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Money Stock Targeting with Alternative Reserve Requirement Systems

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The significance of alternative reserve requirement systems for the performance of short-run money stock targeting is examined. It is shown that different accounting rules for required reserves enforce different dynamic structures on optimal bank behaviour. This result is used subsequently to demonstrate that the conventional conjecture of a priori superiority of current over lagged reserves accounting does not hold in general. Conditions are identified under which lagged reserves accounting is preferable. Finally, the possibility of conflict between the policy goals of stable money and stable interest rates is evaluated.

1. Introduction

The imposition of a reserve requirement on commercial banks is one of the most important and traditional regulatory constraints by which central banks are enabled to control the supply of money. The design of a required reserves system therefore plays an important role in the determination of the efficiency of money stock control achievable by a central bank and is of particular interest under a monetary policy regime of money stock targeting. Alternative regulations of required reserves systems have been discussed in a number of papers[1] with regard to short run monetary control efficiency during recent years. This literature has been mainly concerned with the question whether required reserves should be related to current or lagged deposits. It seems to be commonly accepted by now that lagged reserve accounting (LRR) unambiguously leads to a higher variance of unexpected fluctuations of the money stock than current reserve accounting (CRR). The basic argument for this is that LRR enforces a recursive structure on the financial system, in which the banking sector's demand for reserves and, therefore, the current rate of interest is independent of current money demand and supply shocks, while under CRR the system is fully interdependet and the interest rate does respond to money market disturbancies. This implies that the information content of the interest rate as a signal of current money stock fluctuations is lower under LRR than under CRR, which in

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[1] See e.g. *Coats* (1976), *LeRoy* (1979), *McCallum / Hoehn* (1983), *McCallum* (1985).

turn leads to a lower degree of control efficiency with respect to the money stock[2].

A common feature of the financial sector models used in this literature is that they assume all behavioural relations in the banking sector to be invariant to changes in the required reserves system. The present paper criticizes this assumption on the basis of an application of *Lucas'* (1976) critique of comparisons of alternative policy regimes. Using a dynamic optimization approach to characterize bank behaviour, we will show that a major consequence of LRR as opposed to CRR is that the constraints to a bank's profit maximization problem contain an intertemporal element. Under LRR, commercial banks' supply of deposits and demand for earning assets will therefore depend on future interest rate expectations. This result is used in a second step to show that with rational expectations the recursivity of the financial system under LRR vanishes. It follows that it is impossible to determine a priori which of the two reserve accounting schemes will be preferable with respect to short-run money stock control.

The paper proceeds as follows. In the next section, we present our arguments about bank behaviour under alternative reserve accounting schemes and a model of short run money stock control. Section III derives the joint equilibrium process for the money stock and the rate of interest. Conditions for the superiority of CRR or LRR are evaluated. The final section summarizes the main conclusions. Throughout the paper, we will not argue about the desirability of money stock targeting itself, but rather concentrate on the question of implementing a money stock rule.

2. The Model

2.1 Commercial Bank Behaviour under CRR and LRR

In this section, the impact of alternative designs of reserve requirements on the behaviour of commercial banks is analysed. The framework we use is a simplified version of a conventional micro model of the banking firm[3]. Consider a risk neutral bank planning to maximize expected profits over several periods $(t, t+1, \ldots, T)$. The bank is assumed to issue deposits D and lend central bank money to finance the purchase of earning assets L. For simplicity, we assume the bank to be a price taker on both the credit and deposit markets, so that i_t, the interest rate on earning assets and r_t, the interest rate paid on deposits are exogenous to the bank. The bank is subject to a reserve requirement RR_t, which may be either CRR or LRR. We assume

[2] *McCallum* (1985), 575.

[3] See *Baltensperger* (1980).

that the bank never holds any excess reserves, though excess reserves could be incorporated in the model following the lines of e.g. *Poole*. However, the bank never plans to violate the minimum reserve constraint. Loans and deposits are assumed to be renewable at the beginning of each period and both the discount rate z paid on central bank credit and the minimum reserve ratio are held constant. Finally, the bank has an operating cost function which depends positively on the structure of its current liabilities and the size of the balance sheet[4].

With these specifications, the bank's balance sheet is

$$L_t + RR_t = D_t + RF_t \tag{2.1}$$

where RF_t denotes credit from the central bank. The bank's profit function is

$$\Pi_t = i_t L_t - r_t D_t - zRF_t - \Omega_t \tag{2.2}$$

wehre Ω_t denotes operating cost.

Let $A_t = L_t + RR_t = D_t + RF_t$ be the size of the bank's balance sheet in period t, $b_t = D_t/A_t$ and $\Omega_t = q(A_t, b_t)$ the bank's operating cost function with derivatives q_A, $q_b > 0$ and q_{AA}, $q_{bb} > 0$. We state the bank's maximizing problem as follows:

$$H_t = E_t \sum_{t=t}^{T} \phi^{\tau - t} \Pi_\tau = \max_{A_\tau, b_\tau} \quad \tau = t, \dots, T \tag{2.3}$$

where ϕ is the bank's internal discount factor and E_t denotes the expectations operator with expectations conditioned on information available to the bank in period t. Thus, the bank simultaneously determines the profit maximizing size of the balance sheet as well as the optimal liability structure to finance its assets.

