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Self-Fulfilling Prophecies, Demand-Composition Effects and Economic Fluctuations*

By Mark Weder**

1. Introduction

The hypothesis whether or not business cycles are generated by volatile optimistic or pessimistic expectations which ultimately become self-fulfilling is perhaps as old as the study of the business cycle itself. Until recently the notion of extrinsic uncertainty had never appeared in fully formulated models in modern macroeconomics. This was previously viewed as a theoretical curiosity outside the scope of equilibrium models. Yet as new developments have shown, the presence of a multiplicity of equilibria may not be fully unrealistic.

Recently, the concept of animal spirits made its way into well defined equilibrium business cycle theory.[1] These models have one specific feature in common: the calibrated versions of their respective models possess a continuum of rational expectation solutions which all converge to the steady state. This indeterminacy arises because of some market imperfection which may come from increasing returns or market power. Recent empirical work, especially that conducted by Basu and Fernald (1994, 1997), questions the assertion of these models to be realistic theories of the business cycle, however. Though data for U.S. industry points to the presence of scale economies and market power, the extent thereof seems to be rather modest and, more importantly, too low to give most existing models of indeterminacy a sound foundation as realistic models of the business cycle. This

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1 See for example Farmer and Guo (1994) as well as by Gali (1994). For a recent survey of the literature on indeterminacy refer to Benhabib and Farmer (1998) or Weder (1999).

problem has led researchers to pursue alternative structures where indeterminacy can arise at lower scale economies. Benhabib and Farmer (1996) and Weder (1998, 2000), for example, use multi-sector optimal growth models. This strategy allows the necessary degree of increasing returns to be reduced dramatically.

The underlying structure of the model in this paper, however, is based on a one production-sector growth model of monopolistic competition with endogenous entry and exit of firms. There are two final goods in the economy.[2] It is similar to the model that is presented by Chatterjee and Cooper (1993). The distinctive feature of their model is that firms can practice price discrimination on the prospective use of the products, namely, for investment or consumption purposes.[3] Therefore, the composition of final demand affects the market power of each supplier. It will be demonstrated in this paper that it is possible to generate indeterminacy at modest increasing returns to scale. That is, the minimum extend of scale economies (or the inverse of the markups if pure profits are zero) is in the range that is empirically plausible (generally at 1.10 or lower). Moreover, the size thereof is lower than in most other existing models of indeterminacy. Also, the cyclical properties of the model are similar to those found in data. The model possesses a strong internal propagation mechanism which generates highly persistent time series. This persistence arises without the help of highly autocorrelated forcing variables.[4]

Perhaps most closely related to the present work is Gali (1994), in which he too constructs a model that has stationary sunspot equilibria in the presence of a sufficient difference in the rate of substitution between goods in consumption and investment. As a result of this asymmetry, a monopolistic firm faces a variable demand elasticity depending on the composition of aggregate demand. If the difference of the markups is large enough, fluctuations arise as a consequence of self-fulfilling revisions of expectations. Gali (1994) assumes, however, that firms are not able to price discriminate between investment and consumption goods markets.

My exposition unfolds as follows. Firstly, the model will be presented. The second part studies the equilibrium dynamics, in particular the possibility of sunspot equilibria. This is followed by an analysis of the stochastic properties of the model's variables, which will be compared to corresponding real world time series.

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2 The model is therefore a blend of the one and two sector setups.

3 The model in Chatterjee and Cooper (1993) has a different production technology. It is of regular form and, therefore, it is driven by fundamental shocks only.

4 See Cogley and Nason (1995) for related problems found in most Real Business Cycles models.

2. The model

2.1 The household

The economy consists of one representative agent with lifetime utility

$$(1) \qquad E[\sum_{t=0}^{\infty}\beta^t U(C_t, \mathbf{l_t}) \mid \mathcal{I}_0] \quad 0<\beta<1$$

where C_t is consumption, $\mathbf{l_t}$ leisure and β the discount factor. $\mathcal{I}_0$ is the set of information that is available to the household at period 0. Households are endowed with one unit of time which they can either use for work L_t or leisure:

$$(2) \qquad 1 = L_t + \mathbf{l_t} \,.$$

The following functional form for instantaneous utility is assumed:

$$(3) \qquad U(C_t, L_t) = \log C_t + \frac{1}{1+\chi}(1-L_t)^{1+\chi} \quad \chi \leq 0 \,.$$

Consumption of the households is defined by a CES-aggregator over differentiated intermediate goods:

$$(4) \qquad C_t = \left(\int_0^{N_t} C_{j,t}^{\upsilon} dj\right)^{1/\upsilon} \quad 0<\upsilon\leq 1 \,.$$

Thus, C_t is a function of the level of consumption of an assembled variety of the N_t input goods $C_{j,t}$. Each of these goods enters the aggregator symmetrically. For the case $\upsilon<1$, the goods are imperfect substitutes which will be the source of market power in the model. The aggregator for the investment good I_t is defined as

$$(5) \qquad I_t = \left(\int_0^{N_t} I_{j,t}^{\theta} dj\right)^{1/\theta} \quad 0<\theta\leq 1$$

where the parameter θ has the analog interpretation as υ. Gali (1994) notes that an *a priori* reasoning for an equality of θ and υ does not exist: The substitutability of the two goods originates from two unrelated technologies. We follow this track here.

