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Abhandlungen

Understanding the Predictability of Excess Returns*

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Abstract

A seminal paper by Fama and Bliss (1987) showed that a simple regression model could explain a significant portion of 1-year ahead excess returns. Cochrane and Piazzesi (2005) showed that their regression model could explain a significantly larger portion of excess returns than Fama and Bliss'model and that a single return-forecasting factor essentially encompassed the predictability of excess returns for all of the bonds considered. This paper makes several contributions to the literature. First, I show why excess return models based solely on bond prices are unlikely to provide information about the predictability of excess returns and, in so doing, show that neither FB's model nor CP's model provides information about the predictability of excess returns. Second, I show that the "predictive power" of FB's model is due solely to the high correlation between excess returns and changes in bond prices, and that this correlation accounts for half of the "predictability" reported by CP. Third, I show that forecasting excess returns out of sample is identical to forecasting future bond prices. Consequently, the FB and CP models can be compared with any model that forecast future bond prices (or, equivalently, bond yields).

JEL classification: G0; G1; E0; E4.

Keywords: excess returns, bond prices, predictability, bond risk premia.

Zum Verständnis der Prognostizierbarkeit von Überrenditen

Zusammenfassung

Eine wegweisende Arbeit von Fama und Bliss (1987) legt dar, dass ein schlichtes Regressionsmodell einen signifikanten Anteil der Überrendite des nachfolgenden Jahres erklären kann. Cochrane und Piazessi (2005) demonstrieren, dass ihr Regressionsmodell einen signifikant größeren Anteil der Überrendite erklären kann, als das Modell von Fama und Bliss (1987) und, dass im Wesentlichen ein einziger Prognosefaktor die Vorhersage der Überrenditen der berücksichtigten Anleihen erfasst. Diese Forschungsarbeit be-

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inhaltet mehrere Beiträge zur Literatur. Zuerst wird dargelegt, warum Renditemodelle, die lediglich auf Anleihepreisen beruhen, ungeeignet sind, Informationen zur Prognose von Überrenditen zu liefern und, dass dadurch weder das Modell von FB, noch das Modell von CP, Informationen zur Prognostizierbarkeit der Überrenditen liefert. Zum Zweiten wird gezeigt, dass die Prognosekraft des Modells von FB lediglich auf die hohe Korrelation zwischen der Überrendite und der Preisänderung der Anleihen zurückzuführen ist und, dass diese Korrelation für die Hälfte der Prognosefähigkeit des Modells von CP verantwortlich ist. Zum dritten wird demonstriert, dass die out-of-sample-Prognose von Überrenditen der Prognose zukünftiger Anleihepreise gleicht. Infolgedessen können die Modelle von FB und CP mit jeglichem Modell verglichen werden, das zukünftige Anleihepreise modelliert (oder, gleichbedeutend, Anleiherenditen).

“True wisdom is knowing what you don’t know.” – Confucius

I. Introduction

The predictability of bond excess returns has occupied the attention of financial economists for many years. In their seminal work Fama and Bliss (1987, hereafter, FB) found that a simple regression model, base on Fama (1984, 1986), could explain a significant portion of 1-year ahead excess return on Treasuries with maturities from one to five years and that excess returns were related to the business cycle; “mostly positive during good time and mostly negative during recessions” (p. 689).

Cochrane and Piazzesi (2005, henceforth CP) extended FB’s work by estimating a regression of excess returns on the current 1-year bond yield and four forward rates. Their model produced estimates of R^2 more than twice as large as those from FB’s model. Moreover, they found that a single return-forecasting factor, commonly referred to as the CP factor, encompassed the predictive power of their model. CP interpreted their findings as strong evidence against the expectations hypothesis of the term structure, which requires excess returns to be unpredictable. CP also found that their return-forecasting factor had significant predictive power for bond yields that was unrelated to the ‘level,’ ‘slope,’ and ‘curvature’ factors that are used in conventional 3-factor term structure models.[1] Specifically, they found that yield curve models must include their return-forecasting factor in addition to the traditional three factors despite the

[1] FB and CP models only provide information about the in-sample fit of the data, one should be cautious about using the word predictability. However, since this is routinely done in this literature, I do it too. But I use it only in the context of in-sample fit. Later in the paper when I discuss true out-of-sample predictablitity I will use the word forecastability. The exception is when I discuss CP’s return *forecasting* factor. I call it this becasue that is what they called it and because that is how it is referred to in subsequent literature.

fact that the return-forecasting factor improves the conventional model's fit only marginally.