Within this general set-up, we may consider the consequences of alternative reserve accounting schemes. Unter CRR, required reserves are $RR_t = rrD_t$ and the balance sheet of the bank can be expressed as follows:

$$L_\tau = (1 - rrb_\tau) A_\tau = (1 - rr) D_\tau + RF_\tau. \tag{2.4}$$

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[4] Alternatively, one may consider the following set-up, which may be more familiar to some readers: Neglecting operating cost, the bank operates as a local monopolist on the loan and deposits markets and as a price taker on the market for government bonds, paying an exogenous interest rate. Within this framework, banks select optimal interest rates on credits and deposits. This alternative set-up will, however, not change the conclusions with respect to CRR and LRR.

Solving (2.3) subject to (2.4) yields the following optimality conditions:

$$
\begin{aligned}
(2.5)\qquad q_{A,\tau} &= i_\tau (1 - rrb_\tau) - r_\tau b_\tau - z(1 - b_\tau) \\
q_{b,\tau} &= (z - r_\tau - i_\tau rr) A_\tau . \\
\tau &= t, \ldots, T.
\end{aligned}
$$

The optimal programm (2.5) states some familiar results about the behaviour of commercial banks. In each period, the balance sheet is expanded up to the point where the marginal net yield is equal to marginal cost $q_{A,\tau}$. Simultaneously, the liability structure is chosen such that the marginal cost $q_{b,\tau}$ equals the marginal profit incurred by substituting deposits by central bank credit to finance A_τ. The optimal size of the balance sheet depends positively on the rate of return on earning assets and negatively on the deposit rate, while the ratio of deposits to central bank credit increases with the discount rate and decreases with both the deposit and the credit rates and the reserve requirement ratio. The reserve requirement essentially constitutes a tax on the bank's income from deposits.

The main analytical feature of (2.5) in our context is that the solution under CRR is equivalent to a series of solutions of one period problems independent of each other. We will now show that this property of separability does not hold in the case of LRR. Let the accounting lag be one period, so that $RR_t = rrD_{t-1}$. With this reserves accounting rule, the bank's budget constraint contains a dynamic relation, namely

$$(2.6)\qquad L_\tau = A_\tau - rrb_{\tau-1} A_{\tau-1} = D_\tau - rrD_{\tau-1} + RF_\tau .$$

Solving (2.3) subject to (2.6) yields the following programm:

$$
\begin{aligned}
(2.7)\qquad q_{A,\tau} &= i_\tau - r_\tau b_\tau - z(1 - b_\tau) - \theta_\tau \phi \, rrb_\tau i_{\tau+1|t} \\
q_{b,\tau} &= (z - r_\tau - \theta_\tau \phi \, rri_{\tau+1|t}) A_\tau \\
\theta_\tau &= 1, \tau \leqq T-1; \theta_T = 0 \\
\tau &= t, \ldots, T
\end{aligned}
$$

with $i_{\tau+1|t} = E_t i_{\tau+1}$. As before, these conditions state that the size and the structure of the bank's balance sheet are adjusted such that marginal profits from expanding either total earning assets or the volume of deposits equal expected marginal cost. Conditions (2.7) differ from (2.5), however, in that both the optimal size of the balance sheet and the profit maximizing structure of liabilities in each period depend on the expected rate of interest on earning assets one period ahead. The reason for this is that with LRR, the bank's current decision to issue deposits has an impact, through next period's reserve constraint, on its future profit opportunities. The bank's maximization problem therefore contains a true intertemporal optimization in this case, which basically consists of planning next period's reserve constraint given this period's interest rates and the bank's expectation about future loan rates. From (2.7), we see that the intertemporal element has two aspects. First, there is an implicit trade between current and next period's earning assets held by the bank. Given current loan and deposit rates and constant discount and required reserves rates, the bank will purchase less earning assets in the current period in response to a rise in the expected rate $i_{t+1 \mid t}$ and thus cut back A_t in order reduce next periods' reserve constraint. Secondly, the optimal structure of liabilities changes towards an increasing part of central bank credit, if $i_{t+1 \mid t}$ increases. Therefore the supply of deposits depends positively on the current rate on earning assets, but negatively on the expected future rate, while the bank's demand for central bank credit depends positively on both the current and the expected rates under LRR[5].

The conclusion we draw from these considerations is that the dynamic structure of commercial banks' decision making processes is not invariant to changes between LRR and CRR as reserve accounting schemes. With LRR, current decisions are taken conditionally upon expectations about future interest rates on the credit market, while with CRR the planning process is separable in time and therefore current decisions are exclusively based on current market conditions. Consequently, and in contrast to the conventional approach in the literature, the behaviour of the banking sector should be modelled in a different way for LRR than for CRR regimes in a financial sector model used for the analysis of alternative reserves accounting schemes. This result will be incorporated in the following model of short-run monetary control.

2.2 A Model of Short-Run Money Stock Control

In this section, we present a macro model of the financial sector of a closed economy, appropriate to analyse the significance of alternative reserve

[5] The above discussion is based on the assumption that the bank actually obtains any central bank credit demanded. In a recent paper, *Goodfriend* (1983) has argued that if the central bank exerts quantitative rationing at the discount window, the bank's demand for central bank credit derived from a dynamic optimization problem will depend on loan rate expectations regardless of the required reserves system. This argument will not be pursued here, since it does not alter our main point about money stock control under LRR versus CRR. In our context, what matters is not that under CRR bank decisions are independent of interest rate expectations, but that under LRR they are not.

requirement systems for monetary targeting in the short-run. The model is set up in a way to concentrate on the control and information problems of the central bank. In particular, we will simplify the information structure of the model such that problems of limited information within the private sector of the economy can be disregarded to a large extent. Furthermore, the links between the financial and the real sectors will not be made explicit, and all magnitudes are defined in nominal terms. The model is innovative comparing to previous models in the way it accounts for the differences in bank behaviour under CRR und LRR.