The consumer's capital holdings evolve as

$$(6) \qquad K_{t+1} = (1-\delta)K_t + I_t \quad 0<\delta<1 \,.$$

K_t denotes the stock of capital and is the rate of depreciation. Finally, the period-by-period budget constraint of the household is given by

$$\int_0^{N_t} p_{c,j,t} C_{j,t} dj + \int_0^{N_t} p_{i,j,t} I_{j,t} dj = w_t L_t + q_t K_t + \Pi_t \,. \tag{7}$$

Here $p_{c,j,t}$ is the price of the consumption good j and $p_{i,j,t}$ the price for the investment good j.[5] Furthermore, the household receives profit income from all N_t existing firms, Π_t. Households own the stock of capital and rent it out to the firms at the rental price q_t. w_t is the wage rate. Factor markets are perfectly competitive.

The conditional demand for $C_{j,t}$ can be derived as

$$C_{j,t} = \left(\frac{p_{c,j,t}}{p_{c,t}}\right)^{\frac{1}{v-1}} C_t N_t^{-1/v} \tag{8}$$

which has, as can be seen, a constant price elasticity where

$$p_{c,t} \equiv \left(\frac{1}{N_t}\int_0^{N_t} p_{c,j,t}^{\frac{v}{v-1}} dj\right)^{\frac{v-1}{v}}$$

is the exact price index for the consumption goods. The same can be conducted for the investment goods. The conditional demand for $I_{j,t}$ becomes

$$I_{j,t} = \left(\frac{p_{i,j,t}}{p_{i,t}}\right)^{\frac{1}{\theta-1}} I_t N_t^{-1/\theta} \,. \tag{9}$$

In symmetric equilibrium, the consumption good is used as the numeraire and the price for the investment goods will be denoted by p_t.[6] The period-by-period budget constraint now becomes

$$q_t K_t + w_t L_t + \Pi_t \geq C_t N_t^{\frac{v-1}{v}} + (K_{t+1} - (1-\delta)K_t) p_t N_t^{\frac{\theta-1}{\theta}}$$

This equation shows that the household's value for the two aggregate goods is an increasing function of the number of input goods – captured in the budget constraint by the terms $N_t^{\frac{v-1}{v}}$ and $N_t^{\frac{\theta-1}{\theta}}$ respectively.

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5 Note that these prices may differ since firms are allowed to price discriminate perfectly on the respective markets (see below).

6 Symmetric equilibrium implies that each firm charges the same price(s) for their product. The equilibrium will be formally derived in the next subsection.

Let λ_t denote the current value Lagrange multiplier associated with the household's resource constraint. The household maximizes utility by choosing a sequence $\{C_t, L_t, K_{t+1}\}_{t=0}^{\infty}$ subject to a given K_0 and the constraints. The sequence of future states of technology Z_t is not completely known at $t = 0$ and agents form rational expectations over these variables. The first order conditions are

$$\frac{1}{C_t} - N_t^{\frac{v-1}{v}} \lambda_t = 0 \tag{10}$$

$$(1 - L_t)^{\chi} - \lambda_t w_t = 0 \tag{11}$$

$$\beta E\left[\lambda_{t+1}\left(q_{t+1} + (1-\delta)p_{t+1}N_{t+1}^{\frac{\theta-1}{\theta}}\right) \mid \mathcal{I}_t\right] - \lambda_t p_t N_t^{\frac{\theta-1}{\theta}} = 0 \tag{12}$$

plus the household's budget constraint and the usual transversality condition. (10) and (11) describe the households consumption-leisure trade off and (12) is the intertemporal optimality condition.

2.2 The firms

There are N_t monopolistic competitive firms supplying their specific good j every period t. Endogenous entry and exit of firms will be allowed and this process is modeled in the simplest possible fashion: purely static decision making is considered. Each firm decides to enter the economy (or to stay out of the market) every period. An active firm observes that its profit opportunities exceed its overhead costs, ϕ. However, since free entry and exit is possible, any profits are instantaneously dissipated. Overhead costs are 'lost in space', that is, they are not associated with any income. Each firm j solves

$$\max \Pi_{j,t} = p_{c,j,t} Y_{c,j,t} + p_{i,j,t} Y_{i,j,t} - w_t L_{j,t} - q_t K_{j,t} \tag{13}$$

subject to its production function

$$Y_{j,t} = Y_{c,j,t} + Y_{i,j,t} = Z_t (K_{j,t}^{\alpha} L_{j,t}^{1-\alpha})^{\gamma} - \phi \quad 0 < \alpha < 1, \quad \gamma > 0, \quad \phi \geq 0 \tag{14}$$

and to the given demand functions. $Y_{c,j,t}$ ($Y_{i,j,t}$) denotes the amount of output to be sold as a consumption (investment) good. $K_{j,t}$ and $L_{j,t}$ are capital and labor input of firm j at t. $p_{c,j,t}$ and $p_{i,j,t}$ are the prices of good j if sold as a consumption or investment good. Given the possibility price discrimination, these prices need not be equal. Total factor productivity Z_t evolves as

$$\log Z_{t+1} = \rho_z \log Z_t + (1 - \rho_z) \log Z + z_{t+1}\ .$$

The sequence $\{z_t\}_{t=0}^{\infty}$ is white noise, $Z \equiv 1$ and $0 < \rho_z < 1$. From (8) and (9), the inverse demand functions for the intermediate good j can be derived as

$$p_{cj,t} = \left(\frac{Y_{cj,t} N_t^{1/\upsilon}}{C_t} \right)^{\upsilon - 1} p_{c,t} \tag{15}$$