This paper makes several contributions to the literature on predicting bond excess returns. Specifically, I show why excess return models based solely on bond prices are unlikely to provide information about the predictability of excess returns and, in so doing, show that neither FB's nor CP's model provides information about the predictability of excess returns. The predictability of excess returns reported by CP is entirely do to their model's ability to predict future bond prices and not excess returns. FB's model provides no information about the predictability of excess returns. The "predictive power" of FB's model is due entirely to the correlation between excess returns and changes in bond prices, which FB's model cannot predict.

I then show how the above noted results account for CP's findings that: (i) their return-forecasting factor encompasses the predictive power of their model, (ii) their return-forecasting factor increases the predictive power of standard three-factor term structure model, in spite of the fact that it provides a small improvement in the traditional model's fit, and (iii) long-term forward rates add significantly to the predictability of excess returns on short-term bonds.

The paper concludes with a discussion of how the high correlation between excess returns and changes in prices complicates the interpretation of the in-sample fit of other excess return models. I also show why forecasting excess returns is identical to forecasting future bond prices. This fact means that the forecasting performance of excess return models can be compared with any model that is designed to forecast bond prices (or bond yields). I note that this fact also has implications for market efficiency, the relationship between bond yields and excess returns, and the expectations hypothesis.

The reminder of the paper is divided into four sections: Section 2 replicates CP's and FB's findings using CP's data and sample period. Section 3 explains why it is so difficult to find evidence of the predictability of excess returns using these models, and shows why neither CP's nor FB's model provides information about the predictability of excess returns. Section 4 uses the results in Section 3 to explains the source of CP's other findings. Section 5 presents the conclusions and several implications of paper's findings.

II. CP's and FB's Models and Findings

Following CP and FB, the log yield of a n-year bond is defined as

(1) $$y_t{}^{(n)} \equiv -\frac{1}{n} p_t{}^{(n)},$$

and where $p_t^{(n)}$ is the log price of an n-year zero-coupon bond at time t, i.e. $p_t^{(n)} = \ln p_t^{(n)}$, and where $P^{(n)}$ is the nominal dollar-price of zero coupon bond paying \$1 at maturity. The forward rate of maturity n is defined as

$$f_t^{(n)} \equiv p_t^{(n-1)} - p_t^{(n)}. \tag{2}$$

The excess return of an n-year bond is computed as the log holding-period return from buying an n-year bond at time t and selling it at time $t + 1$ less the log return on a 1-year bond at time t, i.e.,

$$rx_{t+1}^{(n)} = p_{t+1}^{(n-1)} - p_t^{(n)} - y_t^{(1)}.[2] \tag{3}$$

CP's predict excess returns by regressing the 1-year ahead excess return on an n-period bond on the current 1-year yield and the four forward rates, i.e.,

$$rx_{t+1}^{(n)} = \beta_0 + \beta_1 y_t^{(1)} + \beta_2 f_t^{(2)} + \ldots + \beta_5 f_t^{(5)} + \varepsilon_{t+1}^{(n)}. \tag{4}$$

FB's excess return model is

$$rx_{t+1}^{(n)} = \alpha + \beta\left(f_t^{(n)} - y_t^{(1)}\right) + \upsilon_{t+1}^{(n)}. \tag{5}$$

CP estimate both models using monthly data on the prices of zero coupon bonds with maturities of one to five years. The sample period is January 1964 through December 2003.

