Price and Output Determination in an Economy with Two Media of Exchange and a Separate Unit of Account

By Michael Parkin*

Modern monetary theory of a closed economy analyzes a world which has one money — a single medium of exchange which also serves as the only unit of account. In contrast, the world which we inhabit today differs from that featured in standard monetary models in two interesting and perhaps important respects. First, economic agents have a choice of holding, denominating contracts in, and transacting with, a variety of alternative media of exchange. For example, the U. S. dollar circulates freely in Canada and many parts of Western Europe. Eurodollars (or more generally Xeno-currencies) are now an important part of the world money supply. Secondly, agents may keep accounts and denominate constracts in abstracts units of account the media of exchange values of which are variable over time. Examples of these units of account are S. D. R's, the European Unit of Account and a variety of index numbers.

The distinction between the unit of account and medium of exchange functions of money is, of course, an old one. For example, *Jevons* (1875) drew attention to its importance in the following words: "Money ... performs two distinct functions of high importance, acting as — (1) A medium of exchange [and] (2) A common measure of value ..." (*Jevons* 1875, p. 13). "It is in the highest degree important that [we] should discriminate carefully and constantly between [these] ... functions which money fulfils, at least in modern societies. We are so accustomed to using the one same substance [for the two functions] that they tend to become confused together in thought." (*Jevons* 1875, p. 16).[1] The

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David Laidler and *Russell Boyer* have provided invaluable stimulation on the topic of this paper. I am grateful also to *Allan Meltzer* and *Peter Howitt* for their comments and criticism of an earlier draft.

[1] Although the quotation from page 13 of *Jevons* (1875) distinguishes only the two main functions of money, that from page 16 (selectively adjusted here) refers to the four-fold division which arises when the two functions are extended to the intertemporal dimension.

potential confusion to which *Jevons* draws our attention arises from the fact that "today, each country has only one monetary unit: the lira, franc, mark, pound sterling, or dollar. This is the system established by the French assemblies at the end of the eighteenth century ... [In contrast] ... prior to the French Revolution [and going back at least as far as the eighth century A. D.], the monetary system of most European countries was based on altogether different principles ... There was, then, a monetary unit used only as a standard of deferred payments or for the purpose of keeping accounts. This was the function of a money of account, an imaginary or ideal money ... Although it was possible to make contracts or to keep accounts in imaginary money ... it was impossible to make actual payments in these monetary units, since they had not been coined ... Payment was made in real currency, that is, in gold, silver ... vellan or copper coins." (*Einaudi* 1936, pp. 234 - 6).

Modern monetary theory has made little of the distinction which *Jevons* thought to be "in the highest degree important". Indeed, the only modern work which has paid careful attention to the distinction between 'imaginary" and "actual" money, and the corresponding distinction between "accounting" and "money" prices, did so in order to demonstrate that "the accounting price of a given good is distinctive in having no operational significance for the market". (*Patinkin* 1965, p. 16).

One purpose of this paper is to re-examine the distinction which *Jevons* thought to be so important and which, as a descriptive matter, was important for the world in which "imaginary monies" were used, prior to the French Revolution.[2] A second purpose is to examine the implications for price and output determination of another feature of that earlier world, that of the existence of more than one medium of exchange, the relative prices of which were not immutably fixed.[3] This re-examination is, however, more than a matter of historical interest and intellectual curiosity. It is relevant to the post Bretton Woods world which we now inhabit. From the middle 1950s to 1971, the world had, for most practical purposes, one money, the United States dollar, with the price of all other monies fixed in terms of U. S. dollars. That world was highly amenable to analysis using the basic concepts and insights which *Hume* (1741) applied to the gold standard world and which were subsequently refined by such scholars as *Robert A. Mundell* (1971) and *Harry G. Johnson* (1972). Since the demise of the I. M. F.-Bretton Woods Dollar (gold exchange) standard, two developments have taken place

[2] For an account of the use of "imaginary" or "ghost" monies, see both *Einaudi* (1936) and *Cipolla* (1956).

[3] Typically Gold and Silver: in more recent history, the Greenback and Gold. On this latter case, see especially *Newcomb* (1865).

