Some Spatial Aspects of Currency Areas

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1. All questions concerning the possibility and size of currency areas contain a spatial component. This is true even though money represents one of the most mobile of resources. An indication that distance is relevant in monetary affairs is for instance the observation of *Lösch* (1944) that interest rates tends to rise with distance from financial centers. While it would be naive to trace this to a transportation cost for money, this fact shows clearly that something like communications cost over distance is involved here.

Such communications costs are rather elusive. To show that space matters and that distance imposes some real limitation on the potential size of currency areas, we will look into the transportation cost proper of funds. The analysis of these will be carried out in the spirit of *pars pro toto.*

To get an idea of the physical magnitude of the job of moving banknotes from places of issue to the locations of users, we cite the following statistics.

In the Federal Republic of Germany 1000 banknotes have an average weight of 1 Kg. Total circulation is 940 tons. The amount of coins issued is approximately DM 100.— per capita, representing an average weight of 0.5 Kg per capita.

2. Now the transportation problem for money arises because there is a regional imbalance between cash payments received and cash payments made. As a general rule, big cities are deficit areas, since more cash is paid out in wages and salaries than is received by local business. Recreation areas are net recipients and hence surplus areas. Thus broadly speaking, cash must flow back from rural areas to city centers. This movement is seasonal and the pattern may be reversed at times: Thus during the pre-holidays shopping season, cities are surplus areas instead.

Consider now the same problem for a foreign currency say French Francs circulating in West Germany. With respect to Francs, Germany

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is a deficit area and France a surplus area and the pattern is reversed for German Mark circulating in France and in West-Germany. If a single currency were adopted in both countries, the two transportation problems could be reduced to a single one. Only in the exceptional case when every surplus (deficit) region for Deutsche Mark is also a surplus (deficit) region for French Francs would the total transportation cost be the sum of the transportation costs for Deusche Mark and French Francs as before. But we have seen that surplus areas in Deutsche Mark and Franc do not coincide. Therefore there is a partial offset of surplusses and deficits, so that the net shipments required after currency union are considerably less. From the point of view of minimizing the costs of money shipments, a currency union could never be disadvantageous and sometimes clearly advantageous. The only question would seem to be: how large is the saving?

3. On closer inspection, i. e. when trying to measure the alleged savings, this formulation turns out to be a misspecification of the problem. Local surplusses and deficits in banknote availability exist and must be taken care of. This is not done, however, by making transfers between various distant points but rather more economically in the framework of Central Bank Districts and their respective Central Banks. In the case of West-Germany, each District Central Bank holds sufficient reserves of notes to draw on in case of a deficit. Thus transfers need never be made between districts, and all money shipments are either shipments from the printing press to the District Central Banks, or from the banks to points within their district, or conversely. In the following we completely ignore the shipping problem between printing press and District Central Banks and concentrate on the flow within each Central Bank District. It turns out, that the supply of cash to deficit areas is a small problem compared to another one. This is the return of old notes for new ones. Bank notes in West Germany have an average life of $1^1/_2$ years (larger in the South than in the North). Old bank notes must be returned to District Central Banks (and eventually to Frankfurt) to be exchanged for new ones.

4. The following assumptions may seem strange to monetary economists, but are more or less accepted as conventional in location theory.[1] Consider a region with a uniform average population density. The various districts are assumed circular and to cover in the aggregate a very large area. The percentage of the area that is left out in corner between adjacent circles is actually $1 - \frac{\pi}{2} / \sqrt{3}$ = about 9 %. To treat market areas as circles is an approximation. A more rigorous treatment in

[1] See *Launhart* (1963), *Lösch* (1944) and *Beckmann* (1968).

terms of an exhaustive system of hexagonal market areas is also possible. The calculations are more cumbersome and affect only the size of some coefficients in a small way.

We consider the following costs incurred by the circulation of banknotes and coins in one district.

(i) The cost of transporting banknotes and coins to various points in the region as replacement of old notes and coins. For simplicity of formulation we shall speak of notes only.

Let a population density,

k average amount of cash held per capita. This cash is measured in terms of the number of banknotes rather than their face value,

μ the replacement rate. E. g. for an average life time of $1^1/_2$ years $\mu = \frac{2}{3}$,

t transportation cost for one unit of notes.

The annual shipping cost for notes used in a unit area at distance r from the center is then

$$\mu\, aktr$$

In a circular region the area in a ring of width dr and radius r is $2\pi r\, dr$. The total transportation cost per annum in the entire district equals

$$T = \mu\, akt\, 2\pi \int_0^R r^2\, dr \qquad \text{or}$$

(1) $$T = \frac{2\pi}{3} \mu\, a\, kt\, R^3$$

A more sophisticated treatment is as follows. Money distribution (as replacement of old cash) is done by sending a vehicle under guard on a route that is determined along the lines of a travelling salesmen problem. In a circular region such a trip would cover a sequence of concentric rings. The entire length that must be traversed is then (roughly) proportional to the area πR^2, rather than to the third power of the radius.

If more than one round trip is needed to cover the area, then in view of the larger legs that must be traversed to reach the more distant zones, total transportation cost tends to go up by a power larger than 2 but less then 3 of the radius R. In the following we fix for simplicity the exponent at the level 3 of formula (1). The more general case that transportation cost is proportional to R^{α} where $2 \leq \alpha \leq 3$ does not change the results qualitatively but only quantitatively.

(ii) The cost of keeping money reserves. Since there may be a shortage in this district vis à vis the other regions, reserves must be kept to replenish local deficits in the district. The size of the reserves considered necessary may be calculated in different ways.

