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Pareto-improving transition from a pay-as-you-go to a fully funded pension system in a model with differing earning abilities*

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1. Introduction

This paper addresses the question whether it is possible to transform an existing pay-as-you-go (PAYG) pension system into a fully funded system in a Pareto-improving way. Breyer (1989) has shown that such a transformation does not exist if contributions to the PAYG system are lump sum. The mere fact that the interest rate exceeds the growth rate, i.e. that a competitive equilibrium is *dynamically* efficient in the absence of a PAYG system does not imply that it is Pareto-improving to switch from PAYG to a fully funded system. On the other hand, a Pareto-improving conversion policy is possible if there is an additional *static* inefficiency (i.e., there are Pareto-improvements which change the allocation in a finite number of periods only). For example, if contributions are levied as distorting wage income taxes as in Homburg (1990), reducing the volume of the PAYG system also eliminates part of the excess burden caused by the wage tax. This efficiency gain can be used to pay back in finite time the debt incurred to finance pensions in the transition period, making all future generations strictly better off. The relevance of this result has been questioned by Brunner (1994). He generalizes Homburg's model, assuming that there are two individuals in each generation who differ with respect to their abilities and wage rates. In the PAYG-system, they contribute proportionally to their wage income but obtain the same pension. Two kinds of redistribution are implied: one from the young generation to the old and another from the rich individual to the poor. Restricting attention to equal pensions, a reduction in the wage tax rate also eliminates part of the intragenerational redistribution and hence harms the poor individual. Brunner gives a sufficient

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condition implying that a small reduction of the wage tax rate and the pension cannot increase the utility of both individuals of a generation whithout reducing the net payments to other generations. Hence under this condition, there is no Pareto-improving transition from a PAYG-system to a fully funded system.

The purpose of this paper is twofold: First, we reformulate Brunner's assumption so as to clarify its economic interpretation: It requires that the difference between the earning abilities of the two individuals is large or that the distortion caused by the wage tax is low. Otherwise, the gains from reducing the distorting wage tax are large relative to the intragenerational redistribution effect and may therefore be sufficient to compensate the poor individual for cancelling this redistribution. Second, we extend Brunner's result by showing that this sufficient condition is also necessary, i.e., whenever it is not met, a local reduction of the contribution rate and the pension payment is Pareto-improving. Moreover, we give a global condition based on the same economic effect which implies that a Pareto-optimal allocation requires that the PAYG system is totally abolished, i.e., that the contribution rate or the pension payment is reduced to zero. This implies that whenever the distortion caused by the wage tax is large and the income differential is low, a Pareto-improving transition from PAYG to a fully funded system is possible.

The trade-off underlying these considerations is wellknown from the theory of optimal linear income taxation (cf. Sheshinski 1972). A proportional income tax has redistributive effects which increase social welfare. With respect to efficiency, however, lump-sum taxation (or a reduction of lump-sum transfers) would be preferable for raising government revenue. In this paper we apply this trade-off to the analysis of social security.

The paper continues in section 2 with a brief outline of the underlying overlapping generations model, and in section 3 we present the result. Since the distinctions between the key arguments in the work of Breyer, Homburg and Brunner as well as in this paper relate only to static considerations, section 3 deals only with one generation. It is straightforward to use the procedure described by Homburg (1990, corollary 2, pp. 645 - 646) in order to see that a Pareto-improving transition for the whole intertemporal model is possible. In the concluding section 4, we show how the result can be generalized to an economy with more than two agents per generation and briefly discuss its empirical relevance.

2. The model

We use an overlapping generations model of a small open economy, where the wage rate w and the interest factor R are exogenous and constant. Every generation lives for two periods and consists of two individuals who differ in their abilities $h^1 < h^2$. Individual i receives a wage rate $w^i = w \cdot h^i$, $i = 1, 2$. The decision problem for every individual in period t is:

$$\text{(1)} \qquad \begin{aligned} \text{Max} \quad & U^i(c_t^i, z_{t+1}^i, 1 - l_t^i) \\ \text{s.t.} \quad & c_t^i + s_t^i = w^i \cdot l_t^i \cdot (1 - \tau) \\ & z_{t+1}^i = R \cdot s_t^i + p_{t+1}, \end{aligned}$$

