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Export Decision and Risk Sharing Markets*

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In this paper we consider an international firm under exchange rate risk. If firms are risk-averse, then more exchange rate uncertainty will reduce export volume when there are no risk sharing markets. The question is, how can currency futures and currency options be used as hedging instruments by international firms to undo the inverse effect of uncertainty upon export volume?

1. Introduction

In the last decade exchange rates of the major industrial countries have shown substantial volatility. Exchange rate uncertainty became a concern of international firms and, therefore, affected and is affecting international trade and direct foreign investments, although the empirical results are not obvious (see, e.g., *Cushman* (1988), *Krugman* (1989), (1992), *Baltensperger* (1992), *Broll/Wahl* (1992a), *Broll/Zilcha* (1992), *Gagnon* (1993)). This may be explained because risk sharing markets such as currency forwards or futures markets and currency options markets offer important hedging instruments for international firms. The aim of this paper is to study the interaction between exchange rate uncertainty and the production decision of an exporting firm when such hedging instruments are available.

To illustrate the use of currency forwards and currency options as financial hedging instruments, let us consider the situation that a domestic exporting firm receives at some point in the future a payment in foreign currency, in US-Dollars, say. If the firm hedges against exchange rate uncertainty in the forward market, then it is committed to receive a certain amount of domestic currency. This situation can also be achieved if the firm purchases US-Dollar put options and writes US-Dollar call options on the underlying currency, both with equal maturity and strike prices. If the Dollar depreciates and the exchange rate falls below the

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strike price, the exporter has an incentive to exercise the put option, whereas the buyer of the call option will let the call option expire. Alternatively, if the exchange rate does not reach the strike price, the firm will let the put option expire, whereas the buyer of the call option will have an incentive to exercise the call option implying that the firm is committed to deliver the currency.

The main rationale of our study is as follows. If exporting firms are risk-averse, then more exchange rate uncertainty will reduce export volume when there are no risk sharing markets. The benchmark for this observation is the so-called *certainty (equivalent) case,* which implies that the uncertain spot exchange rate $\tilde{e}$ is replaced by its expected value $E(\tilde{e}) = \bar{e}$. Optimal export under a certain exchange rate $\bar{e}$ will then be larger than optimal export under an uncertain exchange rate with expectation $\bar{e}$. Hence the question is, how can currency futures and currency options be used as hedging instruments by international firms to undo the inverse effect of uncertainty upon export volume?

We organize the paper as follows. In *section 2* we present a partial equilibrium model. The effect of exchange rate uncertainty on exports in the absence of risk sharing markets is briefly analyzed. Then we examine the impact of currency futures markets *(section 3)* and currency options markets *(section 4)* upon the firm's export decision. We show that the "separation theorem" holds, i.e., the export amount does neither depend upon the utility function nor upon the spot exchange rate's distribution function. In *section 5* we discuss international hedging policy. With unbiased futures markets, optimal futures contracting implies a riskless profit ("full-hedge theorem"). Under the market setting of currency options export revenue is fully hedged if put and call options' prices incorporate the same risk premia.

2. Export without Hedging Markets

Consider a competitive exporting firm under exchange rate uncertainty. Production and exports give rise to a deterministic cost function $C(x)$ denominated in domestic currency, where x represents export volume. We assume that the function C is strictly convex, increasing and twice differentiable, and that the firm always produces a positive amount.

The export decision is made at time 0 and output will be exported and sold at the fixed foreign currency price p, yielding an uncertain revenue in domestic currency at time 1. Omitting time subscripts the uncertain profit from exports can be written[1]

[1] The certain costs $C(x)$ are compounded to time 1.

$$\tilde{\Pi} = \tilde{e}px - rC(x),$$

where $\tilde{e}$ denotes the uncertain spot rate of foreign exchange at time 1 and $r \equiv (1 + i)$ is the one-period interest factor, i.e., one plus the certain interest rate i. Note that the exchange rate is defined in domestic currency per unit of foreign currency.