There are three types of agents in the model: the central bank, private commercial banks, and non-banks. For analytical convenience, we assume that there are large numbers of identical banks and non-banks, such that asset markets are competitive. The financial assets and liabilities held and issued by these agents are money, central bank money and credit. Money consists of deposits non-banks hold within commercial banks. Banks are assumed to pay interest r_t on these deposits. Central bank money is held only by commercial banks and does not pay interest. Finally, there is a credit market, on which bonds and commercial bank loans are traded as perfect substitutes at the credit market rate i_t. The supply of central bank money consists of borrowed reserves, borrowed at the constant discount rate z, and of non-borrowed reserves. Commercial banks are subject to a required reserves constraint, which is either CRR or LRR. The accounting lag is one period in the latter case.

Denote by m_t, nr_t, and br_t the logs of the stocks money, non-borrowed and borrowed reserves respectively. Eliminating the credit market by Walras' Law, our model has the following two equations determining the equilibria of supply of and demand for reserves (2.8) and money (2.9):

$$R_t^s = nr_t + \alpha_1' (i_t - z) + \alpha_3\, i_{t+1\,|\,t} + \varepsilon_{1t} = \alpha_0 - \alpha_1''\, i_t + \alpha_2\, m_{t-j} + \varepsilon_{2t} = R_t^d \tag{2.8}$$

$$m_t^s = \beta_0 + \beta_1 (i_t - z) - \beta_2\, r_t - \beta_3\, i_{t+1\,|\,t} + \mu_t = \gamma_0 - \gamma_1\, i_t + \gamma_2\, r_t + \xi_t = m_t^d \tag{2.9}$$

$$(\varepsilon_{1t}\ \varepsilon_{2t}\ \mu_t\ \xi_t)' \sim N\ (o, \operatorname{diag}(\sigma_{\varepsilon 1}^2\ \sigma_{\varepsilon 2}^2\ \sigma_\mu^2\ \sigma_\xi^2))\,.$$

All parameters are positive and $(\varepsilon_{1t}, \varepsilon_{2t}, \mu_t, \xi_t,)'$ are white noise stochastic shocks. The parameter α_2 is a positive function of the required reserves ratio, which is assumed to be constant, with $\alpha_2(rr) = 0$ for $rr = 0$ and $\alpha_2(rr) > 0$ for $rr > 0$.

According to the results of section II.1., a comparison of CRR and LRR requires two alternative specifications of the model, to account for the differences in bank behaviour.

This is accomplished by specifying different assumptions about j, the reserves accounting lag and the parameters α_3 and β_3, namely[6]

$$j = \alpha_3 = \beta_3 = 0 \qquad \text{for CRR} \tag{2.10}$$

$$j = 1;\ \alpha_3, \beta_3 \geqq 0 \qquad \text{for LRR.} \tag{2.11}$$

Assume now that the central bank wants to stabilize fluctuations of the money stock around its long run target m^* caused by the current stochastic shocks, i.e. the central bank wants to minimize the conditional variance of money stock innovations, $\text{var}(m_t - m_{t\,|\,t-1})$. However, the central bank is not able to observe the current money stock. Furthermore, assume that the money stock of period $t-1$ is not known at the beginning of the current period, either, since commercial banks report their deposits of period $t-1$ to the central bank only during period t. This is in accordance with actual regulations in many central bank systems. Finally, assume that the interest rate banks pay on deposits is not permanently observed by the central bank but with a similar lag as the money stock. On the other hand the bond rate i_t is currently observable by the central bank. In this set-up, we consider money stock targeting using non-borrowed reserves as the central bank's policy instrument. Two alternative control procedures will be analysed, which differ in the amount of information used for the purpose of money stock control. The first one is the pure non-borrowed reserves strategie which consists of two separate steps. At the beginning of each period, a non-borrowed reserves target nr_t^* is computed according to the rule

$$E\,(m_t \mid m_{t-k}, i_{t+1-k}, r_{t-k}, nr_t^*) = m^*, k \geqq 2\,. \tag{2.12}$$

Note that the expectation (2.12) is conditioned on the information set available to the central bank.

During each period, all central bank actions are entirely oriented at the task of reaching nr_t^* as closely as possible. This is the scenario of money stock control usually considered in the literature and underlying the analysis of required reserves systems considered by most previous authors[7].

From an optimal control point of view, the main feature of the pure strategy is that current central bank actions during each period are based solely on past information, as indicated by (2.12). The current observation of the

[6] Throughout the following discussion it is assumed that no other parameter changes due to a switch in reserve accounting regimes.

[7] See, e.g. *McCallum / Hoehn* (1983), *McCallum* (1985).

interest rate on the credit market is disregarded in the central bank's control procedure. The current information set is therefore not totally exploited and, consequently, the strategy is sub-optimal. An improvement can be achieved by making use of the information about the current money stock contained in the observation of i_t. This is the basic idea of the combination policy proposed by *LeRoy*[8], where an optimal correction nr_t^{**} to the target nr_t^* is computed on the basis of the currently observed interest rate.