Estimates of equations (4) and (5) using CP's data and sample period are summarized in Table 1.[3] CP's model accounts for more than 30 percent of the in sample variation in excess returns for $n = 2; 3; 4; 5$; more than twice that of FB's model. For $n = 5$, CP's estimate of R^2 quadrupoles the estimate from FB's equation.

CP construct their return-forecasting factor by estimating

$$\frac{1}{4}\sum_{n=2}^{5} rx_{t+1}^{(n)} = y_0 + \gamma_1 y_t^{(1)} + \gamma_2 f_t^{(2)} + \ldots \gamma_5 f_t^{(5)} + \bar{\upsilon}_{t+1} = \gamma^{\mathrm{T}} f_t + \bar{\upsilon}_{t+1}. \tag{6}$$

They estimate the equation

$$rx_{t+1}^{(n)} = \varsigma + \lambda\left(\gamma^{\mathrm{T}} f_t\right) + \xi_{t+1}^{(n)}, \tag{7}$$

[2] It is instructive to note that with montly data the one-year excess return on a n-year bond is computed as $rx_{t+12}^{(n)} = p_{t+12}^{(n-1)} - p_t^{(n)} - y_t^{(n)}$. However, for comparability purposes, the notation adopted thoughout the paper follows the one used by CP and FB.

[3] The covariances, for these and all other tests reported in this paper, are estimated using the Newey-West procedure to account of the overlapping data.

Table 1

Estimates of the CP and FB Models, 1964:01–2002:12

Cochrane – Piazzesi Model,								
	$n = 2$		$n = 3$		$n = 4$		$n = 5$	
	Estimated Coefficient	Standard Error	Estimated Coefficient	Standard Error	Estimated Coefficient	Standard Error	Estimated Coefficient	Standard Error
β_0	−1.622	0.525	−2.671	0.980	−3.795	1.353	−4.887	1.706
β_1	−0.982	0.175	−1.781	0.312	−2.570	0.423	−3.208	0.530
β_2	0.592	0.364	0.533	0.638	0.868	0.845	1.241	1.050
β_3	1.214	0.298	3.074	0.538	3.607	0.735	4.108	0.920
β_4	0.288	0.227	0.382	0.421	1.285	0.579	1.250	0.728
β_5	−0.886	0.210	−1.858	0.396	−2.729	0.551	−2.830	0.695
R^2	0.321		0.341		0.371		0.346	
Fama – Bliss Model								
	$n = 2$		$n = 3$		$n = 4$		$n = 5$	
	Estimated Coefficient	Standard Error	Estimated Coefficient	Standard Error	Estimated Coefficient	Standard Error	Estimated Coefficient	Standard Error
α	0.072	0.094	−0.134	0.177	−0.401	0.248	−0.086	0.313
β	0.993	0.106	1.351	0.137	1.612	0.157	1.272	0.193
R^2	0.158		0.174		0.184		0.085	

and find that equation (7) encompasses equation (4). Estimates of equation (7) are presented in Table 2. The encompassing power of the return-forecasting factor is reflected by comparing the estimates of R^2 from equation (7) in Table 2 with those from equation (4) shown in Table 1. The estimates from the two equations are nearly identical. CP conclude that the "single factor explains over 99.5 percent of the variance of expected excess returns" (p. 139).

Table 2

Cochrane – Piazzesi Factor Model, Sample Period 1964:01–2002:12

	$n = 2$		$n = 3$		$n = 4$		$n = 5$	
	Coef.	s. e.	Coef.	s. e.	Coef.	s. e.	Coef.	s. e.
ς	0.125	0.154	0.112	0.277	−0.007	0.367	−0.229	0.446
λ	0.449	0.047	0.852	0.088	1.236	0.122	1.463	0.156
$\overline{R}^2$	0.314		0.337		0.370		0.345	

III. The Predictability of Excess Returns

Figure 1 shows the time series of the five bond prices CP use. It is obvious that these bond prices are highly serially correlated and cross correlated. Table 3 shows the cross correlation, serial correlation, and percent of variance accounted for by each of the five principal components. The five bond prices are highly serially and cross correlated. Importantly, the first principal component accounts for nearly 99 percent of the generalized variance. This is a problem because CP create nine variables, the 1-year yield, four forward rates, and four excess returns, from these five bond prices; they create nine variables out of essentially one independent piece of information. Consequently, it is little wonder that excess returns and forward rates are highly correlated.