(alluded to in the opening paragraph) which make it necessary to develop a different monetary theory from that begun by *Hume*. First, the world has become a highly integrated economic system with individual agents, particularly but not only, large-scale corporations, operating in many different parts of the world and regularly making contracts in, and holding balances of, a multiplicity of national monies, the exchange rates amongst which are variable.[4] Secondly, there have been some limited but potentially important attempts to introduce "imaginary" monies. The most notable of these is the Special Drawing Right (SDR), an international "imaginary" money the value of which is defined in terms of a basket of national currencies. There is also the European unit of account used in a variety of European Communities' regulations and agreements. Additionally, a commonly advocated procedure for moving to European Monetary Union is to begin with the establishment of a common money which only possesses unit of account properties.[5] Finally, the use of cost-of-living escalator clauses, or "indexation", interest in which has been renewed during the "two digit" inflation of the early 1970s, can be viewed as a particular form of imaginary money, the unit of account being a specific basket of goods.

In order to conduct an analysis of an economy which possesses actual and imaginary money, and several, not just one of the former, it is necessary to attempt to refine and make more precise, the age-old but long neglected distinction between the means of payment and unit of account functions of money and to give that distinction analytic content. It is also necessary to modify the standard monetary model in which only one medium of exchange exists, to permit an exploration of the consequences of the existence of more than one such medium.

To facilitate this, four models are developed. The first (Part I) is a model of a closed economy with only one money which serves both as the medium of exchange and the unit of account. This benchmark model acts simply as the specification of a universe of discourse for the development of the subsequent models. The second model (Part II) retains a single medium of exchange but introduces a separate unit of account. The price of the medium of exchange in terms of the unit of account is determined and may only be varied by decree. Agents are free to write contracts either in unit of account or money prices. This model is not offered for its realism but as a simple framework within

[4] The implications for price, output, exchange rate and interest rate determination of one aspect of this phenomenon, what has come to be called "currency substitution", has already attracted some limited attention, see especially *Boyer* (1972, 1973 a, 1973 b, 1976), *Calvo* and *Rodriguez* (1976) and *Girton* and *Roper* (1976).

[5] See, for example, the "All Saints' Day Manifesto for European Monetary Union", *The Economist*, London, November 18, 1975.

which it is possible to give analytic content to the distinction between the unit of account and medium of exchange roles of money. The third model (Part III) has two media of exchange but no separate unit of account. Contracts may be entered into in terms of either medium of exchange and payment made in either. The exchange rate between the two monies is market determined. Finally, the fourth model (Part IV) has all the features of models two and three, namely two media of exchange and a separate unit of account.

The features of the models on which attention centres are the predicted effects on output and inflation of the settings of the various available monetary instruments. The analysis will examine expectational equilibrium effects and impact effects but will not examine the dynamics of the movement from an impact effect to an expectational equilibrium. This neglect of dynamics is designed to achieve simplicity and avoid the heavy taxonomy which would undoubtedly arise from a comparison of the several commonly employed alternative expectations adjustment processes from which most price-output models obtain their dynamic structure.

All the models will deal only with a closed economy. This is a deliberate choice design to focus on the determinants of aggregate output and the general price level (or, with more than one money, the general price levels) in a closed world economy. The individual country is thus completely ignored as are restrictions on the use of money such as those embodied in "legal tender" regulations. Whether or not the results obtained within this framework will be analytically robust or empirically relevant is a matter on which no judgement is offered at this stage.

I. An Economy With One Money

The economy is comprised of a large number of atomistic agents who produce a single (composite) commodity. Money is the only asset in the economy and is fiat money bearing zero interest and issued by a monetary authority which has no other economic function. Each agent sets the money price of his output in a state of less than complete information and, although individual agents may learn, there is a turnover of individual agents (births and deaths of agents) such that on the average, the knowledge available about the price of one agent's output relative to the economy-wide average price, is constant. That is, there is a permanent non-degenerate distribution of relative prices across agents about the economy-wide average price. At the price set, the agent stands ready (is contractually committed) to satisfy demand. Thus, at any moment, sales and output[6] are demand determined. No

[6] Sales and output are identical since goods are assumed to be non-stable and money the only asset.

attempt will be made to state and solve the typical agent's individual optimization problem. Rather, it will simply be assumed that, whatever the details of that problem, its solution is the following set of aggregate behavior propositions:

(i) The demand for money is given by

$$m^d = p + ay - b\,(p^e_{+1} - p)\ ;\quad a, b > 0 \tag{1}$$

where all the variables are natural logarithms and

m^d = nominal balances demanded
y = aggregate real output
p = the price level (average of individual agent's prices)

and where the superscript e denotes 'expectation' an the subscript $\pm i$ denotes time lead (+) or lag (−) of i periods. This is a standard demand for real balances $(m - p)$ as a function of real output (income) and the expected opportunity cost of holding real balances, $(p^e_{+1} - p)$.