Having determined the behaviour of the central bank, we may solve the model to compare the outcomes of money stock control under CRR and LRR. This will be done in the next section. To save additional notations, we set $nr_t = nr_t^*$ or $nr_t = nr_t^* + nr_t^{**}$ to solve the model, so that the stochastic variable ε_{1t} in equation (2.8) picks up the central bank's control error with respect to the non-borrowed reserves instrument.

3. Money Stock Targeting With CRR and LRR

3.1 The Pure Non-Borrowed Reserves Strategy

The formulation of the pure nr strategy requires the definition of the nr money multiplier

$$(3.1) \qquad \begin{aligned} y_t &= m_t^s - nr_t = m_t^s - \alpha_o + \alpha_1 i_t + \alpha_3 i_{t+1 \mid t} - \alpha_2 m_{t-j} - \varepsilon_t \\ \varepsilon_t &= \varepsilon_2 - \varepsilon_{1t} \\ \alpha_1 &= \alpha_1' + \alpha_1''. \end{aligned}$$

Using (3.1), the non-borrowed reserves target is

$$(3.2) \qquad nr_t^* = \mathrm{m}^* - y_{t \mid t-1}^c$$

where the superscript c indicates that expectations are conditioned on the central bank's information set. Next, we eliminate the deposit rate r_t from the system to analyse the joint equilibrium processes of the credit market and the money stock, $x_t = (i_t \, m_t)'$, with $(\beta_2 + \gamma_2) m_t = (\beta_1 \gamma_2 - \beta_2 \gamma_1) i_t - \gamma_2 \beta_3 i_{t+1 \mid t} + \beta_2 \xi_t + \gamma_2 \mu_t$.

Consider the case of CRR, first. The multiplier forecast is derived from (3.1). With this forecast, we insert (3.2) into (2.8). This yields the following system

[8] *Le Roy* (1979), 461.

(3.3)

$$\begin{bmatrix} \alpha_1 & -\alpha_2 \\ -\Delta_1 & (\beta_2+\gamma_2) \end{bmatrix} x_t = A_o + \begin{bmatrix} \alpha_1+\Delta_2 & -\alpha_2 \\ 0 & 0 \end{bmatrix} x^c_{t\,|\,t-1} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} (\varepsilon_t\, \nu_t)'$$

$$\Delta_1 = \beta_1 \gamma_2 - \beta_2 \gamma_1 < 0, \qquad \Delta_2 = \Delta_1 (\beta_2 + \gamma_2)^{-1}$$

$$\nu_t = (\beta_2\, \xi_t + \gamma_2\, \mu_t)$$

where A_o is a matrix of constants which will be neglected for convenience. Assume now that the central bank has rational expectations. The reduced form solutions can be found by standard techniques, as shown in the appendix. Omitting all constants, the reduced forms are

(3.4)

$$\text{a)}\quad i_t = \Gamma_1^{-1}((\beta_2+\gamma_2)\,\varepsilon_t + \alpha_2\, \nu_t)$$

$$\text{b)}\quad m_t = \Gamma_1^{-1}(\Delta_1 \varepsilon_t + \alpha_1 \nu_t)$$

$$\Gamma_1 \equiv \alpha_1(\beta_2+\gamma_2) - \alpha_2\, \Delta_1 > 0\,.$$

Equation (3.4a) shows that the current interest rate responds to ε_t and the current money market shock ν_t if $\alpha_2 > 0$. In other words, the financial sector is a fully interdependent system under CRR as long as the required reserves ratio is strictly positive. With a fixed supply of non-borrowed reserves, money market shocks affect the interest rate positively and their effects on m_t are partly offset by a rise in the interest rate. Thus, current reserve requirements stabilize monetary fluctuations.

The solution of the model for LRR is more complicated, due to the dynamics of the system in this case. Since the central bank ignores m_{t-1} at the beginning of period t, the expectation of the multiplier $y^c_{t\,|\,t-1}$ involves an expectation of m_{t-1}, given the information available in $t-1$. With this expectation, denoted by $\hat{m}_{t-1}$, the multiplier forecast is

(3.5)
$$y^c_{t\,|\,t-1} = m^c_{t\,|\,t-1} - \alpha_o + \alpha_1\, i^c_{t\,|\,t-1} + \alpha_3\, i^c_{t+1\,|\,t-1} - \alpha_2\, \hat{m}_{t-1}\,.$$

When (3.5) is used for the derivation of the non-borrowed reserves target, the processes of i_t and m_t are

(3.6)
$$\alpha_1\, i_t = -\alpha_3\, i_{t+1\,|\,t} + \beta_4\, i^c_{t+1\,|\,t-1} + \beta_5\, i^c_{t\,|\,t-1} - \alpha_2\, \hat{m}_{t-1} + \alpha_2\, m_{t-1} + \varepsilon_t$$

$$(\beta_2+\gamma_2)\, m_t = \Delta_1\, i_t - \beta_3\, \gamma_2\, i_{t+1\,|\,t} + \nu_t$$

where $\beta_4 = \beta_3\, \gamma_2\, (\beta_2+\gamma_2)^{-1}$ and $\beta_5 = \alpha_1 + \Delta_2$. Note that the expectation $i_{t+1\,|\,t}$ is conditional on the information available to private agents in the economy. Equation (3.6) shows that, in contrast to the CRR case, under LRR the money stock and the interest rate are determined by a dynamic process involving expectations about future interest rates. It is straightforward to show that, when expectations are neglected in (3.6), i.e. $\alpha_3 = \beta_3 = \beta_4 = 0$ is assumed, the financial sector is necessarily a recursive system: in this special case, the rate of interest depends only on the current shocks to non-borrowed reserves demand and supply and the predetermined variables m_{t-1} and $\hat{m}_{t-1}$, and is independent of the current money market shock v_t.