The risk-averse exporting firm[2] has a von Neumann-Morgenstern utility function U and maximizes expected utility of profits in domestic currency. Hence, if there are no hedging markets, the firm's decision problem reads

$$\max_x E\left\{U(\tilde{\Pi})\right\}, \tag{1}$$

where E is the expectations operator. An interior solution of the decision problem requires

$$E\left\{U'(\tilde{\Pi})\left[\tilde{e}p - rC'(x)\right]\right\} = 0. \tag{2}$$

In order to reveal the influence of the uncertain exchange rate we use Eq. (2). Since the firm's profit increases with the exchange rate and marginal utility U' is decreasing with regard to profits, we have $Cov(\tilde{e}, U'(\tilde{\Pi})) < 0$. We therefore obtain from Eq. (2) $E\left\{U'(\tilde{\Pi})\right\} rC'(x) = E\left\{U'(\tilde{\Pi})\right\} \bar{e}p + Cov(\tilde{e}, U'(\tilde{\Pi}))\,p$ which leads to

$$\bar{e}p - rC'(x) > 0, \tag{3}$$

since expected marginal utility is positive and where $\bar{e} = E(\tilde{e})$ is the expected spot exchange rate at time 1, and $rC'(x)$ denotes compounded marginal costs.

Certainty (equivalent) case. Let us compare the firm's optimal export decision under uncertainty with the certainty case, i.e., the uncertain spot rate $\tilde{e}$ is replaced by the certain spot rate $\bar{e}$. Then from Eq. (3) and the optimality condition for the certainty case we can state:

Proposition 1: (Effect of Uncertainty) If the spot rate of foreign exchange is uncertain, then the firm's optimal export is lower than its optimal export in the certainty case.

Proof: Let x_c denote optimal export level when $\bar{e}$ is the certain spot exchange rate. Since marginal costs are increasing with output it follows from Eq. (3) and the certainty (equivalent) case $rC'(x_c) = \bar{e}p$ that $x_c > x$.

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[2] We assume that the firm is risk-averse as in the model of *Sandmo* (1971). From financial theory this can be rationalized by assuming that the owner of the firm is risk-averse, for example.

Introducing exchange rate uncertainty causes the well-known effect that a risk-averse firm reduces export volume to deal with the uncertain exchange rate. Suppose that exchange rate uncertainty is measured by the magnitude of the exchange rate's volatility. Then without hedging markets the firm's optimal export is inversely related to this volatility, other things being equal. Hence a risk-averse firm has an incentive to engage in hedging activities and to use instruments like futures and options contracts.

3. Currency Forwards Markets

Suppose that forward contracts for foreign exchange are available. Then the firm can accomplish its optimal hedging by selling currency forward in the futures market, where e_f denotes the forward exchange rate and z_f the futures contract amount in foreign currency.[3] The firm chooses both, x and z_f, so as to

$$\max_{x, z_f} E\left\{ U(\tilde{\Pi}) \right\}, \tag{4}$$

where

$$\tilde{\Pi} = \tilde{e}px - rC(x) + z_f(e_f - \tilde{e}).$$

The first order conditions are given by

$$E\left\{ U'(\tilde{\Pi})\left[\tilde{e}p - rC'(x)\right] \right\} = 0, \tag{5}$$

$$E\left\{ U'(\tilde{\Pi})(e_f - \tilde{e}) \right\} = 0. \tag{6}$$

Owing to the assumed strict concavity of the utility function and strict convexity of the cost function, these are also sufficient conditions for a unique maximum. Now we can state the impact of futures markets on the firm's export decision. From Eqs. (5) and (6) we obtain:

Proposition 2: (Separation with futures) When futures markets are available, the firm's optimal export x_f satisfies

$$rC'(x_f) = pe_f. \tag{7}$$

The *proof* is a direct result from Eqs. (5) and (6).

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[3] The futures price for one unit of foreign currency at contracting time 0 for delivery at time 1 is denoted by e_f.