(ii) Agents set their prices in money terms, in accordance with:

$$p = p^e + \delta\,(y^e - y^*)\ ;\quad \delta > 0 \tag{2}$$

where y^* is equilibrium real output in the steady state.

This is a standard *Phelps* (1968) price adjustment rule with agents setting their own prices at the level which they expect, on the average, other firms will set theirs, plus an adjustment, which is greater the greater is the expected excess demand facing the agent $(\delta\,(y^e - y^*))$.

A commonly used proposition reverses the direction of causation in (2) and postulates instead that supply is a positive function of the difference between the actual and expected price level. The actual price level is then determined by an 'as if' auctioneer who sets the price to clear the market, given expectations. This alternative, first advanced by *Irving Fisher* (1911) and used in much modern work (e. g., *Lucas* 1975, *Sargent* 1976) is not employed here. In the context of the deterministic, single money, model of this section, the two yield identical predictions. However, in the model with more than one money, the two approaches would require different developments. The *Phelps* price setting rather than the *Fisher* supply response story is employed partly because it seems more reasonable and partly because it makes it possible to give analytic content to the distinction between the unit of account and the medium of exchange aspects of money in the next section.

Money is the only asset, the supply of which, m, is exogenous. Stock equilibrium prevails in the money market, i. e.,

$$m^d = m \tag{3}$$

The following assumptions will be made about expectations:

(1) at a moment in time, expectations are given;

(2) over time, expectations adjust such that the expected value and actual value of a variable are equal (expectational equilibrium).

The expectational equilibrium of this model is very simple and familiar. With $p_{+i} = p^e_{+i}$ and $y_{+i} = y^e_{+i}$ $i = 0, \ldots,$ it is clear from (2) that,

$$y = y^* \tag{4}$$

and, imposing the standard rational expectations requirement that agents expect the inflation rate to converge on the rate of monetary expansion and not to explode (*Sargent* and *Wallace* 1973 b), for a constant money stock,

$$p = m - ay^* \tag{5}$$

or, for a constant rate of monetary expansion, Δm,

$$p = m + b\,\Delta m - ay^* \tag{6}$$

The impact multipliers of an unanticipated change in m (dm) in this model are equally very simple and familiar and are:

$$\frac{dy}{dm} = \frac{1}{a} > 0 \tag{7}$$

and

$$\frac{dp}{dm} = 0 \tag{8}$$

Thus, the price level is unresponsive on impact, to change in the money supply but real output responds. The excess demand thereby generated eventually starts prices rising via equation (2) and, once an expectational equilibrium has been reached, real output will be restored to its previous level and prices will have risen proportionately to the rise in the money stock. The particular paths taken by the variables *en route* to this expectational equilibrium will depend on the dynamics of expectations, one example of which is presented in *Laidler* (1973).

DOI https://doi.org/10.3790/schm.99.1-2.95 | Generated on 2025-07-25 04:58:28

It is within the framework of this simple model, that the role of a separate unit of account and more than one medium of exchange will now be analyzed.

II. An Economy With a Medium of Exchange and a Separate Unit of Account

The economy described and analyzed in the preceding section will now be modified to incorporate a unit of account which is different from the units in which the medium of exchange is expressed. Define x as the (natural logarithm of the) number of units of money per unit of account. Thus, with money units always representing the unit of account, $x = 0$. Define the (natural logarithm of the) price level of the economy expressed in unit of account as z, thus,

$$p \equiv z + x \tag{9}$$

defines the relation between the two price levels, p in money units and z in accounting units. The money price of unit of account, x, is fixed by decree and, as shown in the preceding section, the money price level p is determined by m so that the price level expressed in accounting units is uniquely determined given m and x. Change x, and z will change but p will not change. This is essentially *Patinkin's* proposition on the irrelevance of accounting prices.[7] As regards a state of expectational equilibrium, that proposition is clearly correct. However, it will not, in general, be correct concerning impact effects of either a change in the money supply, m, or a change in the accounting price of money, x.