The derivation of the reduced form solutions and LRR is left to the appendix. Omitting all constants, again, they are

$$(3.7) \quad \text{a)} \quad i_t = -(\alpha_2 \Delta_2 / \alpha_1)\, i_{t-1} + (\alpha_2 / \alpha_1)\, m_{t-1} + (1/\alpha_1)\, \varepsilon_t - (\alpha_2 \alpha_3 / \alpha_1^2 \Gamma_2)\, v_t$$

$$\text{b)} \quad m_t = -(\alpha_2 \Delta_2 / \alpha_1)\, i_{t-1} + (\alpha_2 \Delta_2^2 / \alpha_1)\, m_{t-1} + $$

$$+ \ (\Delta_2 / \alpha_1)\, \varepsilon_t + ((\alpha_1 - \Delta_2 \alpha_2 \alpha_3 / \alpha_1) / \alpha_1 \Gamma_2)\, v_t$$

with $\Gamma_2 = (\beta_2 + \gamma_2)\ (1 + \beta_4 \alpha_2 \alpha_3 / \alpha_1) > 0$. Again, for a non-zero required reserves ratio the financial system is fully interdependent. The current interest rate reacts to the money market shock as long as $\alpha_3 > 0$ together with $\alpha_2 > 0$. Under LRR, therefore, the role of the banking sector's expectations of future interest rates is crucial for the determination of the stochastic structure of the system in the short run. It follows that the common argument of a priori inferiority of LRR for money stock control, which is based on the conjecture of recursiveness of the system under LRR, does not hold when the implications of LRR for optimal bank behaviour are incorporated in the model.

The question then arises, under which conditions CRR will be preferable to LRR. The natural criterion for comparison in this context is the conditional variance of money stock fluctuations, caused by current shocks ε_t and v_t, $\mathrm{var}(m_t - m_{t\,|\,t-1})$.

These variances may be computed from the reduced form solutions under the alternative regimes, (3.4) and (3.7). Comparing the two variances, the condition for CRR to perform better than LRR is found to be

$$(3.8) \qquad (\Gamma_1^2 (\alpha_1 - \Delta_2 \alpha_2 \alpha_3 / \alpha_1)^2 / \Gamma_2^2 - \alpha_1^4)\, \sigma_v^2 > (\alpha_1^2 (\beta_2 + \gamma_2)^2 - \Gamma_1^2)\, \Delta_2^2\, \sigma_\varepsilon^2 .$$

Since $\alpha_1 (\beta_2 + \gamma_2) < \Gamma_1$, the right hand side of (3.8) is always negative. The left hand side may be positive or negative, depending on the magnitude of the expectations elasticities α_3 and β_3. Note, first, that (3.8) always holds if expectations are neglected under LRR, so that $\alpha_3 = \beta_3 = 0$, because this makes the left hand side positive. In this case CRR unambiguously outperforms LRR. This is simply a restatement of the result shown by previous authors in the literature. In general, however, the expectations elasticities will not be zero. Their role in condition (3.8) is twofold. First, since Γ_2 increases in β_3, a necessary condition for LRR to be preferable is that β_3 be large. On the other hand, the left hand side increases in α_3, and therefore this parameter should be small in that case.

The intuition of this result is as follows. A positive money market shock v_t leads to an increase in the expected interest rate, so that commercial banks' expected opportunity cost of supplying deposits goes up. This causes a direct reduction of deposit supply via β_3 and an indirect reduction of money demand by the induced rise in the current rate of interest. Both effects are stabilizing. They are partly offset, however, by the substitution effect of an increasing expected interest rate, i.e. the increase in commercial banks' demand for borrowed reserves, which is represented by α_3. Finally, if the left hand side of (3.8) is negative, the preferability of CRR decreases as the ratio of variances $\sigma_v^2 / \sigma_\varepsilon^2$ increases. The relative performance of the two regimes then depends on the stochastic structure of the underlying shocks. If money demand shocks are the dominant shocks in the financial sector, i.e. σ_v^2 is relatively large, money stock targeting with LRR will outperform money stock targeting with CRR. On the other hand, since $\sigma_\varepsilon^2 = \sigma_{\varepsilon 1}^2 + \sigma_{\varepsilon 2}^2$ and ε_{1t} contains the central bank's control error over non-borrowed reserves, (3.8) shows that a central bank which does not have much control over non-borrowed reserves for purposes of monetary targeting[9] is likely to prefer a regime of CRR.

Fluctuations of the money stock are typically not the only problem, central bank policy is concerned with in practice. In addition to a stable money supply, most central banks and monetary policy makers have strong preferences for stable interest rates, too[10]. An alternative criterion for comparing the performances of monetary control regimes therefore is the conditional variance of changes in the interest rate caused by the current stochastic shocks, $\mathrm{var}(i_t - i_{t\,|\,t-1})$. With respect to this criterion, CRR will be preferable, if

$$(3.9) \qquad (\alpha_3^2\, \Gamma_1^2 / \alpha_1^2\, \Gamma_2^2 - \alpha_1^2)\, \alpha_2^2\, \sigma_v^2 > (\alpha_1^2\, (\beta_2 + \gamma_2)^2 - \Gamma_1^2)\, \sigma_\varepsilon^2 .$$

The right hand side of (3.9) is known to be negative from above. As before, the sign of the left hand side depends on the magnitude of the expectations elasticities. Note, first, that if expectations effects are neglected, (3.9) will be violated when the variance of money market shocks is relatively large. In this special case, the problem of choosing a required reserves system creates

[9] This includes the case where ε_{1t} contains unpredictable variations of non-borrowed reserves due to commitments of the central bank to other policy targets different from money stock control.

