

# Is the Neoclassical Growth Economy a Market Economy?

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This paper introduces time explicitly into the neoclassical growth model. It creates a problem in the pricing of production and consumption flows, on the one hand, and of the physical quantities transacted, on the other.

It appears that the explicit introduction of a market into the neoclassical growth model creates an inconsistency in the pricing of its good.

The property required of the market is that it transacts non-infinitesimal *physical* quantities  $Q(t)$  of the good at the model's price  $p(t)$  at any instant  $t$  in continuous time. The analysis is in continuous time, to follow *Samuelson's* stability requirement that timeless economic magnitudes must be treatable as special values of functions of continuous time.<sup>1</sup>

Now it is noted that an individual's market decision at time  $t$  remains a discontinuous decision. The quantity  $Q(t)$  he sells will have accrued at instantaneous rates  $Q'(\tau)$  at the model's prices  $p(\tau)$  over the production period  $t - \varepsilon \leq \tau \leq t$ , where  $\varepsilon$  is the length of time between transactions. The buyer will consume  $Q(t)$  at the instantaneous rates  $Q'(\tau)$  that are priced at  $p(\tau)$  over  $t < \tau < t + \varepsilon$ .

If all decisions now were to occur periodically,  $\varepsilon$  time units apart, at the same instants  $t = t_i, i = 0, \dots, n$ ,  $p(\tau)$  would be constant between transactions. Time would pass as a sequence of static states that  $t$  would merely date continuously.

But as *Hahn* has pointed out: "First, it must be admitted that period analysis is highly artificial since while people may take decisions discontinuously, not all people take decisions at the same time".<sup>2</sup> That is, the timing of an individual's decision, and of transactions must be functions of continuous time. It can occur at any arbitrary point in time and cannot be taken to coincide with the terminal point of a process that is assumed independent, in its timing, of continuous time.

This means a seller's decision at  $t$ , or a buyer's decision at  $t - \varepsilon$ , involve an intertemporal price  $\Pi(t)$  of a flow across time at time  $t$ , of

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<sup>1</sup> *Samuelson* (1970).

<sup>2</sup> *Hahn* (1955).

$\frac{1}{\varepsilon} \int_{t-\varepsilon}^t Q'(\tau) d\tau$  that is no longer equal to  $p(t)$ . Rather,  $\Pi(t)$  is the average<sup>3</sup>

$$\Pi(t) = \frac{1}{\varepsilon} \int_{t-\varepsilon}^t p(\tau) d\tau.$$

In the growth model  $p(\tau)$ , the price of the good relative to the wage rate, declines at the constant proportionate rate  $-g$  from an initial value  $P_0$  as the function

$$p(\tau) = p_0 e^{-g\tau}.$$

Thus,

$$\Pi(t) = p(t) \frac{e^{g\varepsilon} - 1}{g\varepsilon} \text{ for } \varepsilon > 0,$$

i.e. the prices of production and of consumption and the model's price all differ at transaction time  $t$ .

As a defense, it may be argued that, if the discontinuous decision interval were eliminated, the intertemporal price would disappear because as  $\varepsilon \rightarrow 0$ ,  $\Pi(t) = p(t)$ .

However, as  $\varepsilon \rightarrow 0$ ,  $Q(t) \rightarrow 0$ , also. That is, this argument destroys the market as a mechanism that is capable of transacting non-infinitesimal physical quantities at a price. In what sense can the market, as we know it, and as we like to rationalize it, be an allocative device?

### Summary

Continuous time enters the growth model through the stability requirements on the model. Its presence creates a problem of transforming the transaction price of the good, which can only be a price of physical units of the good at a point in time, into the price of a flow of the same good across time, as it enters into a transactor's decision, at a point in time. Growth models treat these prices as interchangeable. The present paper tries to show that these prices diverge in the growth model.

### Zusammenfassung

Kontinuierliche Zeit wird durch Stabilitätsbedingungen in das Wachstumsmodell aufgenommen. Sie führt zum Transformationsproblem des Transaktionspreises der Güter — ausgedrückt in physischen Einheiten zu einem bestimmten Zeitpunkt — in den Preis einer Stromgröße, die in die Entscheidung

<sup>3</sup> The cost of a flow of a unit accrues at  $p(\tau)$  at  $\tau$  over  $t - \varepsilon \leq \tau \leq t$  and it will total  $\int_{t-\varepsilon}^t p(\tau) d\tau$  at  $t$ . The value per unit of time at time  $t$  of the flow of one unit over  $t - \varepsilon \leq \tau \leq t$  is, thus,  $\Pi(t)$ .

derung des Wirtschaftssubjekts zu einem bestimmten Zeitpunkt eingeht. Die Arbeit versucht zu zeigen, daß diese Preise in Wachstumsmodellen divergieren.

### References

- Clower, W. R. and J. F. Due* (1972), *Microeconomics*. Homewood, Illinois,  
*Hahn, F.* (1955), The Rate of Interest and General Equilibrium Analysis. *Economic Journal*, 65 (1955), 64.  
*Samuelson, P. A.* (1970), *Foundations of Economic Analysis*. New York.  
*Solow, R. M.* (1956), A Contribution to the Theory of Economic Growth. *Quarterly Journal of Economics*, 70 (1956), 65 - 94.