The way in which the model of the preceding section needs modification to allow for the existence of a unit of account and exogenous accounting price of money depends on the units in which agents write prices into their contracts. If prices are contracted only in terms of money units then the model needs no modification at all and accounting prices, including that of money, x, are indeed irrelevant. If, however, prices are set in terms of accounting prices and contracts entered into which are fixed in accounting prices, then the model of the preceding section does stand in need of modification. That some contracted prices were set in terms of accounting prices when a separate unit of account existed is very well established and documented (see *Einaudi* 1936).

Assume that fraction λ of agents set their prices in money terms and fraction $(1 - \lambda)$ set them in accounting units, standing ready to trade at the accounting price converted into money prices at the current accounting price of money. An agent which sets its price in money units will set the price at

[7] Cited above; see *Patinkin* (1956) p. 16.

DOI https://doi.org/10.3790/schm.99.1-2.95 | Generated on 2025-07-25 04:58:28

(10) $$p_i = p_i^e + \delta (y_i^e - y_i)$$

which is equation (2) for an individual agent. An agent which sets its price in accounting units will set the price at

(11) $$z_i = p_i^e - x_i^e + \delta (y_i^e - y_i^*) \; .$$

However, the money price at which this second agent will actually trade is

(12) $$p_i = z_i + x = p_i^e + x - x_i^e + \delta (y_i^e - y_i^*)$$

Aggregation over the economy and applying the weight λ to (10) and $(1 - \lambda)$ to (12) gives the price index for the economy in money units as

(13) $$p = p^e + (1 - \lambda)(x - x^e) + \delta (y^e - y^*)$$

This makes the obviously correct statement that the price level in money units will change, not only as a result of a change in the expected money price level and excess demand, but also by the difference between the actual accounting price of money, x, and the accounting price which was expected, x^e, when the accounting prices were set, times that fraction of agents $(1 - \lambda)$ who set their prices in accounting units.

In an expectational equilibrium, $x = x^e$, (13) becomes (2) in the preceding model and, therefore, a fully anticipated change in the accounting price of money has no effects on anything except the price level expressed in accounting units. However, if prices were contracted in accounting units for a fixed period, there would be impact effects of such a change. The impact effects are calculated by holding expectations constant and using the system (1), (3), (13) and are:

$$\frac{\partial y}{\partial m} = \frac{1}{a} > 0 \; ; \qquad \frac{\partial y}{\partial x} = \frac{-(1 - \lambda)(1 + b)}{a} \leq 0 \; ;$$

$$\frac{\partial p}{\partial m} = 0 \qquad \frac{\partial p}{\partial x} = (1 - \lambda) \geq 0 \; .$$

In addition, the price level expressed in accounting prices, z, which is

(14) $$z = p - x$$

will change on impact as x changes by

$$\frac{\partial z}{\partial x} = -\lambda \; .$$

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These results are now discussed. First, the impact effects of a change in m are exactly the same in this case as in the model with no separate unit of account and for fairly obvious reasons. However, there are now some additional impact effects which arise from a change in the accounting price of money. A rise in the accounting price of money will raise prices expressed in money units by $(1 - \lambda)$ and lower prices expressed in unit of account terms by λ. Further, it will lower real output by $(1 - \lambda)(1 + b)/a$. The central parameter in the interpretation of these results is λ. If no one sets prices in unit of account then $\lambda = 1$ and changing the accounting price of money changes the price level when expressed in terms of unit of account but has no other effects. If everyone sets prices in unit of account then $\lambda = 0$ and a rise in the accounting price of money has a full unitary effect on the price level in money terms, which, via a reduction in real balances, causes a reduction in real income of $(1 + b)/a$. For the more general case where some fraction of agents set prices in money; $0 < \lambda < 1$ and some in unit of account, $0 < (1 - \lambda) < 1$, the intermediate impact multipliers set out above hold. The practical relevance of these results is, of course, highly limited. If the monetary authority were repeatedly changing the value of x, it is hard to see why anyone should get tied into contracts denominated in units of z. These impact effects would, therefore, be extremely short-lived. However, if x had been stable over a long period with a large fraction of fixed term price contracts denominated in z, then the effect of a once and for all change in x could be longer lasting.

In the above exercise, and throughout this paper, λ is treated as a parameter. It is of great importance to ask, what determine λ? Will it be anything other than either 0 or 1? How is its value affected by the variability of m and x? These questions, whilst important, are not addressed here.