Proposition 2 claims that the optimal export level is chosen at a point where compounded marginal cost is equal to marginal revenue. Except for the costs of production all parameters affecting the export level are market data: the forward rate, the foreign commodity price and the certain interest rate. Neither the distribution parameters of the random exchange rate nor the firm's risk aversion have any effect on the quantity of exports. The implication is that any two exporting firms with identical technologies but with different attitudes towards risk and different probability beliefs will produce an equal amount of exports.[4] Suppose the forward rate e_f is unbiased, i.e., e_f is equal to the expected exchange rate $\bar{e}$. This implies that the optimal export x_f is equal to the export level in the certainty (equivalent) case (i. e., $x_f = x_c$). Hence by introducing an unbiased futures market international trade will increase, if there are no hedging markets before.

4. Currency Options Markets

Let us now consider currency option contracts which represent another financial hedging instrument. Such contracts provide the holder with the right to sell (put) or buy (call) the underlying currency at a prefixed strike price and expiration date. We consider a market that offers put and call options for every desired strike price k and the associated put and call prices p_o and c_o. The put price is the maximum amount the firm can lose from a put option contract, and the call price is the maximum amount the firm can gain from a call option contract.[5] Thus a hedge by the firm will be self-financing if and only if put and call prices coincide.

We introduce the following definitions. Let $z_p\,(z_c)$ define the European put (call) option contract amount in foreign currency under strike price k, and let e denote a realization of the exchange rate at time 1. Then $(k - e)^+ = \max\{0, k - e\}$ and $(e - k)^+ = \max\{0, e - k\}$ signifies the cash inflow if the put option and the call option is rationally exercised, respectively.

The firm purchases (writes) put options if z_p is positive (negative). On the other hand the firm purchases (writes) call options if z_c is negative (positive). With currency options the decision problem of the exporting firm is then given by

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4 Similar results have been derived by *Danthine* (1978) with price uncertainty. The separation property for international firms are discussed in *Benninga/Eldor/Zilcha* (1985), *Kawai/Zilcha* (1986), *Broll/Wahl* (1992b), *Zilcha/Broll* (1992).

5 If incidentally the exchange rate at time 1 equals the strike price, then the firm sells the foreign currency in the spot market. Note that we neglect transaction costs, margin requirements, taxes and the like.

(8) $$\max_{x, z_p, z_c} E\left\{U(\tilde{\Pi})\right\},$$

where

$$\tilde{\Pi} = \tilde{e}px - rC(x) + z_p\left[(k - \tilde{e})^+ - rp_o\right] + z_c\left[rc_o - (\tilde{e} - k)^+\right].$$

The first order conditions are

(9) $$E\left\{U'(\tilde{\Pi})\left[\tilde{e}p - rC'(x)\right]\right\} = 0,$$

(10) $$E\left\{U'(\tilde{\Pi})\left[(k - \tilde{e})^+ - rp_o\right]\right\} = 0,$$

(11) $$E\left\{U'(\tilde{\Pi})\left[rc_o - (\tilde{e} - k)^+\right]\right\} = 0.$$

From these conditions we prove the separation property for currency options markets.

Proposition 3: (Separation with options) When options markets are available, the firm's optimal export x_o satisfies

(12) $$rC'(x_o) = p\left[k + r(c_o - p_o)\right].$$

Proof: Let $\hat{U}'(\tilde{\Pi}) \equiv U'(\tilde{\Pi})/E\left\{U'(\tilde{\Pi})\right\}$, and note that $k - \tilde{e} = (k - \tilde{e})^+ - (\tilde{e} - k)^+$. Then substracting Eq. (11) from (10) it follows after some manipulations that $E\left\{\hat{U}'(\tilde{\Pi})\tilde{e}\right\} = k + r(c_o - p_o)$. Combining this result with $E\left\{\hat{U}'(\tilde{\Pi})\tilde{e}\right\}p = rC'(x)$ from Eq. (9) implies the claim.