Indeed, the high estimates of R^2 that they obtain is entirely due to the high degree of correlation among the five bond prices. This can be shown by rewriting equation (4) in terms of the five bond prices used to construct bond yields, forward rates, and excess returns, i. e.,

$$\begin{aligned} p_{t+1}^{(n-1)} - p_t^{(n)} + p_t^{(1)} = \beta_0 + \beta_1\left(-p_t^{(1)}\right) + \beta_2\left(-p_t^{(1)} - p_t^{(2)}\right) + \ldots \\ + \beta_5\left(-p_t^{(4)} - p_t^{(5)}\right) + \varepsilon_{t+1}^{(n)}. \end{aligned} \tag{8}$$

Because $-p_t^{(n-1)}$ and $p_t^{(1)}$ are on both the left- and right-hand sides of equation (8), it can be written solely in terms of $p_{t+1}^{(n-1)}$. This is easily seen when

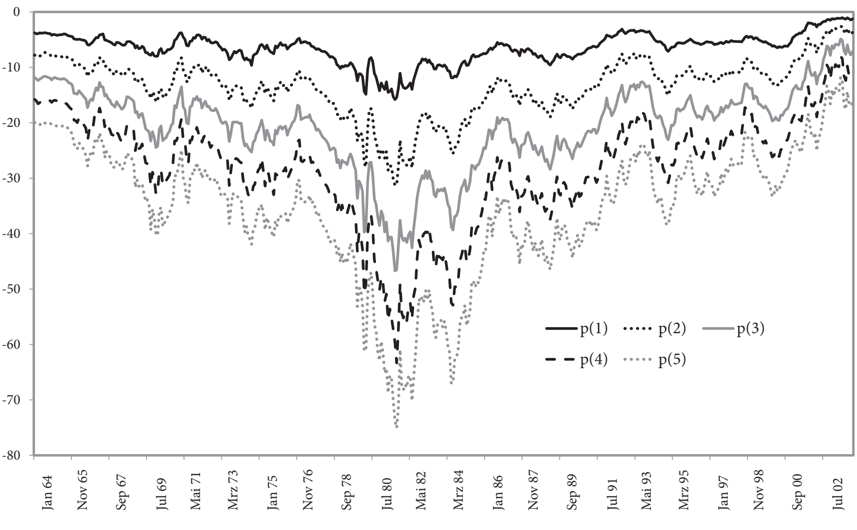

Figure 1: Log of Bond Prices

Table 3

Correlations Among Bond Prices

	$p^{(1)}$	$p^{(2)}$	$p^{(3)}$	$p^{(4)}$	$p^{(5)}$
	Cross Correlation				
$p^{(1)}$	1.0000	0.9928	0.9824	0.9713	0.9625
$p^{(2)}$	0.9928	1.0000	0.9967	0.9910	0.9859
$p^{(3)}$	0.9824	0.9967	1.0000	0.9979	0.9950
$p^{(4)}$	0.9713	0.9910	0.9979	1.0000	0.9986
$p^{(5)}$	0.9625	0.9859	0.9950	0.9986	1.0000
	Serial Correlation				
Correlation	0.9858	0.9885	0.9893	0.9893	.09901
	Principle Components				
Percent of Var.	0.9861	0.0133	0.0003	0.0002	0.0001

$n = 2$. In this case, $\left(p_t^{(1)} - p_t^{(2)}\right)$ appears on both the right- and left-hand sides of equation (8) so it can be written equivalently as

(9) $$p_{t+1}^{(1)} = \beta_0 + \beta_1(-p_t^{(1)}) + (\beta_2 - 1)\left(p_t^{(1)} - p_t^{(2)}\right) + \ldots + \beta_5\left(p_t^{(4)} - p_t^{(5)}\right) + \varepsilon_{t+1}^{(n)}.$$