III. An Economy with Two Media of Exchange

In this section, the economy of Section I will be modified, not to incorporate a separate unit of account but *two* media of exchange. There are nominal dollars, m, and nominal $\delta\varrho\alpha\chi\mu\alpha\iota$ (drachmas), μ. There are two price levels; that with all prices expressed in dollar terms, p, and that with all prices expressed in drachma terms, π. The exchange rate between dollars and drachmas is ε, i. e., the dollar price of a drachma is ε. (All the above variables are natural logarithms.)

The demand for dollars is as in equation (1) above except that there is substitution between dollars and drachmas as well as between dollars and goods and hence is:

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(15) $$m^d = p + ay - b\,(p^e_{+1} - p) - c\,(\varepsilon^e_{+1} - \varepsilon)$$

where all the variables and parameters have been defined above except for c which is the semi-elasticity of substitution[8] between dollars and drachmas. The demand for drachmas is given by:

(16) $$\mu^d = \pi + \alpha\, y - \beta\,(\pi^e_{+1} - \pi) + \gamma\,(\varepsilon^e_{+1} - \varepsilon)$$

where all the parameters have an obvious interpretation. It will be noticed that γ, the semi-elasticity of demand for drachmas with respect to the expected change in the dollar-drachma exchange rate, will not in general be equal to $-c$. This arises because agents may substitute across the two monies and goods and only the substitution effects across all three necessarily offset each other.

One feature of these demand functions is unusual and needs a comment: that is that the demand for both dollars and drachmas are functions of the same income variable — i. e., aggregate (world) income. Dollars are issued by the central bank of America and Drachmas by the central bank of Europa but the citizens of these two countries are free to hold (and use) either money. Hence, on the usual conventions and aggregation assumptions, it is *world* not *national* income which enters the demand functions for real balances. It is true that this ignores the distribution of income between regions but this is a standard simplification of all monetary models.

The supplies of dollars and drachmas are exogenous and money market equilibrium prevails, i. e.,

(17) $$m^d = m$$

(18) $$\mu^d = \mu$$

Prices are set by firms, some fixed in dollars and some in drachmas. Let Φ be the fraction set in dollars. Then, following the discussion in the preceding section, the general price level in dollars will be given by:

(19) $$p = p^e + (1 - \Phi)\,(\varepsilon - \varepsilon^e) + \delta\,(y^e - y^*)$$

and in drachmas by

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[8] It is the semi-elasticity and not the elasticity because $\varepsilon^e_{+1} - \varepsilon$ is the absolute not logarithmic opportunity cost of holding dollars at the dollar-drachma margin. It might be thought that the expected rate of inflation in terms of drachma should also appear in the demand function for dollars. However, if purchasing power parity holds and is exepected to hold, the expected drachma inflation rate is implicitly contained in the expected dollar inflation rate and the expected rate of change of the exchange rate.

DOI https://doi.org/10.3790/schm.99.1-2.95 | Generated on 2025-07-25 04:58:28

$$\pi = \pi^e - \Phi(\varepsilon - \varepsilon^e) + \delta(y^e - y^*) \tag{20}$$

The interpretation of the above is that, at the beginning of the period of analysis, fraction of agents Φ set dollar prices equal to their expectation of the dollar price level while fraction $(1 - \Phi)$ set their prices equal to their expectation of the drachma price level, both groups adding an adjustment for expected excess demand. After prices have been set, the exchange rate is determined and those prices set in drachmas are converted to dollar and those set in dollars converted into drachmas at the going exchange rate.

The assumptions concerning expectations are the same as those stated above (Section I). Equations (15) - (20) determine $\{p, \pi, y, m^d, \mu^d, \varepsilon\}$ with all expectations constant in the short run and those same six variables with fulfilled expectations in the long run.