The optimal export can be determined independently of the utility function and of the probability distribution of the spot exchange rate. Our result follows because we implicitly consider a forward contract hedge: the hedging is realized by building a portfolio of currency put and call options such that we obtain a synthetic forward contract (see *Cox/Rubinstein* (1985) and *Broll/Wahl* (1992a)). If a futures contract is considered (see *section 3*), then the put-call parity relationship for European options forces the forward rate to be equal to $k + r(c_o - p_o) = rc_o + r(k/r - p_o)$, which represents the compounded call premium from writing the call plus the compounded net gain from buying the put. This relationship must hold, for otherwise arbitrage possibilities exist in the hedging markets. Note that arbitragefree risk sharing markets imply $x_f = x_o$. Hence optimum export does not depend upon the hedging market we introduce.

What we have derived with futures and options, therefore, is a powerful separation theorem. In a world with uncertain exchange rates, currency futures or (and) currency options markets allow for a separation of the production decision for exports from the financial decision for hedging transactions.

5. Hedging Policy

We have seen that if separation holds, export and hedging decisions of the international firm can be optimized separately. Therefore, we now assume w.l.o.g. that optimal export volume is given. How will then the firm choose its optimal hedging policy?

Let us first consider futures markets. Then the hedging policy of the firm will depend upon whether or not the futures market is unbiased. In other words, the amount of hedging depends upon the magnitude of the (expected) risk premium $q_f = E(\tilde{e}) - e_f$ in the forward rate. This is summarized in:

Proposition 4: (Hedging with futures) If the forward rate contains no risk premium ($q_f = 0$), then the firm sells currency futures up to the foreign export revenue. If the risk premium is positive ($q_f > 0$) or negative ($q_f < 0$), respectively, then the amount of currency futures sold is lower or higher than foreign export revenue, respectively.

Proof: The result follows immediately from Eq. (6) and the definition of profit in Eq. (4). Rewriting Eq. (4) as $\tilde{\Pi} = \tilde{e}(px - z_f) + z_f e_f - rC(x)$ shows: if $px = z_f$, profit is riskless so that $Cov(\tilde{e}, U'(\tilde{\Pi})) = 0$ and, hence, from Eq. (6) e_f must equal $E(\tilde{e})$. Since marginal utility is monotonically decreasing with profit, in the optimum $z_f > (<)\, px$ can only occur if $e_f > (<)\, E(\tilde{e})$.

Proposition 4 is well-known in the literature (see *Kawai/Zilcha* (1986), *Zilcha/Eldor* (1991), *Broll/Wahl* (1992a), (1992b)). It shows that in unbiased futures markets a *full hedge* represents the firm's optimum hedging policy. Hence the firm completely avoids profit risk.

Let us now turn to the options markets setting. In this case the firm's optimal hedge contracting will be determined by the risk premia in the put and in the call option prices. We present the result in the following proposition by making use of the (expected) risk premia notation $q_p = E\{(k - \tilde{e})^+\} - rp_o$ for the put option and $q_c = E\{(\tilde{e} - k)^+\} - rc_o$ for the call option. Note that both options have the same strike price.

Proposition 5: (Hedging with options) If option prices contain identical risk premia ($q_p = q_c$), then the firm completely hedges export revenue in the options market. If the risk premium in the put price is higher than the risk premium in the call price ($q_p > q_c$), then the firm overhedges its export revenue. An underhedging decision is optimal if the inequality is reversed.

Proof: The equation $e_f = E(\tilde{e})$ is equivalent to the equation $e_f - k = E\{\tilde{e} - k\} = E\{(\tilde{e} - k)^+\} - E\{(k - \tilde{e})^+\}$ which is equivalent to $e_f - [k + r(c_o - p_o)] = q_c - q_p$. It follows from the put-call parity that $q_c = q_p$. Hence, in general $sign(q_f) = sign(q_c - q_p)$. The result then follows from Proposition 4.

It shows that the exporting firm completely hedges its uncertain export revenue if the risk premium in the put price is equal to the risk premium in the call price. The intuition simply is that the call price works in favour of the firm's profit, whereas the put price works in disfavour. Hence the net advantage from the risk premia is zero. Translated to the futures price it means that the futures market is unbiased. Note that for this result to hold it is not necessary that the risk premia in the options markets be zero. If they are zero, then trivially they are identical.