While less obvious, equation (4) can be written for any value of n as,

(10) $$p_{t+1}^{(n-1)} = \delta_0 + \delta_1 p_t^{(1)} + \delta_2 p_t^{(2)} + \ldots + \delta_5 p_t^{(5)} + \varepsilon_{t+1}^{(n)},$$

where $\delta_1 = (\beta_2 - \beta_1 - 1)$ for all n, $\delta_i = (1 - \beta_i + \beta_{i+1})$ for i equal to n, $\delta_i = (\beta_{i+1} - \beta_i)$ for $i \geq n \neq 5$, $\delta_5 = \beta_5$ for $n \neq 5$ and $\delta_5 = (1 - \beta_5)$ for $n = 5$. This establishes the econometric equivalence of equations (4) and (10). The error term from both equations is measured in terms of bond prices, not excess returns. The estimate of R^2 that CP report is merely the sum of squared errors from equation (10) relative to the total sum of squares of excess returns. As such, it provides no information about the in-sample predictability of excess returns *per se*.

Equation (4) can also be expressed equivalently in terms of bond yields. This can be seen by multiplying both sides of the equation (10) by $-(1/n - 1)$, to obtain

$$y_{t+1}^{(n-1)} = \tau_0 + \delta_1((1/n-1)y_t^{(1)}) + \delta_2((2/n-1)y_t^{(2)}) + \ldots + \delta_5((5/n-1)y_t^{(5)}) + (-1/(n-1))\varepsilon_{t+1}^{(n)},$$

which can be written more compactly as

$$(11) \qquad y_{t+1}^{(n-1)} = \tau_0 + \tau_1 y_t^{(1)} + \tau_2 y_t^{(2)} + \ldots + \tau_5 y_t^{(5)} + (-1/(n-1))\varepsilon_{t+1}^{(n)}.$$

Note that the error term of equation (11) is merely the error term of equation (4) or (10) expressed in terms of bond yields rather than bond prices.[4] Nevertheless, equations (4), (10) and (11) are econometrically equivalent – no information can be obtained from any one of these equations that cannot be obtained from the others. Moreover, it means that in spite of CP's claim that "we're forecasting one-year excess returns, and not the spot rates" (p.140), they are predicting spot rates; their conclusions about excess returns depend solely on the model's ability to explain the future spot price (or equivalently, the future yield).

An analogous econometric equivalence result holds for FB's excess return model. Specifically, equation (5) is econometrically equivalent to

$$(12) \qquad p_{t+1}^{(n-1)} - p_t^{(n-1)} = \alpha' + \theta\left(f_t^{(n)} - y_t^{(1)}\right) + \upsilon_{t+1}^{(n)},$$

or, equivalently, in terms of bond yields,

$$(13) \qquad y_{t+1}^{(n-1)} - y_t^{(n-1)} = \alpha' + \theta\left(f_t^{(n)} - y_t^{(1)}\right) + (-1/(n-1)) + \upsilon_{t+1}^{(n)}.$$ [5]

The error term in equation (13) is merely the error term from either equation (5) or equation (12) expressed in terms of the change in bond yields, rather than the change in bond prices. Equation (5) econometrically equivalent to equations (12) or (13).

The fact that CP's and FB's models are econometrically equivalent to models of bond prices (or bond yields), means that neither CP's nor FB's model pro-

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[4] This can be done because the total sum of squares of excess returns is also measured in term of bond prices: Note that $rx_{t+1}^{(n)} \equiv p_{t+1}^{(n-1)} - p_t^{(n)} - y_t^{(1)}$ can be written as $rx_{t+1}^{(n)} \equiv p_{t+1}^{(n-1)} - p_t^{(n)} + p_t^{(1)}$. It turns out that the total sum of squares of excess returns is larger than that for $p_{t+1}^{(n-1)}$, otherwise, the estimated R^2 would have been greater than 1.