The simplest approach is to consider reserve requirements as a fixed proportion of total circulation. Let n be this proportion. The necessary reserves S are then given by

$$S = \pi\, nR^2\, ak \tag{2}$$

A more sophisticated view considers the demands on the reserve fund by any place to be a normally distributed random variable with mean zero and variance σ^2. The aggregate demand of the entire region has then mean zero and variance

$$\text{var} = \pi\, ka\, \sigma^2\, R^2$$

if these demands are independent.

When currency demands in adjacent locations are correlated — as one should expect them to be — the aggregate variance is increased but remains proportional to R^2.

Let now a security level be fixed in such a way that the reserves must be adequate to demand with a given probability p. This probability is then achieved by fixing reserves as a certain multiple of a standard deviation. Let m be this multiple. Then the required reserve are, in the case of independant demands,

$$S = m\, \sqrt{ak\, \pi}\, \sigma\, R \tag{3}$$

They are thus proportional to the radius R of the district. Combining both views, reserve costs appear to be proportional to a power R^{β} of the radius R of the district where $1 \leq \beta \leq 2$.

The economic cost of reserves will be small, actually, since it is no more than a storage cost for paper money and coin. It may some times be neglected in a first approach.

(iii) There is finally a fixed cost of administering the currency department of the District Central Bank. This cost F is not negligible, but may be difficult to determine by an outsider.

Given the three costs it is now straightforward to determine the optimum size of a currency district. Before we do this it may be asked if there is not a further cost to be considered i. e. the cost of returning old banknotes to the District Central Banks of issue from other districts where they may be found. What we assume here (and show elsewhere) is that a regular flow exists between the different districts governed by certain rules. This determines an average passage time for the first return of a note to each district of origin, and an equal average time interval between successive returns. If we assume that this is smaller than the average life time, no special return, i. e. shipments of notes to their home district for trade-in, is necessary.

5. The total cost of currency management is now as follows. Let A be the total area considered. Then $\frac{A}{\pi R^2}$ is the number of districts. In each district we have the costs T, S, F as given by (1), (2), (3) totalling

$$C = \frac{A}{\pi R^2} \cdot [T + S + F]$$

$$C = \frac{A}{\pi R^2} \left[\frac{2}{3} \mu\, akt\, \pi R^3 + m\, \sigma \sqrt{ak\, \pi}\, R + F\right]$$

$$= \frac{\alpha}{2 R^2} + \frac{\beta}{R} + \gamma R \qquad \text{(say)}$$

with $\alpha, \beta, \gamma > 0$.

This function is convex. A necessary and sufficient condition for a minimum is that the derivative with respect to R vanishes.

(4) $$0 = -\frac{\alpha}{R^2} - \frac{\beta}{R^3} + \gamma \qquad \text{or}$$

(5) $$\gamma R^3 - \beta R = \alpha$$

Equation (4) may be solved graphically. The graph of the left hand side is of the form of figure 1.

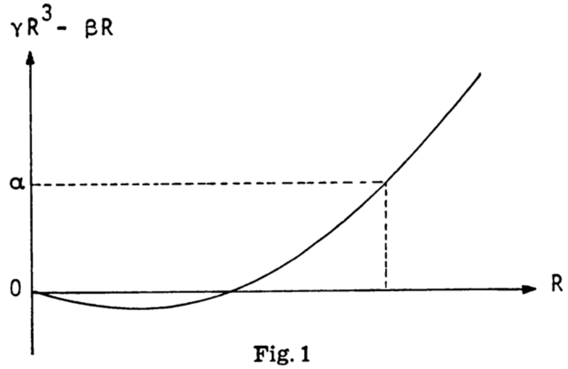

Fig. 1

We note the following: the optimal size (radius) R of a currency management district is unique, increasing with α and β and decreasing with γ. Thus the optimal size increases with the fixed cost and the reserve costs and decreases with the transportation cost coefficient.

To prove this consider equation (4) to define implicitly the function $R(\alpha, \beta, \gamma)$. Implicit differentiation of (4) yields the assertion.

A more interesting question is how an increase in population density a or in transaction cash demand k per capita affects the optimum district size.

If the cost of reserces can be neglected, a definite answer is possible. In fact, the optimal size $\hat{R}$ may then be determined explicitly

$$\hat{R} = \sqrt[3]{\frac{\pi \mu a k t}{3 F}} \tag{6}$$

An increase in population density or cash demand per capita permits a decrease in district size. However, the larger the population density a, the larger should be the population $\pi a R^2$ served by one District Central Bank since

$$\pi a R^2 = (\pi a)^{1/3} \left(\frac{3 F}{\mu k t}\right)^{2/3} \tag{7}$$

A decrease in transportation cost or the replacement rate permits an increase in the size of district areas.

If population densities or transportation costs (larger in Bavaria than in Hessia, for instance) or if cash demand per capita differs between regions, then the resulting optimal sizes differ and we have a case for unequal sizes of districts.

6. The following calculation is illustrative. Let

transportation cost be	1.— DM per t/km $= 10^{-6}$ DM/km per bill
transaction cash per capita	$k = 3$ banknotes
population density	$a = 240$ persons per km^2
replacement rate	$\mu = \frac{2}{3}$
fixed cost of currency management per annum	DM 100 000.—

Then

$$R = \sqrt[3]{\frac{3 \cdot 10^5}{\pi \cdot \frac{2}{3} \cdot 240 \cdot 3 \cdot 10^{-6}}}$$

$$= \sqrt[3]{\frac{10^{10}}{\pi \cdot 10}}$$

$$\approx 583 \text{ km}$$

As far as currency management is concerned, if these figures are at all representative, a single Landeszentralbank would be sufficient for the entire Federal Republic of Germany.

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