Remark: As one of the referees pointed out we discuss the impact of the risk premium on the hedging volume but we do not investigate its impact on export production. To examine this question we have to separate two problems: from the separation property it follows, that expectations and, therefore, the expected risk premium does not matter for optimal exports. On the other hand we have the effect of uncertainty which depends upon the expected risk premium if futures markets are available. This effect disappears if and only if the expected risk premium vanishes. Consider Eq. (7) and insert the risk premium q_f. It follows $(\bar{e} - q_f)p = rC'(x_f)$. Hence $x_f < (>) x_c$ if and only if $q_f > (<) 0$.

Conclusions

We have analyzed export and hedging decisions of an international firm facing exchange rate uncertainty. We extended previous work by proving a separation result, i.e. by deriving preference and probability independent export production rules when currency futures or (and) currency options are available.

Since futures markets imply the separation result it follows that this result also holds in options markets if the traded options allow for constructing an economically equivalent futures contract. For if there are no arbitrage opportunities in risk sharing markets, selling a futures contract is equivalent to a portfolio of European put and European call options on the underlying currency, both with the same date of expiration and equal to the delivery date of the futures contract, and both with a common strike price.[6] Fully hedging the export revenue by currency

[6] Note that the strike price need not be equal to the forward rate. If it is, then put and call option prices coincide.

options will be an optimal hedging policy if (and only if) the risk premia implied in the prices of puts and calls are identical (given a specified strike price). This is equivalent of saying that futures markets are unbiased.

The separation result reveals that the development of hedging markets facilitates the rational decision making of exporting firms. In the absence of hedging markets, exporters have to form expectations about future spot exchange rates and they also have to specify risk preferences. This implies high information costs. From an institutional point of view the paper shows that the availability of hedging markets allows for a substantial reduction in the complexity of the export decision making of international firms. The point is that in a world without uncertainty the rational exporting firm equates marginal costs to an observable market price. This procedure is also applicable under uncertainty if futures or options are available. The only difference is that the futures price of its put-call parity equivalent is the relevant, but still observable market data to be used.[7]

Finally we showed that the hedging decision of the firm is influenced by firm-related data like expectations and risk behaviour. Hence separation results provide theoretical justification for the real-world division of production and financial decision making processes in international firms. That is to say that the 'agent' may decide on the optimal export, whereas the 'principal' must decide on the optimal hedging.

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[7] For a general overview and discussion of (non)existing risk sharing markets see *Newbery* (1989). The economics of institutional arrangements are addressed in *Vosgerau* (1989).

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Zusammenfassung

Exportentscheidung und Risikomärkte: Maximiert ein exportierendes risikoscheues internationales Unternehmen seinen erwarteten Gewinnutzen, dann führt ein Hedging des Wechselkursrisikos mittels Futures oder Optionen dazu, daß die optimale Exportentscheidung unabhängig ist sowohl von dem Ausmaß der Risikoscheu des Unternehmens, als auch von seinen Wechselkurserwartungen (Separationstheorem). Vielmehr bestimmen Grenzkosten, ausländischer Güterpreis und Devisenterminkurs das optimale Exportvolumen. In bezug auf die optimale Hedgingpolitik gilt, daß bei unverzerrten Märkten der gesamte Exporterlös abgesichert wird (Full-hedge Theorem). Dieses Ergebnis mag erklären, warum die erheblichen Wechselkursschwankungen der Vergangenheit den internationalen Handel nicht spürbar beeinflußt haben, denn die Risikoprämien auf den Hedgingmärkten scheinen nahe null zu sein.

Abstract

If we consider a risk-averse international firm under exchange rate risk in the absence of risk sharing markets, then exchange rate uncertainty has an inverse effect on export production of the firm. Provided that currency futures and currency option markets are available, the firm can undo the inverse effect of exchange rate uncertainty. The firm is able to realize even a riskless profit if the hedging markets are unbiased.

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