[10] See e.g. the statements by central bank policy makers in *Meek* (1983).

a conflict between the two policy goals of the central bank. In the general case, however, CRR dominates LRR from the point of view of stable interest rates, if β_3 and thus Γ_2 is small and α_3 is large. Conversely, for LRR to be preferable under this criterion it is necessary that the substitution effect of an expected rise in the interest rate be small, while the direct effect via β_3 should be large. If this is the case, then again the attractiveness of LRR increases with the predominance of money market shocks in the financial system. Thus, if direct expectations effects are strong and money market shocks are relatively volatile, there is no conflict between the two policy goals of stable money and stable interest rates, as far as the choice of an optimal design of the required reserves system is concerned. Both criteria lead to the same decision of adopting LRR under such circumstances.

3.2. The Combination Policy

So far, we have assumed that the central bank disregards any current information about money market conditions which becomes available to her after the choice of the current non-borrowed reserves target. The basic idea of the combination policy is to use the currently observable rate of interest i_t to improve control performance. This is achieved by adding a correction term nr_t^{**} to nr_t^*, which is derived from the condition that

$$E\,(m_t - m_{t\,|\,t-1} \mid i_t, nr_t^{**}) = 0 \tag{3.10}$$

or, equivalently,

$$E\,(\psi_{21}\,\varepsilon_t + \psi_{22}\,v_t \mid g_t, nr_t^{**}) = 0 \tag{3.10'}$$

where $g_t = i_t - i_{t\,|\,t-1} = \psi_{11}\,\varepsilon_t + \psi_{12}\,v_t$ is the current interest rate signal and ψ_{ij}, $i, j = 1, 2$ are the coefficients for the two current shocks in the reduced form solution of the model. It is straightforward to show that the optimal correction of the non-borrowed reserves target is

$$nr_t^{**} = (\theta\,/\,\psi_{21})\,g_t\,. \tag{3.11}$$

In equation (3.11), θ is the regression coefficient of $(\psi_{21}\,\varepsilon_t + \psi_{22}\,v_t)$ on g_t. The derivation of the optimal correction and the resulting conditional variance of the money stock are shown in the appendix. As before, it is possible to derive a condition of superiority of CRR over LRR from the point of view of stabilizing unexpected fluctuations of the money stock. For the combination policy, this condition is

$$\alpha_2\,\sigma_v^2\,(1 - \alpha_3^2\,/\,\alpha_1^2) > \beta_4\,(2\,\alpha_1 + \beta_4\,\alpha_2)\,\sigma_\varepsilon^2\,. \tag{3.12}$$

The right hand side of this condition is always positive and vanishes with $\beta_4 = 0$. Thus, if expectations effects play no role, (3.12) is always fulfilled and CRR always leads to a more precise control of the money stock. The reason for this is that with $\alpha_3 = 0$ the rate of interest contains no information about the current money market shock when LRR is adopted, and therefore the central bank has no chance to react to this shock. In the general case, however, a sufficient condition for LRR to perform better than CRR is found to be $1 < \alpha_3^2 / \alpha_1^2$. This is because the ratio of α_3 / α_1 determines the quality of the interest rate as a signal of the current money market shock v_t under LRR relatively to CRR. For any $\alpha_2 > 0$, the responsiveness of the current interest rate to current money market shocks increases as α_3 / α_1 increases, and therefore movements in the interest rate convey more information about v_t. Thus, (3.12) shows that under a combination policy the substitution effect of an expected rise in the interest rate should be large and the direct effect relatively weak for LRR to be preferable to CRR.

Turning to the criterion of interest rate stability, the condition of superiority of CRR is

$$\sigma_\varepsilon^2 (\beta_2 + \gamma_2)^2 (\alpha_3^2 - \alpha_1^2 (1 + \beta_4 \alpha_2 / \alpha_1)^2) > \alpha_2^2 \alpha_3^2 \sigma_v^2 (1 - (1 + \beta_4 \alpha_2 / \alpha_1)^2) . \tag{3.13}$$

The right hand side of this condition is always negative, indicating that LRR can only be preferrable if α_3 is relatively small. In particular, if $\beta_4 = 0$, (3.13) becomes $1 > \alpha_3^2 / \alpha_1^2$, which is just the opposite condition of (3.12) in that case. Furthermore, (3.13) never holds if $\alpha_3 = 0$. This generalizes the result stated by *LeRoy*[11], that the variance of interest rate fluctuations will be higher under the regime, for which the quality of the interest rate as a signal of the money market shock v_t is higher. (3.12) and (3.13) together imply that the choice of a reserves accounting scheme under a combination policy always leads to a conflict between the two policy goals of stable money and stable interest rates.