where c_t^i and z_{t+1}^i denote consumption in the respective periods, s_t^i denotes savings in period t and l_t^i labour supply in period t. We assume that U^i is strictly quasiconcave and twice differentiable, that the first partial derivatives are positive and that leisure is a normal good. The variable τ is the income tax rate and p_{t+1} is the pension paid by the social security system. We restrict attention to PAYG systems where pensions have to be equal for both individuals, implying with the proportional wage income tax a redistribution within a generation. Denoting the optimal labour supply by $l^i(\tau, p_{t+1})$, we assume "agent monotonicity", i.e., $h^1 < h^2 \Rightarrow w^1 \cdot l^1(\tau, p_{t+1}) < w^2 \cdot l^2(\tau, p_{t+1})$ for all (τ, p_{t+1}). In the initial situation the pension is financed by a PAYG system with parameters $\bar{\tau} > 0$ and $\bar{p}_{t+1} > 0$. In equilibrium every individual i contributes $\bar{\tau} \cdot w^i \cdot l^i(\bar{\tau}, \bar{p}_{t+1})$ during the working period and receives a pension

$$\text{(2)} \qquad \bar{p}_{t+1} = \frac{\bar{\tau} \cdot [w^1 \cdot l^1(\bar{\tau}, \bar{p}_{t+2}) + w^2 \cdot l^2(\bar{\tau}, \bar{p}_{t+2})]}{2}$$

when old. The indirect utility of individual i in PAYG is $V^i(\bar{\tau}, \bar{p}_{t+1})$.

Is it possible to vary the parameters of the pension system (τ, p_{t+1}) so as to improve generation t without making any other generation worse off? To see this, we consider the problem

$$\text{(3)} \qquad \begin{aligned} \text{Max} \quad & V^1(\tau, p_{t+1}) \\ \text{s.t.} \quad & \text{(3.1)} \;\; 2 \cdot p_{t+1} - R \cdot \tau \cdot [w^1 \cdot l^1(\tau, p_{t+1}) + w^2 \cdot l^2(\tau, p_{t+1})] \\ & \qquad\quad \leq 2 \cdot \bar{p}_{t+1} - R \cdot \bar{\tau} \cdot [w^1 \cdot l^1(\bar{\tau}, \bar{p}_{t+1}) + w^2 \cdot l^2(\bar{\tau}, \bar{p}_{t+1})] \\ & \text{(3.2)} \;\; V^2(\tau, p_{t+1}) \geq V^2(\bar{\tau}, \bar{p}_{t+1}) \\ & \text{(3.3)} \;\; p_{t+1} \geq 0. \end{aligned}$$

The constraint (3.1) says that the net payments from the state to the individuals in generation t remain the same as under PAYG. By choosing an appropriate level of government debt in period t which is entirely paid back in period $t+1$ this net payment can be shifted in time so as to leave the net wealth (and hence the utility) of both older and younger generations unchanged. With constraint (3.2) we maintain the utility of the rich individual at the level of PAYG. Constraint (3.3) requires that pensions should not be levied as a lump-sum tax. Since in the present model, introducing a uniform lump-sum tax is Pareto-improving, the agents would agree to introduce it. However, while this is convincing theoretically, one does not observe any lump-sum taxes in reality. Therefore, it can be argued that such taxes are politically infeasible in practice, and we prefer to maintain constraint (3.3). Anyway, as can be seen from the theorem, the constraint (3.3) is binding. Hence, even if we drop it so as to allow for lump-sum taxation, the optimal choice cannot imply a positive pension. Thus, PAYG is optimally abolished (and potentially replaced by a "negative pension").

A solution (τ, p_{t+1}) to problem (3) describes a Pareto-optimal allocation subject to the constraint that consumption and labour of all other generations than t are as under the existing PAYG system. We call it a Pareto-optimum for short.