and

$$p_{ij,t} = \left(\frac{Y_{ij,t} N_t^{1/\theta}}{I_t} \right)^{\theta - 1} p_{i,t} \ , \tag{16}$$

if the good is used as a consumption good or an investment good respectively. Given the constant price elasticity of demand, profit maximization requires that

$$p_{cj,t} = \frac{1}{\upsilon} \frac{\partial \mathbf{C}(q_t, w_t, Y_{j,t})}{\partial Y_{j,t}} = \frac{1}{\upsilon} \frac{1}{\gamma Z_t} A q_t^{\alpha} w_t^{1-\alpha} \left(\frac{Y_{j,t} + \phi}{Z_t} \right)^{\frac{1}{\gamma} - 1} \tag{17}$$

and

$$p_{ij,t} = \frac{1}{\theta} \frac{\partial \mathbf{C}(q_t, w_t, Y_{j,t})}{\partial Y_{j,t}} = \frac{1}{\theta} \frac{1}{Z_t} A q_t^{\alpha} w_t^{1-\alpha} \left(\frac{Y_{j,t} + \phi}{Z_t} \right)^{\frac{1}{\gamma} - 1} \tag{18}$$

hold.[7] These are the standard pricing rules for monopoly pricing as a markup over marginal unit costs. Note that marginal costs are decreasing for $\gamma > 1$ and increasing for $\gamma < 1$.

Implicitly assumed here is that arbitrage through the household sector is not possible. For example, intermediate goods that are sold as consumption goods cannot be transformed into investment goods. This can be defended as follows. Suppose $> \theta$.[8] {abs}{page}{abs}Therefore, it would not make sense to the household to buy good j on the investment market and use it as a consumption good. However, the opposite act would be advantageous. This arbitrage could be ruled out, however, if one imposes some form of (full)

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[7] A is some positive constant. The cost function of firm j is given by

$$\mathbf{C}(w_t, q_t, Y_{j,t}) = A q_t^{\alpha} w_t^{1-\alpha} \left(\frac{Y_{j,t} + \phi}{Z_t} \right)^{\frac{1}{\gamma}} .$$

When $\gamma > 1$, these first order conditions are not necessarily sufficient for optimality. However, in each of the following calibrations, the second-order conditions are satisfied.

[8] This is the only case that is considered in this work. It implies that the investment goods aggregator-technology is the more complex.

depreciation of the intermediate good unless it is welded together immediately after the purchase. That is, the depreciation does not take effect when the good is attached with the other consumption goods according to technology (4).[9] An alternative rationale is a putty-clay assumption: each firm determines *ex ante* how the output is allocated across the two types of goods.[10]

At every period in time the number of active firms is determined by the zero profit condition

$$p_{cj,t}Y_{cj,t} + p_{ij,t}Y_{ij,t} = Aq_t^{\alpha}w_t^{1-\alpha}\left(\frac{Y_{j,t}+\phi}{Z_t}\right)^{\frac{1}{\gamma}}. \tag{19}$$

Equations (17) and (18) together imply

$$\upsilon p_{cj,t} = p_{ij,t}\theta\,. \tag{20}$$

Given the choice of the numeraire, the last equation implies that the price for the investment goods becomes in symmetric equilibrium

$$p_{ij,t} = p_t = \frac{\upsilon}{\theta}\,. \tag{21}$$

Inserting the optimal pricing rules into equation (19) yields

$$\frac{1}{\upsilon}Y_{cj,t} + \frac{1}{\theta}Y_{ij,t} = \gamma Z_t(K_{j,t}^{\alpha}L_{j,t}^{1-\alpha})^{\gamma}\,. \tag{22}$$

Now, (22) can be rewritten in terms of aggregate variables:

$$\frac{1}{\upsilon}\frac{C_t}{N_t^{1/\upsilon}} + \frac{1}{\theta}\frac{I_t}{N_t^{1/\theta}} = \gamma Z_t(K_t^{\alpha}L_t^{1-\alpha}N_t^{-1})^{\gamma} \tag{23}$$

which can be combined with the firms' technology to yield implicitly the equilibrium number of active firms as

$$\left(\frac{1}{\upsilon}-\frac{1}{\theta}\right)\frac{C_t}{N_t^{1/\upsilon}} = \left(\gamma-\frac{1}{\theta}\right)Z_t(K_t^{\alpha}L_t^{1-\alpha}N_t^{-1})^{\gamma} + \frac{1}{\theta}\phi\,. \tag{24}$$

In the special case of $\theta = \upsilon$, the first term on the left hand side vanishes and the model's zero profit condition collapses back into the "standard"

9 This is analogous to the assumption of extremely high conversion costs.

10 I would like to thank Richard Rogerson for pointing out this possibility to me.

form. It is this deviation from existing works (e.g. Devereux, Head and Lapham, 1996) that will deliver indeterminacy at modest degrees of scale economies.

Finally, by combining the optimal pricing rule with the conditional demand for labor, it is possible to derive the wage rate as

$$w_t = \upsilon\gamma(1-\alpha)Z_t(K_t^{\alpha}L_t^{1-\alpha})^{\gamma}L_t^{-1}N_t^{1-\gamma} \,. \tag{25}$$

Analogously the rental rate of capital is given as

$$q_t = \upsilon\gamma\alpha Z_t(K_t^{\alpha}L_t^{1-\alpha})^{\gamma}K_t^{-1}N_t^{1-\gamma} \,. \tag{26}$$

Note that this simple aggregation of the conditional demands does not yet yield the actual rental prices. These demands must be combined with the equilibrium value for N_t as given by the zero profit condition. Our measure of overall aggregate output S_t is defined as

$$S_t = \upsilon\gamma Z_t K_t^{\alpha\gamma}L_t^{(1-\alpha)\gamma}N_t^{1-\gamma} \tag{27}$$

which is simply the sum of capital and labor income.

