Economic Review* 86, pp. 112–32.
- Backus, D. K. / Kehoe, P. J. (1992), International Evidence on the Historical Properties of Business Cycles. *American Economic Review* 82, pp. 864–88.
- Boucekkine, R. (1995), An Alternative Methodology for Solving Nonlinear Forward-Looking Models. *Journal of Economic Dynamics and Control* 19, pp. 711–34.
- Burda, M. / Wyplosz, C. (1994), Gross Worker and Job Flows in Europe. *European Economic Review* 38, pp. 1287–1315.
- Burdett, K. / Wright, R. (1989), Unemployment Insurance and Short Term Compensation: The Effects of Layoffs, Hours per Worker and Wages. *Journal of Political Economy* 97, pp. 1479–96.
- Burnside, C. / Eichenbaum, M. (1996), Factor Hoarding and the Propagation of Business-Cycle Shocks. *American Economic Review* 86, pp. 1154–1174.
- Chada, B. / Prasad, E. (1994), Are Prices Countercyclical? Evidence from the G-7. *Journal of Monetary Economics* 34, pp. 239–57.
- Cogley, T. / Nason, J. M. (1995), Output Dynamics in Real Business Cycle Models. *American Economic Review* 85, pp. 492–511.
- Cooley, T. F. / Ohanian, L. E. (1991), The Cyclical Behaviour of Prices. *Journal of Monetary Economics* 28, pp. 23–60.
- Cooley, T. F. / Hansen, G. D. (1994), Money and the Business Cycle. in T. F. Cooley (Ed.): *Frontiers of Business Cycle Research*. Princeton University Press.
- (1998), The Role of Monetary Shocks in Equilibrium Business Cycle Theory. *European Economic Review* 42, pp. 605–617.
- Correia, I. / Neves, J. C. / Rebelo, S. (1995), Business Cycle in a Small Open Economy. *European Economic Review* 39, pp. 1089–1114.
- Danthine, J. P. / Donaldson, J. B. (1993), Methodological and Empirical Issues in Real Business Cycle Theory. *European Economic Review* 37, pp. 1–36.

- Devereux, M. B. / Gregory, A. W. / Smith, G. W.* (1992), Realistic Cross-Country Consumption Correlations in a Two-Country Equilibrium Business Cycle Model. *Journal of International Money and Finance* 11, pp. 3 – 16.
- Dixit, A. / Stiglitz, J.* (1977), Monopolistic Competition and Optimum Product Diversity. *American Economic Review* 67, pp. 297 – 308.
- Dornbusch, R.* (1987), Exchange Rates and Prices. *American Economic Review* 77, pp. 93 – 106.
- Erdmann, K.* (1990), Arbeitsmarktstatistische Zahlen in Zeitreihenform. Jahreszahlen für die Bundesrepublik Deutschland – Ausgabe 1990 – Beiträge aus der Arbeitsmarkt – und Berufsforschung. Nürnberg.
- Fuhrer, J. C. / Moore, G. E.* (1995), Monetary Policy Trade-offs and the Correlation Between Nominal Interest Rates and Real Output. *American Economic Review* 85, pp. 219 – 39.
- Hairault, J. O. / Portier, F.* (1993), Money, New-Keynesian Macroeconomics and the Business Cycle. *European Economic Review* 37, pp. 1533 – 68.
- Hayashi, F.* (1982), Tobin's Marginal Q and Average Q: A Neoclassical Interpretation. *Econometrica* 50, pp. 213 – 24.
- Howitt, P.* (1988), Business Cycles With Costly Search and Recruiting. *Quarterly Journal of Economics* 103, pp. 147 – 65.
- Kydland, F. E. / Prescott, E. C.* (1990), Business Cycles: Real Facts and a Monetary Myth. *Federal Reserve Bank of Minneapolis Quarterly Review*, pp. 3 – 18.
- (1996), The Computational Experiment as an Econometric Tool. *Journal of Economic Perspectives* 10, pp. 69 – 85.
- Laffarque, J. P.* (1990), Resolution d'un Modele Macroeconometrique avec Anticipation Rationnelles. *Annales d'Economie et Statistique* 17, pp. 97 – 119.
- Layard, R. / Nickell, S. / Jackman, R.* (1991), Unemployment. Oxford University Press. Oxford.
- Leeper, E. M. / Sims, C. A.* (1994), Toward a Modern Macroeconomic Model Usable for Policy Analysis. in S. Fisher and J. J. Rotemberg (Ed.): *NBER Macroeconomics Annual*. MIT-Press.
- Lucas, R. J.* (1987), *Models of Business Cycles*. Basil Blackwell. Oxford.
- Mendoza, E. G.* (1991), Real Business Cycles in a Small Open Economy. *American Economic Review* 81, pp. 797 – 818.
- Mendoza, E. G. / Razin, A. / Tesar, L. L.* (1994), Effective Tax Rates in Macroeconomics: Cross Country Estimates of Tax Rates on Factor Income and Consumption. NBER Working Paper No. 4864.
- Matsuyama, K.* (1987), Current Account Dynamics in a Finite Horizon Model. *Journal of International Economics* 23, pp. 299 – 313.
- Merz, M.* (1995), Search in the Labor Market and the Real Business Cycle. *Journal of Monetary Economics* 36, pp. 269 – 300.
- Mortensen, D. T.* (1989), The Persistence and Indeterminacy of Unemployment in Search Equilibrium. *Scandinavian Journal of Economics* 91, pp. 347 – 70.
- Oliveira Martins, J. / Scarpetta, S. / Pilat, D.* (1997), Mark-up Pricing, Market Structure and the Business Cycle. *OECD Economic Studies* 27.