The long-run (expectational equilibrium) predictions depend on the nature of the (fully anticipated) paths of the two money supplies. If the assumption is made that both m and μ are on constant growth paths of Δm and $\Delta\mu$, then the expectational equilibrium predictions are:

$$p = m - ay^* + b\,\Delta m + c\,(\Delta m - \Delta\mu) \tag{21}$$

$$\pi = \mu - dy^* + \beta\Delta\mu - \gamma\,(\Delta m - \Delta\mu) \tag{22}$$

$$\varepsilon = m - \mu + (b + c + \gamma)\,\Delta m - (\beta + \gamma + c)\,\Delta\mu - (a - \alpha)\,y^* \tag{23}$$

$$y = y^* \tag{24}$$

These predictions are noteworthy for they imply that the equilibrium price level in each money is not independent of the rate of growth of *both* monies. A rise in the rate of growth of μ (a rise in $\Delta\mu$), will raise the price level π and lower the price level p and a rise in Δm will raise p and lower π. These effects occur because of the effects of relative money supply growth rates on the rate of change of the exchange rate and the effects of the latter variable on the demand functions for the two monies. A further property of the expectational equilibrium needing comment is that the exchange rate will be affected by real factors if the income elasticities of demand for the two monies differ from each other. This is simply a reflection of the fact that the rate of change of the equilibrium exchange rate is equal to the difference in the rate of growth of the *excess* demand for the two monies.

The impact effects (expectations constant) of changes in m and μ are as follows:

$$dp = (1 - \Phi)\,(\alpha\, dm - a d\,\mu)/h \tag{25}$$

(26) $$d\pi = -\Phi(\alpha\, dm - a d\mu)/h$$

(27) $$d\varepsilon = (\alpha\, dm - a d\mu)/h$$

(28) $$dy = \{\Phi(1+\beta) + \gamma)\, dm + [(1-\Phi)(1+b) + c]\, d\mu\}/h$$

where

(29) $$h = a[\Phi(1+\beta) + \gamma] + \alpha[(1-\Phi)(1+b) + c] > 0$$

All these impact effects are unambiguously and obviously signed. Several features of them are, however, worth highlighting and commenting on. First, a rise in m (or a fall in μ) will raise dollar prices, p, and cut drachma prices, π, via its effect on the exchange rate. That is, it is the fraction setting prices in drachma $(1-\Phi)$, which scales the change in p and the fraction setting prices in dollars, Φ, which scales the change in π. Secondly, the role of the degree of money substitutability is noteworthy. If dollars and drachmas are perfect substitutes, $c = \gamma = \infty$, then $dp = d\pi = d\varepsilon = 0$ and

(30) $$dy = \frac{1}{a+\alpha}(dm + d\mu)$$

These results are strongly intuitive and make the obvious point that the more substitutable the two monies are, the less will be the effect of changing any one of them on the exchange rate and hence on prices. Further, if they are perfect substitutes, their composition is irrelevant and only their aggregate matters as regards impact effects on real output. Thirdly, a rise in the supply of one money, say drachmas, will generate excess demand (28) which will, in the inter-run (not explicitly analyzed), start all prices rising, including those set in dollars and will not, until the steady state is reached (equation (21)), leave the dollar inflation rate immune, even though the exchange rate is flexible.

Finally, notice that if one monetary authority is deflating, the other could only attempt to offset its deflationary effects on output by accepting a depreciation of its money and a rise in prices expressed in units of that money. The more important the deflating money issuer (the bigger its income elasticity of demand), the greater would be the depreciation of the other money required to achieve any given offset to the deflation of output.

IV. An Economy with Two Media of Exchange and a Seperate Unit of Account

This section brings together the two preceding cases and analyzes a world with two media of exchange and a separate unit of account. The basic model specification is not very different from that in the preceding section. Since the unit of account is not an asset, its existence has no effect on the demand for money functions which remain as equations (15) and (16). Money supply and money market equilibrium are given (17) and (18) above. It is short-run price setting behavior which is modified by the existence of both a unit of account and two monies. Let λ be the fraction of agents who set their prices in dollars, Φ drachmas and $(1-\lambda-\Phi)$ in unit of account. There are three price levels in this economy which, in the short run will be determined as:

$$p = p^e + \Phi(\varepsilon - \varepsilon^e) + (1-\lambda-\Phi)(x - x^e) + \delta(y^e - y^*) \tag{31}$$

$$\pi = \pi^e - (1-\Phi)(\varepsilon - \varepsilon^e) + (1-\lambda-\Phi)(x - x^e) + \delta(y^e - y^*) \tag{32}$$

$$z = z^e + \Phi(\varepsilon - \varepsilon^e) - (\lambda + \Phi)(x - x^e) + \delta(y^e - y^*) \tag{33}$$

where ε is, as before, the dollar price of drachmas and where x is the accounting price of dollars (both expressed as natural logarithms).

The meaning of (31) - (33) is analogous to (15) and (20) above and needs no further discussion here.