[5] FB are aware of this. Indeed, they begin their analysis by writing their model as shown in equation (14), noting that "evidence that b_1 (θ in equation 14) is greater that 0.0 implies that the forward-spot spread observed at time t has power to forecast the changes in the 1-year spot rate" (p. 682). They then note that equation (14) is "complimentary" to equation (5) and present estimates of equation (5). What they fail to note is that equations (13) or (14) explains almost none of the variation of changes in bond prices or bond yields.

vides informations about excess returns: CP's model provides information about excess returns only to the extent that it provides information about future bond prices. FB's model provides information about excess returns only to the extent that it provides information about changes in the bond's price.

The surprising thing is that FB's model explains virtually none of the change in bond prices (or, equivalently, changes in bond yields). The estimates of R^2 from equation (12) (or equation 13) are 0.000, 0.014, 0.031, and 0.004, for $n = 2$, 3, 4, 5, respectively.

The intriguing question is: How can the residuals from a model that has no explanatory power for the change in bond prices generate estimates of R^2 in terms of excess returns ranging from 0.085 to 0.184? Not surprisingly, the answer lies in the fact that these bond prices are highly correlated. Specifically, the answer lies in the fact that $rx_{t+1}^{(n)}$ and $\left(p_{t+1}^{(n-1)} - p_t^{(n-1)}\right)$ are highly correlated; the correlations range from 83.8 percent to 91.8 percent for the four excess returns. Of course, the high correlation is due to the fact that $p_t^{(n)} \approx p_t^{(n-1)}$, so that, $\left(p_{t+1}^{(n-1)} - p_t^{(n)} - y_t^{(1)}\right) \approx \left(p_{t+1}^{(n-1)} - p_t^{(n-1)} - y_t^{(1)}\right)$, so that $rx_{t+1}^{(n)}$ is highly correlated with $\left(p_{t+1}^{(n-1)} - p_t^{(n-1)}\right)$. In the case of FB's model, the high correlation between $rx_{t+1}^{(n)}$ and $\left(p_{t+1}^{(n-1)} - p_t^{(n-1)}\right)$ accounts for essentially all of the estimates of R^2 reported in Table 1.[6]

The high degree of correlation between $rx_{t+1}^{(n)}$ and $\left(p_{t+1}^{(n-1)} - p_t^{(n-1)}\right)$ also accounts for a significant percentage of the estimates of R^2 from CP's model. To understand why, consider a simple AR(1) model of bond prices, i. e.,

$$p_{t+1}^{(n-1)} = \phi_0 + \phi_1 p_t^{(n-1)} + \eta_{t+1}^{(n-1)}. \tag{14}$$

Equation (14) is nothing more than a restricted version of equation (10). If equation (14) provides no information about the future bond price beyond its current level, i. e., $\phi_0 = 0$ and $\phi_1 = 1$, the residuals from equation (14) would be $\left(p_{t+1}^{(n-1)} - p_t^{(n-1)}\right)$. This means that estimates of equation (4) would generate relatively high estimates of R^2 in terms of excess returns even though bond prices (or, equivalently, bond yields) could not be predicted beyond their current level. Consequently, the estimates of R^2 from CP's model can be partitioned into three source; the estimate of R^2 that would occur if CP's equation had no predictability for bond prices, the R^2 due to the serial correlation of bond prices – the estimates from the AR(1) model, and the R^2 associated with cross correlation of bond prices – the estimate of R^2 obtained from equation (4).

[6] Fichtner and Santa-Clara (2012) note that the FB model generates estimates of R^2 up to 15 percent despite the fact that it performs no better than the random walk model; however, they fail to undetstand the source of the anomaly.