3. The equilibrium dynamics

This section describes the equilibrium dynamics around the economy's steady state. The stationary state will be discussed first. Then, the calibration will be outlined. Finally, indeterminacy conditions will be derived.

3.1 The steady state

The steady state exists and is unique. Using the zero profit condition and the pricing rule, the steady state number of firms is given by

$$N = \left[(\theta-\upsilon)CS - \theta + \frac{1}{\gamma}\right]/(\upsilon\phi/S) \tag{28}$$

where omission of the time index implies steady state values. S is the steady state aggregate output and we denote the consumption share on output as $CS \equiv CN^{\frac{\upsilon-1}{\upsilon}}/S$. It can be shown by implicit differentiation that

$$\frac{\partial \log N}{\partial \log(\phi/S)} < 0, \quad \frac{\partial \log N}{\partial \log \gamma} < 0, \quad \frac{\partial \log N}{\partial \log \upsilon} < 0, \quad \frac{\partial \log N}{\partial \log \theta} < 0 \,,$$

and

$$\operatorname{sign}\left[\frac{\partial \log N}{\partial \log(CS)}\right] = \operatorname{sign}\left[\frac{\theta}{\upsilon} - 1\right] .$$

As could be expected, a rise in the respective market power measures $1/\upsilon$ and $1/\theta$ lowers the steady state number of firms since the profit margin decreases. Also, the impact of demand composition, CS, depends on the market power. If market power is greater in the investment sector, that is $\upsilon > \theta$, an increase in the consumption share lowers the equilibrium number of firms.

3.2 The solution mechanism and indeterminacy

The solution method which is used here was first introduced into the \RBC literature by King, Plosser and Rebelo (1988). The linearized model reduces to the following matrix difference equation:

$$\begin{bmatrix} E[\hat{\lambda}_{t+1} \mid \mathcal{I}_t] \\ \hat{K}_{t+1} \\ E[\hat{Z}_{t+1} \mid \mathcal{I}_t] \end{bmatrix} = \mathbf{J} \begin{bmatrix} \hat{\lambda}_t \\ \hat{K}_t \\ \hat{Z}_t \end{bmatrix} \tag{29}$$

where the matrix $\mathbf{J}$ is 3×3 and hats over variables denote percentage deviations from steady states. The system contains one predetermined endogenous variable (the stock of capital), one predetermined exogenous variable (the state of technology) and one endogenous nonpredetermined variable (the shadow value of wealth). In the standard rational expectations case with a unique equilibrium, the model displays the saddle point behavior only if exactly one eigenvalue is strictly outside the unit circle. This property must be checked for the present model because of its imperfect market structure since the First Welfare Theorem no longer applies. If all three eigenvalues are inside the unit root, the model's adjustment path is no longer unique and sunspot equilibria may arise. This possibility is typically coined indeterminacy of rational expectations.

3.3 Calibration

Parameter value determination is in accord with the Real Business Cycle tradition: steady state values of the model will be matched with estimates of average growth rates and *great ratios*. First a baseline model structure will be defined.

Consistent with Real Business Cycle practice, δ will be set equal to 0.025 on a quarterly basis. The production function is Cobb-Douglas, hence the parameter α equals the capital share. The capital share of GNP net of housing in the United States is about 30 percent for the period from 1954 to 1989. We set α accordingly. We calibrate the consumption share at 0.75, which is the same value as in Schmitt-Grohe (1997). Furthermore the quarterly discount factor β will be set to 0.99. For the case $\chi = 0$, the model's labor market corresponds to the Hansen (1985) and Rogerson (1987) indivisible labor market formulation.[11]

Let us now turn to the modeling of market imperfections. Basu and Fernald (1994) report estimates for increasing returns between one and 1.26. However, their preferred point estimate is 1.03. Market power as measured by markups over costs is reported by Morrison (1990) to be around 1.14. Burnside (1995) and Burnside, Eichenbaum and Rebelo (1995) report evidence for constant returns.[12] On a more disaggregated level, Basu and Fernald (1997) find evidence for scale economies only in the durable goods producing sector of the U.S. economy. Similarly, Harrison (1997) offers evidence that modest scale economies are only present in the investment sector. These results should act as the ballpark figure for the following calibration of scale economies.

3.4 Indeterminacy results

3.4.1 Eigenvalues

In light of the mentioned empirical work, the model must be checked to see if it is capable of generating indeterminacy without the assumption of high increasing returns to scale and sharply decreasing marginal costs. First $\gamma = 1.00$ will be set, which implies constant marginal costs. Table 1 considers alternative (but identical) values for υ and θ.[13]

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[11] Interestingly, the numerical choice of the ϕ/S ratio does not affect the steady state as a consequence of the zero profit condition.

[12] See Benhabib and Farmer (1996) for a discussion of recent empirical results.

[13] The third eigenvalue of **J** is the persistence parameter of the technology sequence ρ_z. It is not reported in the Tables. The Tables are to be read as follows: the leftmost column(s) depict alternative parameter spaces, the columns denoted by Roots 1 and 2 refer to the numerical eigenvalues of **J**. The rightmost column denotes the qualitative dynamics of the model. *IRS* indicates the implied returns to scale.