The situation is illustrated in figure 1, where I^1 and I^2 are the indifference curves of V^1 and V^2 in (τ, p)-space which go through the status-quo point $(\bar{\tau}, \bar{p}_{t+1})$:

$$\text{(4)} \qquad \left.\frac{\mathrm{d}p_{t+1}}{\mathrm{d}\tau}\right|_{V^i} = -\frac{V^i_\tau}{V^i_p} = R \cdot w^i \cdot l^i(\tau, p_{t+1}), \quad i = 1, 2$$

implies with agent monotonicity:

$$\text{(5)} \qquad 0 < \left.\frac{\mathrm{d}p_{t+1}}{\mathrm{d}\tau}\right|_{V^1} < \left.\frac{\mathrm{d}p_{t+1}}{\mathrm{d}\tau}\right|_{V^2}.$$

Furthermore, we have:

$$\text{(6)} \qquad \left.\frac{\mathrm{d}^2 p_{t+1}}{\mathrm{d}\tau^2}\right|_{V^i} = R \cdot w^i \cdot \left(\frac{\partial l^i}{\partial \tau} + \frac{\partial l^i}{\partial p_{t+1}} \cdot \left.\frac{\mathrm{d}p_{t+1}}{\mathrm{d}\tau}\right|_{V^i}\right) = R \cdot w^i \cdot (l^i_\tau)^c < 0,$$

where $(l^i_\tau)^c$ denotes the compensated wage tax effect on labour supply.

Hence, the indifference curves are strictly monotonically increasing, strictly concave, and at any intersection of two indifference curves the

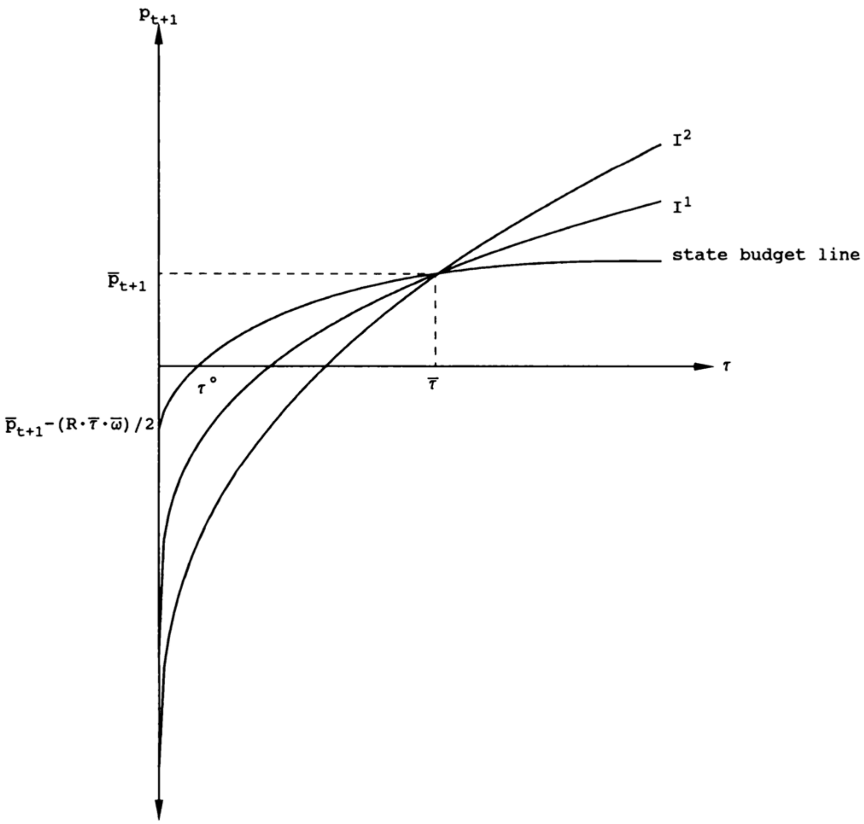

Figure 1: A Pareto-improving transition from PAYG to a fully funded system is possible if the state budget line cuts I^1 from above.

slope of I^2 exceeds the slope of I^1. We consider only the case where under the PAYG system, generation t pays a net payment to other generations, i.e., where the r.h.s. of (3.1) is negative. We call generation t a loser of PAYG in this case. This restriction is motivated by the fact that otherwise, the interest factor exceeds the growth rate of the wage bill. If this were true in all periods, an equilibrium without a PAYG system is (dynamically) inefficient and hence it is a good idea to have a PAYG system. If generation t is a loser, there exists some $0 < \tau^\circ < \bar{\tau}$ such that the state budget (3.1) holds for a pension $p_{t+1} = 0$ and $\tau = \tau^\circ$, as shown in figure 1.