3.3. Money Stock Targeting with a Total Reserves Strategy

The results of the previous sections can be straightforwardly extended to the case of the total reserves strategy, where the central bank uses total reserves instead of non-borrowed reserves as the policy instrument. The model can be adapted to this case by changing equation (2.8) to

$$R_t^s = r_t^* + \varepsilon_{1t} = \alpha_o - \alpha_1'' i_t + \alpha_2 m_{t-j} + \varepsilon_{2t} = R_t^d . \tag{3.14}$$

[11] *LeRoy* (1979), 465. LeRoy only considers the special case of $\alpha_3 = 0$.

The reserves target r_t^* is derived from a rule similar to (2.12). The important difference between the demand for total reserves and the demand for non-borrowed reserves in our context in that the former does not depend on the expected rate of interest $i_{t+1|t}$. With a total reserves strategy, expectations effects of LRR only operate directly on the deposit supply function.

Comparison of the reserve requirement systems can be made with the help of the conditions of III.1. and III.2., where α_3 now has to be set equal to zero and α_1 equal to $\alpha_1'{}'$. The results are as follows: Under a pure total reserves strategy, a sufficient condition for LRR to outperform CRR is that direct expectations effects are strong enough. Again, there is no conflict between the two policy goals in this case, and the preferability of LRR increases with the volatility of money market shocks. Under a combination policy, however, CRR unambiguously leads to a smaller conditional variance of money supply and a higher variance of interest rate fluctuations, because the interest rate reveals no information about current money market shocks when total reserves are chosen as policy instrument under LRR.

4. Summary and Conclusions

We have presented a model of short run money stock targeting to analyse the performance of monetary control procedures under alternative required reserves systems. In contrast to the common approach in the literature, we have stressed the importance of interest rate expectations in commercial bank behaviour under a lagged reserve accounting scheme. Expectations effects are introduced into the money supply mechanism under lagged reserve accounting because the reserves system imposes a dynamic structure on commercial banks' planning processes.

It has been shown that the commonly assumed a priori inferiority of a lagged reserves accounting system compared to a current accounting system does not hold when expectations effects are taken into consideration. Expectations effects are a stabilizing element under LRR and play an important role in the determination of the stochastic structure of the financial sector. Money stock control performance can be better under a lagged than under a current accounting regime, depending on the relative magnitudes and structure of such expectations effects as well as on the central bank's monetary control strategy.

The analysis of this paper has taken all parameter values as given under the respective regimes. Presumably, however, the strength of expectations effects in bank behaviour depends on the extent to which the central bank commits herself to money stock control. If the commitment is weak in the short run and the central bank tries to keep the interest rate within a predetermined range, the importance of expected opportunity cost considerations will probably loose importance in commercial banks' planning processes. Expectations effects will then be rather weak. The adoption of a lagged reserves accounting scheme should therefore be combined with a strong commitment of the central bank to short run monetary control. Current reserves accounting, on the other hand, seems to be more appropriate if money stock targeting is understood only as a medium or long run policy rule.

Summary

The significance of current and lagged required reserves accounting schemes for the performance of short-run money stock control is analysed in the framework of a rational expectations model. In contrast to previous studies, we emphasize the implications of alternative accounting rules for the dynamic structure of optimal bank behaviour. Due to the role of interest rate expectations in the money supply and the demand for reserves, the usual conjecture of unambiguous inferiority of LRR fails to hold. LRR is preferable to CRR if expectations effects are strong and money market disturbancies dominate disturbancies in the reserves market.

Zusammenfassung

Wir betrachten die Bedeutung von Mindestreservesystemen mit verzögerter und unverzögerter Reservepflicht für den Erfolg kurzfristiger Geldmengenkontrolle im Rahmen eines Modells mit rationalen Erwartungen. Besonderes Gewicht liegt dabei auf den Implikationen der verschiedenen Reservepflichten für die dynamische Struktur optimaler Verhaltensregeln der Geschäftsbanken. Bei verzögerter Reservepflicht bewirken Zukunftserwartungen der Banken über die Zinsentwicklung, daß das übliche Argument, der stabilisierende Effekt unverzögerter Reservepflicht sei größer als der verzögerter Reservepflicht, in unserem Modell nicht a priori gilt. Verzögerte Reservepflicht führt zu exakterer Geldmengensteuerung, wenn Erwartungseffekte im Bankverhalten ausgeprägt sind und Geldmarktstörungen bedeutender sind als Fluktuationen am Markt für Zentralbankgeld.

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Appendix

Solutions for the Pure Strategy and the Combination Policy

To solve for the CRR case, rewrite equation (3.3) as follows:

(A1) $$A_2\,x_t \;=\; A_0 + A_1\,x^c_{t\,|\,t-1} + \zeta_t$$

with $\zeta_t = (\varepsilon_t\ \nu_t)'$. Apart from expectations as of period $t-1$, x_t depends only on a matrix of constants and a white noise error process. Hence, a possible solution is

(A2) $$x_t \;=\; \Lambda_t + \Lambda_1\,\zeta_t\,.$$

Obviously, $x^c_{t\,|\,t-1} = \Lambda_0$ since $\zeta^c_{t\,|\,t-1} = 0$. The solution can now be derived using Lucas' method of undetermined coefficients.