Table 1

Roots of Model

υ	θ	*IRS*	Root 1	Root 2	
0.90	0.90	1.11	1.088	0.921	saddlepath stable
0.80	0.80	1.25	1.121	0.914	saddlepath stable
0.70	0.70	1.43	1.387	0.889	saddlepath stable
0.6722	0.6722	1.49	– 0.979	0.858	indeterminacy
0.65	0.65	1.54	0.847+0.110i	0.847-0.110i	indeterminacy

Table 1 shows that the roots split around unity unless the markup (and, implicitly, the returns to scale) becomes very large. The value of $\upsilon = \theta = 0.67$ corresponds to a markup of 1.49. This values is far too high empirically. Indeterminacy cannot arise with symmetrical markups at modest increasing returns. Now, Table 2 looks at heterogeneous degrees of market power. In particular, it is assumed that the investment demand is less elastic than consumption demand (holding υ fixed at 0.95). This pattern is indirectly supported by the evidence in Basu and Fernald (1997) and Harrison (1997).[14]

Table 2

Roots of Model

υ	θ	*IRS*	Root 1	Root 2	
0.95	0.90	1.07	1.099	0.927	saddlepath stable
0.95	0.85	1.08	1.341	0.869	saddlepath stable
0.95	0.8398	1.09	– 0.9359	0.1769	indeterminacy
0.95	0.80	1.10	0.992+0.102i	0.992-0.102i	indeterminacy

The assumed asymmetry leads to indeterminacy at modestly low returns to scale. Increasing returns of 1.09 are within the region that is considered in recent empirical work. The Table shows that for small markup differences in the two output markets, the model can be indeterminate. In particular, it is required that the degree of market power in the investment market exceeds the respective degree in the market for consumption goods. For further understanding note that the average steady state markup of the firm can be defined as

$$CS\frac{1}{\upsilon} + (1 - CS)\frac{1}{\theta}\,.$$

[14] These authors examine increasing returns in production. However, in the present model there exists a close connection between scale economies and the markup.

This implies that for market powers $\theta = 0.83$ and $\upsilon = 0.95$, the returns to scale amount to 1.09.[15] This value is well within the range that is reported by Morrison (1990) as well as by Basu and Fernald (1995). Furthermore, marginal costs must not decrease sharply as in related works.

The result are not restricted to the particular numerical choices. If the parameters are set as in King, Plosser and Rebelo (1988) or Schmitt-Grohe (1997a), the results are basically unaltered. If we set $\alpha = 0.42$, $\delta = 0.024$ and the labor supply elasticity at four. Holding fixed $\upsilon = 0.95$ again leads to indeterminacy at $\theta = 0.766$ (or increasing returns to scale of 1.12, as opposed to 1.09). Indeterminacy may still arise with a lower labor supply elasticity yet the returns to scale that are needed to generate this case are higher.

It can be shown that the reverse case of a stronger market power in the consumption goods sector does not lead to indeterminacy.[16]

Until this point it has been assumed that marginal costs are constant. If $\gamma > 1$, indeterminacy is obtained at even lower increasing returns to scale. On the other hand, the presence of increasing marginal costs raises the minimum required sectoral markups. Table 3 visualizes this behavior.

Table 3

Minimum Increasing Returns to Scale for Indeterminacy

γ	θ	*IRS*
0.95	0.826	1.092
1.00	0.839	1.087
1.05	0.875	1.075

υ is fixed at 0.95, $\alpha = 0.30$ and $\chi = 0$.

To summarize this subsection, the model displays indeterminacy at rather modest returns to scale. Multiplicity of rational expectations equilibria occurs at increasing returns to scale that are modestly low. This can be seen as a significant innovation compared to several other one sector indeterminacy models. The model thus seems to escape empirical and theoretical criticism that has been directed at related work. To visualize this claim the following Table offers a comparison to other one-sector and two-sector models.

15 This result follows immediately from the zero profits condition.

16 However, this result can be circumvented if $\gamma > 1$.

Table 4

Minimum Increasing Returns to Scale for Indeterminacy

Author(s)	Model	*IRS*
Benhabib-Farmer (1994)	one-sector	1.43
Schmitt-Grohe (1997), Gali (1994)	one sector	1.37
Benhabib-Farmer (1996)	two-sector	1.08
Weder (1998)	two-sector	1.05

3.5 Interpretation

Intertemporal models of monopolistic competition generally have the potential of irregular, that is, indeterminate solutions. Since the Second Welfare Theorem does not apply, the equilibrium is no longer necessarily unique. Moreover, if the model possesses the above structure, economic fluctuations in response to random events that do not involve any change in the fundamentals may arise. For every agent it is not necessarily suboptimal to form expectations in this manner. In a rational expectations equilibrium, it is correct to follow these animal spirits when all agents expect these to matter (as in Keynes' *beauty contest*).

The economic intuition for the present model is as follows. Suppose that the representative agent expects future output to be high, this means that they also expect a large number of active firms in the economy. In the case of a booming economy, the returns to capital are high (given the scale economies and market participation). Thus, agents start to invest more in the present period. If the elasticity of investment demand is lower than the demand elasticity of consumption ($\theta < v$), this again implies that at given prices revenues of every firm increase. More firms enter the economy to take advantage of these opportunities until profits are dissipated through sufficient entry. This again spreads the decreasing returns of the single inputs (the increasing returns to specialization effect) and thereby increases present output. Also, the increase in product variety shifts the labor supply outwards, ultimately leading to more accumulation, which in turn encourages even more entry in the present and future periods. All of this translates into a higher future return to capital and the expected boom becomes self-fulfilling.