3. A condition for the inefficiency of PAYG

Writing $D(\tau, p) \equiv w^2 \cdot l^2(\tau, p) - w^1 \cdot l^1(\tau, p)$, we can state our main assumption (7) and the result. If (7) holds, the Pareto-optimal solution is to lower the tax rate below the level of PAYG and to pay no pension to both individuals.

(7) For all (τ, p_{t+1}) with $\tau \geq \tau^\circ$ holds:

$$D(\tau, p_{t+1}) \cdot (1 - \tau \cdot R \cdot w^2 \cdot l_p^2) < -\tau \cdot [w^1 \cdot (l_\tau^1)^c + w^2 \cdot (l_\tau^2)^c].$$

Result:

If generation t is a loser and if restriction (7) holds, then $(\tau^\circ, 0)$ is the solution of (3), where $0 < \tau^\circ < \bar{\tau}$.

Proof:

For a Pareto-improving decrease in τ we know from (5) that it is sufficient to improve individual 1 because any point on the left above I^1 is also an improvement for individual 2 (see figure 1).

Applying the implicit function theorem to (3.1), using Roy's identity (4) and writing $\omega = w^1 l^1(\tau, p_{t+1}) + w^2 l^2(\tau, p_{t+1})$, we obtain:

$$\text{(8)} \qquad \frac{\mathrm{d}V^1}{\mathrm{d}\tau} = V_p^1 \cdot \left[-R \cdot w^1 \cdot l^1 + \frac{R \cdot \omega + R \cdot \tau \cdot (w^1 \cdot l_\tau^1 + w^2 \cdot l_\tau^2)}{2 - R \cdot \tau \cdot (w^1 \cdot l_p^1 + w^2 \cdot l_p^2)} \right].$$

Lowering τ is Pareto-improving if and only if (8) is negative. Since $V_p^1 > 0$, this is true if and only if the term in the brackets is negative. Using the Slutzky equation one finds after some calculations that this term is negative if and only if the inequality in (7) holds. Therefore (8) holds for every (τ, p) with $\tau \geq \tau^\circ$, so that τ has to be lowered until (3.3) is binding. Thus for $p_{t+1} = 0$ and $0 < \tau^\circ < \bar{\tau}$ a Pareto-optimum is achieved. τ° can be calculated from (3.1). Q.E.D.

To give an intuition for this proof look at figure 1. The state budget line from (3.1) runs through the intersection of the indifference curves I^1 and I^2 and through the point $(0, \bar{p}_{t+1} - (1/2) \cdot R \cdot \bar{\tau} \cdot \bar{\omega})$ where $\bar{\omega} = w^1 l^1(\bar{\tau}, \bar{p}_{t+1}) + w^2 l^2(\bar{\tau}, \bar{p}_{t+1})$. The slope of the state budget line is the second term of the sum in the brackets in (8). The slope of an indifference curve I^1 is $R \cdot w^1 \cdot l^1$. Since (8) is equivalent to (7), our assumption means that at every point (τ, p) with $\tau \geq \tau^\circ$ the slope of the state budget line is smaller than the slope of I^1 that runs through this point. Thus at every point (τ, p) with $\tau \geq \tau^\circ$ we can locally lower τ remaining on the state budget line and improving individual 1. Because this holds locally at every point (τ, p) with $\tau \geq \tau^{\circ}$, it holds globally, so that we can lower τ along the state budget line until we reach the point where $p = 0$.

To understand condition (7) economically we consider a local change of (τ, p) so that

$$\mathrm{d}p = R \cdot w^1 \cdot l^1 \cdot \mathrm{d}\tau \tag{9}$$

holds. Due to the concavity of I^1 this means that the poor individual and a fortiori the rich individual is not made worse off by this change. (7) now implies that this change decreases the net payments from the state to generation t or, equivalently, increases the budget surplus. Paying out this increase we can improve both individuals as has been proved in the result.