The equations (15) - (18) with (31) - (33), seven in all, determine the seven endogenous variables p, π, z, ε, y, m^d, and μ^d. The expectational equilibrium behavior of this model is exactly as set out in (21) - (24) above except that, additionally, the accounting price level, Δz will inflate at the rate:

$$\Delta z = \Delta m - a\,\Delta y^* - \Delta x \tag{34}$$

where Δx is the exogenous rate of change of the accounting price of dollars.

The impact effects (with given expectations) are (not in reduced form):

$$dp = \{\Phi(\alpha\, dm - a\, d\mu) + (1-\lambda-\Phi)(a(1+\beta) + a\gamma + ac)\, dx\}/s \tag{35}$$

$$d\pi\, \{(1-\Phi)(a\, d\mu - \alpha\, dm) + (1-\lambda-\Phi)(\alpha(1+b) + a\gamma + \alpha c)\, dx\}/s \tag{36}$$

$$\begin{aligned} dz = \{\Phi(\alpha\, dm - a\, d\mu) - [(\lambda+\Phi)(a\gamma + \alpha c) + \\ + \lambda a(1+\beta) + \Phi\alpha(1+b)]\, dx\}/s \end{aligned} \tag{37}$$

$$d\varepsilon = \{\alpha\, dm - a\, d\mu + [a(1+\beta) - \alpha(1+b)](1-\lambda-\Phi)\, dx\}/s \tag{38}$$

$$\begin{aligned} dy = \{[(1+\beta)(1-\Phi) + \gamma]\, dm + [(1+b)\Phi + c]\, d\mu - \\ - (1-\lambda\Phi)[(1+\beta)(1+b) + (1+\beta)c + (1+b)\gamma]\, dx\}/s \end{aligned} \tag{39}$$

where

$$s = \alpha(1+b)\Phi + a(1+\beta)(1+\Phi) + \alpha c + a\gamma > 0 \tag{40}$$

These effects represent a straightforward combination of those of the two preceding models. A rise in m unambiguously has a positive impact effect on p, z, ε and y and a negative effect on π. A rise in μ has the same sign effect on y but opposite sign effects on all the other variables to m. A rise in x raises the two money prices, lowers accounting prices, lowers real income and has a ambiguous effect on the exchange rate ε, depending on the relative magnitudes of $a(1+\beta)$ and $\alpha(1+b)$. The magnitude of the effects of a change in x on all variables (except z) is proportional to the fraction of agents using the unit of account as the price setting unit $(1-\lambda-\Phi)$. The role of currency substitutability in this model is analogous to that in Section III above and, if $c=\gamma=\infty$, the model essentially becomes that of Section II.

A more interesting and policy relevant case arises if the unit of account is defined not as a fixed (and variable by decree) number of dollars but as a weighted average (with predetermined weights) of dollars and drachmas. This corresponds to the definitions of an S. D. R. or European Unit of Account, for example. Suppose we define the unit of account as

$$1\ U \text{ of } A = \psi \text{ Dollars} + (1-\psi) \text{ Drachmas}$$

with ψ a constant. Then since the dollar-drachma exchange rate is ε, the unit of account price of dollars, x, is given by

$$x = (1-\psi)\,\varepsilon \tag{41}$$

and, the accounting price of drachmas,

$$x + \varepsilon = -\,\psi\varepsilon \tag{42}$$

The expected accounting price of the dollars will be

$$x^e = (1-\psi)\,\varepsilon^e\ . \tag{43}$$

Using the above, (41) and (43), in (31), (32) and (33) gives the following price setting equations:

$$p = p^e + (\Phi + (1-\psi)(1-\lambda-\Phi))(\varepsilon-\varepsilon^e) + \delta(y^e - y^*) \tag{44}$$

$$\pi = \pi^e - [(1-\psi)\lambda + (1+\Phi)\psi](\varepsilon-\varepsilon^e) + \delta(y^e - y^*) \tag{45}$$

$$z = z^e + (\Phi - (1-\psi)(\lambda+\Phi))(\varepsilon-\varepsilon^e) + \delta(y^e - y^*) \tag{46}$$