- Pissarides, C. A. (1990), *Equilibrium Unemployment Theory*. Basil Blackwell. Oxford.
- (1998), The Impact of Employment Tax Cuts on Unemployment and Wages; The Role of Unemployment Benefits and Tax Structure. *European Economic Review* 42, pp. 155 – 84.
- Roberts, J. (1987), An Equilibrium Model with Involuntary Unemployment at Flexible, Competitive Prices and Wages. *The American Economic Review* 77, pp. 856 – 74.
- Rotemberg, J. J. (1982), Sticky Prices in the United States. *Journal of Political Economy* 90, pp. 1187 – 1211.
- (1996), Prices, Output and Hours: An Empirical Analysis based on a Sticky Price Model. *Journal of Monetary Economics* 37, pp. 505 – 33.
- Taylor, J. B. (1980), Aggregate Dynamics and Staggered Contracts. *Journal of Political Economy* 88, pp. 1 – 23.
- van Ours, J. C. / Ridder, G. (1992), Vacancies and the Recruitment of New Employees. *Journal of Labor Economics* 10, pp. 138 – 155.

Zusammenfassung

Dieser Artikel untersucht inwieweit ein dynamisches Gleichgewichtsmodell mit Suchkosten auf dem Arbeitsmarkt, Preisanpassungskosten und überlappenden Lohnkontrakten mit wichtigen stilisierten Fakten hinsichtlich des Zusammenhangs von Preisen, Zinssätzen und BIP für die deutsche Wirtschaft übereinstimmt. Läßt man technologische und monetäre Schocks zu, so zeigt sich, daß das Modell sowohl die beobachtete negative Korrelation zwischen Preisniveau und BIP als auch die positive Korrelation zwischen der Inflationsrate und dem BIP abbilden kann. Die Modellergebnisse sind ebenfalls konsistent mit charakteristischen Lead und Lag Mustern, wie z. B. dem Vorlauf der Preise und der nominalen kurzfristigen Zinssätze gegenüber dem BIP und der verzögerten Anpassung der Inflation.

Abstract

The paper addresses the question whether an open economy dynamic equilibrium model with search in the labour market, price adjustment costs and wage staggering is consistent with important stylised facts concerning the relationship between prices, interest rates and output for the German economy over the flexible exchange rate period. It is found that by allowing for technology and monetary shocks, the model can replicate the negative contemporaneous correlation between prices and output, as well as the positive contemporaneous correlation between inflation and output. The results generated by the model economy are also consistent with characteristic lead and lag patterns, such as the lead of prices and nominal short term interest rates relative to output and the lagged response of inflation to movements of GDP.

JEL-Klassifikation: E30, E31, E32, E43

Keywords: Business cycles; Nominal rigidity; Wage staggering; Labour market search; Technology shocks; Monetary shocks.