To see this, we compute the change of the budget surplus. From (9) the change of the net payment to the poor individual is only induced by the change of his labour supply. The tax that he has to pay increases by:

$$R \cdot \tau \cdot w^1 \cdot l^1_{\tau} \cdot \mathrm{d}\tau + R \cdot \tau \cdot w^1 \cdot l^1_p \cdot \mathrm{d}p = R \cdot \tau \cdot w^1 \cdot (l^1_{\tau})^c \cdot \mathrm{d}\tau, \tag{10}$$

since the income effect of the change of the tax rate is offset by the change of the pension. The net payment to the rich individual changes according to two effects. With an inelastic labour supply his tax payment decreases by $R \cdot w^2 \cdot l^2 \cdot \mathrm{d}\tau$ and the pension which is uniform for both individuals decreases only by $\mathrm{d}p = R \cdot w^1 \cdot l^1 \cdot \mathrm{d}\tau$ as assumed in (9). So his net payment to the government changes by:

$$- R \cdot w^1 \cdot l^1 \cdot \mathrm{d}\tau + R \cdot w^2 \cdot l^2 \cdot \mathrm{d}\tau = R \cdot D \cdot \mathrm{d}\tau. \tag{11}$$

Because of the reaction of his labour supply his tax payment increases by:

$$\begin{aligned} & R \cdot \tau \cdot w^2 \cdot l^2_{\tau} \cdot \mathrm{d}\tau + R \cdot \tau \cdot w^2 \cdot l^2_p \cdot \mathrm{d}p \\ & = [R \cdot \tau \cdot w^2 \cdot (l^2_{\tau})^c - R^2 \cdot \tau \cdot w^2 \cdot l^2_p \cdot D] \cdot \mathrm{d}\tau \end{aligned} \tag{12}$$

For the rich individual, the income effect of the change in τ is only partially offset by the change of the pension which explains the last term in brackets. If the sum of these changes (10) + (11) + (12) of the net payments from the individuals to the government is positive, then the budget surplus increases with decreasing tax rate τ:

(13) change of budget surplus

$$= \{R \cdot D + R \cdot \tau \cdot [w^1 \cdot (l^1_{\tau})^c + w^2 \cdot (l^2_{\tau})^c] - R^2 \cdot \tau \cdot w^2 \cdot l^2_p \cdot D\} \cdot \mathrm{d}\tau$$

From $d\tau < 0$, this is positive if and only if (7) holds. Condition (7) is fullfilled, if the difference between the wage incomes is small and/or the distortion of the wage tax on labour supply, i.e., $(l_\tau^i)^c$, $i = 1, 2$, is high.

Brunner (1994, assumption EF on p. 518) assumes that the existing PAYG system with a positive tax rate and a positive pension solves problem (3). This implies that locally at the status quo $(\bar{\tau}, \bar{p}_{t+1})$, condition (7) is not satisfied. Thus, Brunner's assumption EF is the complement to a local version of our assumption (7). Therefore, our result shows that his condition EF is not only sufficient but also necessary for the impossibility to improve on an existing PAYG system by marginally reducing the wage tax and the pension. Moreover, if one assumes (7) for the whole range $\tau \geq \tau^\circ$, a Pareto-optimum requires that the pension is reduced to zero, thereby abolishing the PAYG system. To summarize condition (7), we conclude that a Pareto-improving transition from a PAYG to a fully funded pension system is possible in spite of intragenerational redistribution if the distortion caused by the wage tax is large, or if the labour incomes of both individuals do not differ too much.

4. Conclusion

We demonstrated in this paper that an abolition of a PAYG pension system can be Pareto-improving in spite of intragenerational redistribution if the income differential between the poor and the rich household is low or the deadweight loss caused by the income related contributions is large. This is made precise by condition (7). In order to derive a policy conclusion from this result, one would have to know whether in actually existing pension schemes, the crucial inequality in condition (7) is true, or rather its converse, i.e., condition EF in Brunner (1994). In condition (7), the relevant measure of the amount of redistribution is the income difference between the poor and the rich household. Since it is not obvious how this measure is to be generalized to an actual economy with more than two households per generation, we first state how this can be done. We use this in order to discuss some empirical results which indicate how likely it is that condition (7) holds in actual economies.