Using the system described by equations (15) - (18) together with (44) - (46) which replace (31) - (33), the expectational equilibrium and impact effects of alternative monetary policies may now be analyzed. Clearly the expectational equilibrium properties of the model are exactly as before except that, since x is endogenous and equal to $(1-\psi)\,\varepsilon$, the steady state rate of inflation in unit of account prices is given by

$$(34) \qquad \Delta z = \Delta m - a \Delta y^* - \Delta x$$

with

$$(47) \qquad \Delta x = (1 - \psi) \Delta \varepsilon$$

and

$$(48) \qquad \Delta \varepsilon = \Delta m - \Delta \mu - (a - \alpha) \Delta y^*$$

$$(49) \qquad \Delta z = \psi \Delta m + (1 - \psi) \Delta \mu - [\psi a + (1 - \psi) \alpha] \Delta y^*$$

This states that, with the unit of account defined as a weighted average of currencies, the price level expressed in terms of that unit will inflate by the same weighted average of the growth rates of the supplies of those currencies, less the weighted growth in demand for real balances of those currencies.

The impact multipliers of a change in m (dm) and a change in μ ($d\mu$) are as follows:

$$(50) \qquad dp = \{(\Phi + (1 - \psi)(1 - \lambda - \Phi))(\alpha\, dm - a d\mu)\}/k$$

$$(51) \qquad d\pi = - \{((1 - \psi)\lambda + \psi (1 - \Phi))(\alpha\, dm - a d\mu)\}/k$$

$$(52) \qquad dz = \{(\psi \Phi - (1 - \psi)\lambda)(\alpha\, dm - a d\mu)\}/k$$

$$(53) \qquad d\varepsilon = \{\alpha\, dm - a d\mu\}/k$$

$$(54) \qquad dy = \{[(1 + \beta)((1 - \psi)\lambda + \psi (1 + \Phi)) + \gamma]\, dm + \\ + [(1 + b)(\Phi + (1 - \lambda - \Phi)) + c]\, d\mu\}/k$$

where

$$(55) \qquad k = a((1 + \beta)((1 - \psi)\lambda + \psi (1 - \Phi) + \gamma) + \\ + \alpha((1 + b)(\Phi + (1 - \psi)(1 - \lambda - \Phi)) + c) > 0$$

The signs of these multipliers are:

	dp	$d\pi$	dz	$d\varepsilon$	dy
dm	+	−	?	+	+
$d\mu$	−	+	?	−	+

The unambiguous signs are the same as in (35) - (39) above and the ambiguity on dz arises because a rise in m raises ε thereby raising p and lowering π. Since z is a combination of p and π prices the net change in z depends on the weights ψ and $(1 - \psi)$ assigned to each currency in defining the unit of account and on the fractions λ and Φ setting prices in dollars and drachmas respectively. If $\frac{\psi}{1-\psi} > \frac{\lambda}{\Phi}$ then $\frac{dz}{dm} > 0$ and $\frac{dz}{d\mu} < 0$. Note that if the weight attached to dollars (ψ) is equal to the fraction of agents who set prices in dollars (λ) and the weight attached to drachmas ($1 - \psi$) equal to the fraction of agents who set prices in drachmas (Φ), then the according price level is immune to changes in m and μ.

The sensitivity of these multipliers to the basic parameter are similar to those in the preceding case. The larger the fraction of agents setting prices in a given unit of account, the smaller the impact multiplier on the price level expressed in that unit. The greater the degree of currency substitutability, the smaller the effects of a change in any one money supply. The effect of the weight given to one currency in defining the unit of account, on the impact effects of monetary shocks, is ambiguous and depends in a manner too complicated to be worth setting out, on both the relative numbers of agents setting prices in the different currencies and the income elasticities of demand.

V. Conclusions

This paper will be concluded with a series of questions to which the preceding analysis gives rise rather than present a summary of the main results.

First, there are questions concerning the optimality of setting prices in one or other medium of exchange or in unit of account terms. Under what conditions of information and adjustment costs is each of these optimal? Is there a stable distribution of agents over these alternatives or is there a dynamic process which leads all to converge on one standard?

Secondly there are some questions about policy. Will the distribution of agents over the alternative types of money be independent of the variability (and hence predictability) of m and μ? It may be conjectured that the most stable unit, if one is most stable, will dominate as the unit of account.

Thirdly there are questions concerning the robustness of the above results concerning modifications which recognize legal tender and other restrictions concerning the use of money.

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DOI https://doi.org/10.3790/schm.99.1-2.95 | Generated on 2025-07-25 04:58:28