The solution for the LRR case starts with the definition of the central bank's expectation of m_{t-1}, which is unknown at the end of period $t-1$. However, the central bank knows the period $t-1$ equilibrium interest rate i_{t-1}. We assume that the central bank uses i_{t-1} to compute an estimate of the lagged money stock, namely $\hat{m}_{t-1} = \Delta_2\,i_{t-1} + \beta_4\,i^c_{t\,|\,t-1}$, which is derived from the system of equations (3.6). Using this estimate, we rewrite (3.6) as follows (constants are omitted for convenience):

(A3) $$Ax_t \;=\; Bx_{t+1\,|\,t} + Cx^c_{t+1\,|\,t-1} + Dx^c_{t\,|\,t-1} + Ex_{t-1} + F\zeta_t$$

where the matrices of coefficients are

$$\mathrm{A} \;=\; [(\alpha_1\ \ 0)\ \ (-\Delta_1\ \ (\beta_2+\gamma_2))]'\,, \qquad \mathrm{B} \;=\; [(-\,\alpha_3\ \ 0)\ \ (-\beta_3\,\gamma_2\ \ \ 0)]'\,,$$

$$\mathrm{C} \;=\; [((\alpha_3-\beta_4)\ \ 0)\ \ (0\ \ \ 0)]'\,, \qquad \mathrm{D} \;=\; [(-\,\beta_6\ \ 0)\ \ (0\ \ \ 0)]'\,,$$

$$\mathrm{E} \;=\; [((-\,\alpha_2\,\Delta_2)\ \ \alpha_2)\ \ (0\ \ \ 0)]'\,, \qquad \mathrm{F} \;=\; [(1\ \ \ 0)\ \ (0\ \ \ 1)]'\,,$$

with $\beta_6 = \alpha_1 + \Delta_2 - \alpha_2\,\beta_4$.

Define the rational expectations solution

$$\text{(A4)} \qquad x_t = \Pi_1 \, x_{t-1} + \Pi_2 \, \zeta_t \, .$$

To solve for the coefficients of (A4) it should be noted that the expectation $i_{t+1\,|\,t}$ is formed on the basis of the private sector's information set, which, according to the assumptions of our model includes the current and lagged shocks ζ_t and ζ_{t-1} and therefore is larger than the central bank's information set. As usual, the occurence of forward expectations in (A3) enforces non-linearities on the solution. This means that the model with LRR has multiple solutions. Following the common practice in the rational expectations literature, we retain the stable solution, which is characterized by the condition $\pi_{1,11} = -\Delta_2 \, \pi_{1,12}$. Using the same method as before and noting that the central bank's expectation

$$\text{(A5)} \qquad x^c_{t\,|\,t-1} = \Pi_1 \, Lx_{t-1} + \Pi_1 \, Kx_{t\,|\,t-1} = (\mathrm{I} - \Pi_1 \, K)^{-1} \, \Pi_1 \, Lx_{t-1}$$

where $L = [(1 \;\; 0) \;\; (\Delta_2 \;\; 0)]'$ and $K = [(0 \;\; 0) \;\; (\beta_4 \;\; 0)]'$ from the definition of $\hat{m}_{t-1}$ above, the parameters of the reduced form can be derived as follows:

$$\Pi_1 = [(-\alpha_2 \, \Delta_2 \, / \, \alpha_1 \quad \alpha_2 \, / \, \alpha_1) \;\; (-\alpha_2 \, \alpha_2^2 \, / \, \alpha_1 \quad \alpha_2 \, \Delta_2 \, / \, \alpha_1)]'$$

$$\Pi_2 = [(1 \, / \, \alpha_1 \quad -\alpha_2 \alpha_3 \, / \, \alpha_1^2 \, \Gamma_2) \;\; (\Delta_2 \, / \, \alpha_1 \quad (\alpha_1 - \Delta_2 \, \alpha_2 \, \alpha_3 \, / \, \alpha_1) \, / \, \alpha_1 \, \Gamma_2)]'$$

with $\Gamma_2 = (\beta_2 + \gamma_2) \; (1 + \beta_4 \, \alpha_2 \, \alpha_3 \, / \, \alpha_1)$.

Consider now the solution for the combination policy. The problem is to construct nr^{**}_t which is linearly dependend on the interest rate signal $g_t = i_t - i_{t\,|\,t-1}$. Note that the central bank is able to compute g_t, once banks have reported their deposits of $t-1$ during the current period. Let the current monetary shock be $m_t - m_{t\,|\,t-1} = \psi_{21} \, \varepsilon_t + \psi_{22} \, v_t$, where ψ_{21}, ψ_{22} are the coefficients of the reduced form solutions. If ε_t and v_t were known, nr^{**}_t would be derived from

$$\text{(A6)} \qquad \psi_{21} \, (\varepsilon_t - nr^{**}_t) + \psi_{22} \, v_t = 0 \, .$$

However, since only g_t is observable, the optimal condition is given by (3.11). The conditional variance of money supply is then

$$\operatorname{var}(m_t - E(m_t \mid nr^*_t, nr^{**}_t, \ldots)) = \sigma^2_\varepsilon \sigma^2_v (\psi_{11} \, \psi_{22} - \psi_{12} \, \psi_{21})^2 \, / \, (\psi^2_{11} \, \sigma^2_\varepsilon + \psi^2_{12} \, \sigma^2_v)$$

and the conditional variance of the rate of interest is given by

$$\operatorname{var}(i_t - E(i_t \mid nr^{**}_t, nr^*_t, \ldots)) = \psi^2_{12} \, (\psi_{11} \, \psi_{22} - \psi_{12} \, \psi_{21})^2 \, \sigma^4_v \, / \, \psi^2_{21} \, (\psi^2_{11} \, \sigma^2_\varepsilon + \psi^2_{12} \, \sigma^2_v) \, .$$