It is notable that the model obtains indeterminacy at even lower scale economies than Benhabib and Farmer's (1996) two sector model. The reason for this behavior appears to be rooted in the one sector production setup. It is relatively easy to obtain multiple equilibria in a two sector model since the composition of demand (and output) affects the relative price. A comparison of the Euler equation (12) and the equivalent equation in Benhabib and Farmer (1996) and Weder (2000) show that the two terms involving the number of firms take on the role that was played by the relative price in their model. However, the two sector model predicts that every alternative investment path implies curtailing consumption since input factors must be reallocated across sectors. This effect counters the possibility of indeterminacy due to consumption smoothing. The effect is of much lesser importance in the present model since no reallocation must take place. Hence, indeterminacy arises at lower returns to scale.

The following figure displays the behavior of the investment-consumption ratio and consumer confidence at the 1990-1991 U.S. business cycle peak.[17] A sharp decline in the investment-consumption ratio can be observed. This drop also appears to lead the cycle, a behavior which supports the intuition for indeterminacy that was given here. The same pattern can be reported for consumer confidence.

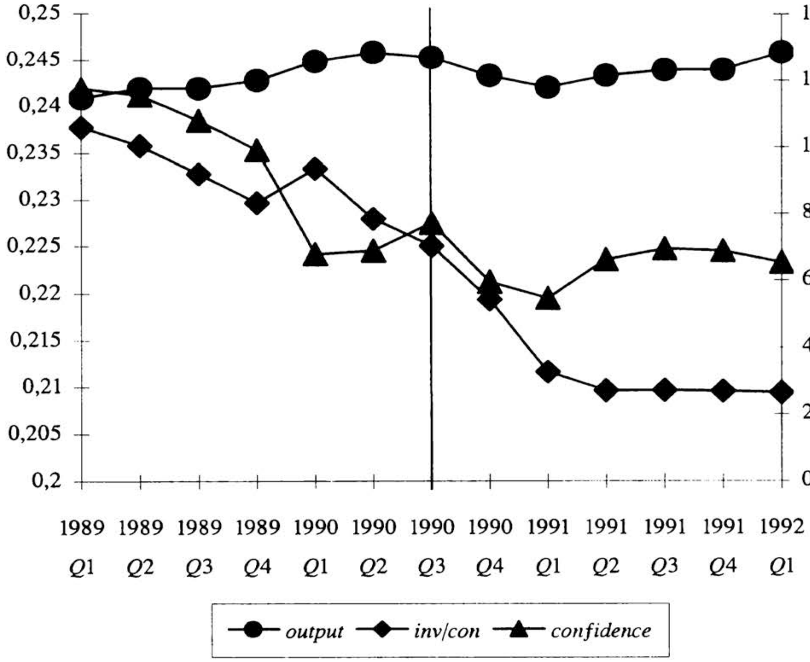

Figure 1

[17] This most recent U.S. recession is widely seen as triggered by a shift in consumption expenditures potentially caused by a decline in animal spirits (see Blanchard, 1993).

4. Second moments

The presented model must be judged on how good it can replicate the variability of the different aggregate macroeconomic time series behavior.

4.1 Population moments

The following Tables report population moments for the U.S. economy. Log levels were detrended by computing deviations from a common estimated linear trend (see King, Plosser and Rebelo, 1988 for details). Table 5 reports the fluctuations in aggregate variables in order to access their relative magnitudes and comovements.

Table 5

Sample and Model Moments

US data				RBC			Farmer and Guo		
variable	σ_x/σ_S	$\sigma_{x,S}$	AC1	σ_x/σ_S	$\sigma_{x,S}$	AC1	σ_x/σ_S	$\sigma_{x,S}$	AC1
Output	1.00	1.00	0.96	1.00	1.00	0.93	1.00	1.00	0.84
Consumption	0.69	0.85	0.98	0.64	0.82	0.99	0.82	0.82	1.00
Investment	1.35	0.60	0.93	2.31	0.92	0.88	2.32	0.82	– 0.08
Hours	0.52	0.07	0.97	0.48	0.79	0.86	0.43	0.56	– 0.24
Real Wage	1.14	0.76	0.97	0.69	0.90	0.98	0.83	0.90	0.97

σ_x/σ_S denotes the relative standard deviation of variable x with output. $\sigma_{x,S}$ denotes the correlation of variable x with output. AC1 denotes the first order autocorrelation of the variable. The table is taken from Schmitt-Grohe (1997a). RBC denotes the baseline model by King, Plosser and Rebelo (1987). Farmer and Guo (1994) is reported here as the standard one-sector sunspot model.}

The data suggest that investment is substantially more volatile than output, and that consumption is less so. The Table also reports a high autocorrelation for all variables. Finally, it is shown that all variables considered are strongly procyclical with the exception of employment.[18] Business cycle properties of a standard version of the Real Business Cycle model and those of the Farmer and Guo (1994) model are reported for comparison. Both models are capable of reproducing the stylized facts to a certain degree.

[18] See King, Plosser and Rebelo (1988) for a discussion of the acyclical behavior of employment and the sensitivity of this result on the detrending method.