In the n-person-economy, we continue to call the agent with the lowest wage rate household 1. Then, it is still true that a reduction of the contribution rate and the pension which is beneficial for household 1 is also an improvement for all other households. Now denote for all $i = 2, \ldots, n$ by $D^i(\tau, p) \equiv w^i \cdot l^i(\tau, p) - w^1 \cdot l^1(\tau, p)$ the difference between the incomes of household i and household 1. Taking account of the contributions and the pensions of all households in the budget constraint (3.1) and proceeding as in the proof of the result, one sees that it holds in the n-person case if the inequality in (7) is replaced by:

$$\sum_{i=2}^{n} [D^i(\tau, p) \cdot (1 - \tau \cdot R \cdot w^i \cdot l_p^i)] < -\tau \cdot \sum_{i=1}^{n} w^i \cdot (l_\tau^i)^c. \tag{14}$$

In the n-person case, the sum of all compensated labour supply effects has to be compared with a weighted sum of all differences between the income of household i and the income of the poorest household. Hence, in order to assess the empirical relevance of the result precisely, one would have to know the deadweight loss caused by the labour tax for all households and the entire distribution of labour incomes. However, it seems that it is a good approximation to consider the difference between the average and the lowest wage income on the one hand and the average deadweight loss caused by labour taxation on the other hand. This is equivalent to interpreting the rich household 2 in our formal analysis as representing the average earner (and not, as one might suspect, the richest one), and household 1 as the one with the lowest wage income.

The quantitative importance of the deadweight loss of labour income taxation has been investigated by Hausman (1985). For prime age males in the US, he estimates that the average deadweight loss of labour income taxation is as much as 22% of the tax revenue raised (Hausman 1985, table 5.5, p. 246). Reducing the contribution rate of the social security system therefore may lead to substantial welfare gains. On the other hand, the difference between the lowest and the average income is obviously quite large (especially in the USA), making it unlikely that (7) holds. Notice, however, that only wage incomes are relevant for the study of pension systems. Those rich people who mainly receive income from other factors do not contribute to the intragenerational redistribution effectuated through the public pension system, and those poor who never work do not profit from it (Verbon 1988, p. 33, reports that in the US, "at least forty quarters of employment are necessary to become eligible for pension payments.") Since non-working households are typically on the low end of the income distribution while many rich have substantial non-wage income, one overstates the redistributional effect of the public pension system if one looks at the distribution of personal incomes. Combining Hausman's result with this observation, it does not seem too unlikely that condition (7) is satisfied in actual economies. It is certainly more likely to be true in countries where the dispersion of labour income is low and social security contribution rates are high such as the Netherlands than in countries like the US where the opposite is true.

It must be admitted, however, that there is substantial controversy about the reliability of Hausman's results (see McLure and Zodrow

1994). Hence, it is premature to draw a definite conclusion from empirical studies of the welfare effect of labour taxation, and more so concerning the empirical validity of our result.

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Zusammenfassung

Diese Arbeit greift die Frage auf, ob ein Paretoverbessernder Übergang von einem Umlage- zu einem Kapitaldeckungsverfahren in der Rentenversicherung möglich ist. Es wird ein Modell mit unterschiedlichen Individuen betrachtet, in dem das Arbeitsangebot endogen ist und die Beiträge zum Umlageverfahren in Form einer Lohnsteuer erhoben werden. Es wird gezeigt, daß die Abschaffung des Umlageverfahrens zu einer Paretoverbesserung führt, wenn die Arbeitseinkommen der beiden Individuen sich nur geringfügig unterscheiden oder die durch die Lohnsteuer verursachte Verzerrung groß ist.

Abstract

This paper resumes the discussion whether a Pareto-improving transition from a pay-as-you-go to a fully funded pension system is possible. In contrast to recent work it is shown that in a model with differing individuals, where labour supply is endogenous and contributions to the pay-as-you-go system are raised as an income tax, its abolition is a Pareto-improvement if the labour incomes of both individuals do not differ too much or if the distortion by the wage tax is large.

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