4.2 Model moments: theory

If the dynamical system (29) is indeterminate, then for a given initial capital stock $\hat{K}_0$, $\{Z_t\}_{t=0}^{\infty}$ and an arbitrary (nonpredetermined) $\hat{\lambda}_0$, one can generate a sequence of probability distributions of the random variables $\{\hat{\lambda}_t, \hat{K}_t\}_{t=1}^{\infty}$ by adding another arbitrary random variable sequence $\{\epsilon_t\}_{t=1}^{\infty}$ of expectational errors. This latter sequence of shocks represents beliefs of agents which act in the very same way as Keynes's animal spirits, as the driving force of the model economy by shocking $\hat{\lambda}_{t+1}$. Equation (29) can be rewritten as a first order vector autoregressive process

$$\begin{bmatrix} \hat{\lambda}_{t+1} \\ \hat{K}_{t+1} \\ Z_{t+1} \end{bmatrix} = \mathbf{J} \begin{bmatrix} \hat{\lambda}_t \\ \hat{K}_t \\ Z_t \end{bmatrix} + \mathbf{R} \begin{bmatrix} \epsilon_{t+1} \\ 0 \\ z_{t+1} \end{bmatrix} \tag{30}$$

where $\mathbf{R}$ is 3×1. This equation describes the equilibrium laws of motion of the model.

4.3 Model moments

The following Tables report the second moments for the model. In order to be able to extract the working mechanism of the model, several versions will be considered which are driven by technology shocks, animal spirits shocks or a combination thereof.

In light of the mentioned plethora of recent empirical work, the calibration sets increasing returns to scale in the model as low as possible. The following Table 6 lists the parameter values.[19] The labor supply elasticity is four, marginal costs are mildly decreasing and the returns to scale are only modest.

Table 7 reports the moments of the model when either driven by sunspot shocks or a combination of sunspot and technology shocks. All variables basically possess the correct relative volatilities and are all highly autocorrelated. The procyclicality of consumption expenditures in data can be replicated by the model even when it is driven by *white noise* sunspot shocks only. Investment appears to be somewhat too volatile. However, when compared to the sample and model moments reported in Table 5, it becomes clear that the model does not perform any less inferior than existing theories. Also, it can be shown that the correlation of productivity and hours is slightly negative in both versions (-0.31 and -0.17 respectively). This finding is consistent with the well-known Dunlop-Tarshis puzzle.

19 This is in line with Schmitt-Grohe (1997a).

Table 6

Model parameters

ELS	α	δ	υ	θ	γ
4.00	0.42	0.024	0.99	0.80	1.03

Table 7

Model Moments

	Sunspot Shocks			Both Shocks		
variable	σ_x/σ_S	$\sigma_{x,S}$	AC1	σ_x/σ_S	$\sigma_{x,S}$	AC1
Output	1.00	1.00	1.00	1.00	1.00	0.99
Consumption	0.72	0.63	0.99	0.81	0.53	0.98
Investment	4.37	0.69	0.99	5.21	0.65	0.97
Hours	0.68	0.50	0.99	0.85	0.95	0.97
Productivity	0.91	0.77	0.99	0.97	0.70	0.98

'Both Shocks' refers to the case in which the model is driven by independent (mildly persistent) technology and white noise sunspot shocks: $\rho_z = 0.50$.

The model is successful in reproducing strong autocorrelations. This is a compelling result, especially considering that the shock sequence was assumed to be i.i.d. in which case output's first order autocorrelation becomes 0.03 in the RBC model. Moreover, even for researchers who do not believe in sunspots as a principal source of business cycles, the present model offers an example which comprises a strong endogenous propagation mechanism.

In Table 8, it is assumed that marginal costs are constant ($\gamma = 1$). All remaining parameters stay unchanged. If marginal costs are constant, the model predicts a less favorable match to U.S. data. The reason for its performance dependency on this model parameter can be understood in the same way as the effect of (persistent) technology shocks. Declining marginal costs imply a wealth-increasing effect of output expansions. This effect pulls along consumption even if technology shocks are absent.[20] The second part of Table 8 reports the case when the same economy is subject to persistent sunspot innovations.[21] This assumption results in more persistent cycles and a procylical consumption pattern.

20 Mathematically, the modulus of the roots of **J** approaches one and (sunspot) shocks become endogenously more persistent. A related finding is reported in Weder (2000) for a two sector model similar to Benhabib and Farmer's (1996).

21 This case is constructed with a certain disregard of notation and does not represent a strict rational expectations solution. However, it appears that measures of household optimism (like consumer confidence indicators) are highly autocorrelated.

Table 8

Model Moments

Sunspots				Persistent Sunspots		
variable	σ_x/σ_S	$\sigma_{x,S}$	AC1	σ_x/σ_S	$\sigma_{x,S}$	AC1
Output	1.00	1.00	0.95	1.00	1.00	1.00
Consumption	0.75	– 0.09	0.94	0.70	0.18	0.99
Investment	6.36	0.81	0.90	5.46	0.78	0.99
Hours	1.19	0.71	0.90	1.00	0.63	0.99
Productivity	0.80	0.27	0.96	0.81	0.46	0.99

The model is the same as in the previous Table, however, marginal costs are constant. 'Persistent Sunspots' refers to the case in which the model is driven by persistent sunspot shocks ($\rho_s = 0.90$).

Even though the main theme of this work is the possibility of sunspot equilibria in general equilibrium, we close off the discussion with a version of the model that is solely driven by fundamental noise. To this end, we take the last model and feed it with white noise technology shocks. Table 9 shows again the strong endogenous propagation mechanism.

Table 9

Model Moments

Technolgy Shocks			
variable	σ_x/σ_S	$\sigma_{x,S}$	AC1
Output	1.00	1.00	0.99
Consumption	0.74	0.60	0.99
Investment	5.69	0.83	0.98
Hours	1.22	0.74	0.98
Productivity	0.82	0.11	0.99

The model is the same as in the previous Table, however, marginal costs are constant and it is driven by white noise technonolgy shocks.

5. Conclusion

In this paper a dynamic model of monopolistic competition with entry and exit has been presented and examined. The number of existing firms in the intermediate sector is determined by a zero profit condition given fixed overhead costs to operate the firm. It is shown that the model displays indeterminacy at modest degrees of increasing returns in cases where the market power in the investment goods market exceeds market power in the consumption goods market. Furthermore, the model is quite successful in replicating major business cycle facts. In contrast to existing Real Business Cycle models, the animal spirits model contains a strong endogenous propagation mechanism.

Appendix

The linearized version of the economy

This Appendix describes the approximated equations of the model economy. These equations refer to (10) to (12) and (23) to (27) (30) and (32) in the text.

$$CS\hat{C}_t + (1-CS)\hat{I}_t + \left(\frac{\upsilon-1}{\upsilon}CS + \frac{\theta-1}{\theta}(1-CS) - 1 + \gamma\right)\hat{N}_t - (1-\alpha)\gamma\hat{L}_t = \alpha\gamma\hat{K}_t + \hat{Z}_t \tag{31}$$

$$CS\hat{C}_t + \frac{\theta}{\upsilon}(1-CS)\hat{I}_t + \left(\frac{\upsilon-1}{\upsilon}CS + \frac{\upsilon}{\theta}\frac{\theta-1}{\theta}\left(1-\frac{C}{S}\right) + N\frac{\phi}{S} - \frac{1-\gamma}{\gamma}\right)\hat{N}_t - \frac{1-\alpha}{\upsilon}\hat{L}_t = \frac{\alpha}{\upsilon}\hat{K}_t + \frac{1}{\gamma\upsilon}\hat{Z}_t \tag{32}$$

$$\left(\upsilon N\frac{\phi}{S} - 1\right)\hat{Y}_t + \hat{S}_t - N\frac{\phi}{S}\hat{N}_t = 0 \tag{33}$$

$$-\chi\frac{L}{1-L}\hat{L}_t - \hat{w}_t = \hat{\lambda}_t \tag{34}$$

$$-\hat{C}_t - \frac{\upsilon-1}{\upsilon}\hat{N}_t = \hat{\lambda}_t \tag{35}$$

$$(1-\gamma)\hat{N}_t + (1-\alpha)\gamma\hat{L}_t - \hat{q}_t = (1-\alpha\gamma)\hat{K}_t - \hat{Z}_t \tag{36}$$

$$(1-\gamma)\hat{N}_t + ((1-\alpha)\gamma - 1)\hat{L}_t - \hat{w}_t = -\alpha\gamma\hat{K}_t - \hat{Z}_t \tag{37}$$

$$-\hat{S}_t + (1-\gamma)\hat{N}_t + (1-\alpha)\gamma\hat{L}_t = -\alpha\gamma\hat{K}_t - \hat{Z}_t\,. \tag{38}$$

$$\beta Q\hat{\lambda}_{t+1} + \beta(1-\delta)\frac{\upsilon}{\theta}\frac{\theta-1}{\theta}N^{\frac{\theta-1}{\theta}}\hat{N}_{t+1} + \beta q\hat{q}_{t+1} = \upsilon N^{\frac{\theta-1}{\theta}}\hat{\lambda}_t + \frac{\upsilon}{\theta}\frac{\theta-1}{\theta}\hat{N}_t \tag{39}$$

$$\delta^{-1}(1-CS)\hat{K}_{t+1} = (1-\delta)\delta^{-1}(1-CS)\hat{K}_t + \hat{S}_t - CS\hat{C}_t - (CS\frac{\upsilon-1}{\upsilon} + \frac{\theta-1}{\theta}(1-CS)\hat{K}_t \tag{40}$$

$$\hat{Z}_{t+1} = \rho_z\hat{Z}_t + z_{t+1} \tag{41}$$

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Abstract

This paper develops a dynamic general equilibrium model of monopolistic competition with entry and exit. It is shown that the model displays indeterminacy at modest degrees of increasing returns in cases when the market power in the consumption goods market and in the investment goods market is asymmetric. Furthermore, the model is successful in replicating major business cycle facts. In contrast to most existing Real Business Cycle models, the animal spirits model contains a strong endogenous propagation mechanism.

Zusammenfassung

In dieser Arbeit wird eine dynamische allgemeine Gleichgewichtsökonomie mit monopolistischer Konkurrenz und endogenem Marktein- und zutritt entwickelt. Es wird gezweigt, daß das Model Nichtdeterminiertheit bei niedrigen Skalenerträgen aufweist, insbesondere wenn die Firmenmarktmacht im Konsumgütersektor unter der entsprechenden im Investitionsgütersektor liegt. Weiterhin kann das Modell stilisierte Konjunkturfakten reproduzieren. Im Vergleich zu herkömmlichen Real Business Cycle Modellen kann vor allem der ausgeprägte endogene Übertragungsmechanismus genannt werden.

JEL-Klassifikation: E32

Keywords: Sunspots, technology shocks, business cycles.