Table 4

Sources of In-Sample Fit of Cochrane and Piazzesi's Model

	$n = 2$	$n = 3$	$n = 4$	$n = 5$
No Predictability	0.158	0.162	0.157	0.082
AR(1) Model	0.223	0.219	0.212	0.142
CP model Equation (4)	0.321	0.341	0.371	0.346

Table 4 reports the estimates of R^2 from these three sources. For $n = 2$, 3, and 4 the no-predictability model accounts for about half of the estimates of R^2 from equation (4). For $n = 5$, no-predictability model accounts for about 25 percent of CP's estimate of R^2. The estimates of R^2 are increased if bond prices are modeled as a simple AR(1) process; however, the percentage increases are relatively modest. This is not surprising because it is widely known that bond prices are well approximated by an I(1) process. The remaining increases are due to the cross correlation of bond prices. The percentage increase in the estimates of R^2 due to the cross correlation of bond prices is much larger than the marginal contribution due to serial correlation. Moreover, the marginal contribution increases monotonically with n, ranging from about 10 percentage points for $n = 2$ to 20 percentage points for $n = 5$.

The question of which bond prices contribute most to the increase in the estimates of R^2 is answered by regressing p_{t+1}^{n-1} on all possible combinations of the five bond prices and calculating the R^2 in terms of excess returns. These estimates, presented in Table 5. While bond prices across the entire term structure

Table 5

Estimates of for All Possible Combinations of Bond Prices

xr_{t+1}^2		xr_{t+1}^3		xr_{t+1}^4		xr_{t+1}^5	
Maturity combination	R^2	Maturity combination	R^2	Maturity combination	R^2	Maturity combination	R^2
AR(1)	0.223	AR(1)	0.219	AR(1)	0.212	AR(1)	0.142
1,2	0.228	2,1	0.244	3,1	0.270	4,1	0.219
1,3	0.236	2,3	0.258	3,2	0.279	4,2	0.222
1,4	0.237	2,4	0.255	3,4	0.244	4,3	0.199

xr^2_{t+1}		xr^3_{t+1}		xr^4_{t+1}		xr^5_{t+1}	
Maturity combination	R^2	Maturity combination	R^2	Maturity combination	R^2	Maturity combination	R^2
1,5	0.230	2,5	0.234	3,5	0.219	4,5	0.148
1,2,3	0.257	2,1,3	0.258	3,1,2	0.279	4,1,2	0.223
1,2,4	0.254	2,1,4	0.255	3,1,4	0.271	4,1,3	0.219
1,2,5	0.230	2,1,5	0.244	3,1,5	0.320	4,1,5	0.332
1,3,4	0.237	2,3,4	0.259	3,2,4	0.279	4,2,3	0.224
1,3,5	0.258	2,3,5	0.287	3,2,5	0.315	4,2,5	0.321
1,4,5	0.295	2,4,5	0.323	3,4,5	0.275	4,3,5	0.249
1,2,3,4	0.259	2,1,3,4	0.259	3,1,2,4	0.279	4,1,2,3	0.225
1,2,3,5	0.280	2,1,3,5	0.296	3,1,2,5	0.330	4,1,2,5	0.333
1,2,4,5	0.309	2,1,4,5	0.328	3,1,4,5	0.360	4,1,3,5	0.338
1,3,4,5	0.296	2,3,4,5	0.335	3,2,4,5	0.362	4,2,3,5	0.333
CP Model	0.321	CP Model	0.341	CP Model	0.371	CP Model	0.346

appear to make an important contribution to the estimates of R^2 reported by CP, long-term prices seem to be more important. For all maturities, the estimates are very close to those of CP's model, only if four of the five bond prices are included. For $n = 2$ or 3, the estimates get close to those of CP's model only when p_t^4 and p_t^5 are included. For $n = 4$ or 5 the estimates of R^2 gets close to those of CP's model when p_t^1 and p_t^5 or p_t^2 and p_t^5 are included. Hence, long-term prices are relatively more important than short-term prices.

IV. CP's Other Findings

1. The Encompassing Power of CP's Factor

The analysis in the previous section shows that the relatively large estimates of R^2 that CP obtain are due to the fact that bond prices (or yields) are very persistent and highly cross correlated. It is perhaps not surprising to find that the high degree of serial and cross correlation in bond prices also accounts for the encompassing power of CP's return-forecasting factor. To see this, it is useful to note that CP's return-forecasting factor can also be expressed solely in terms of bond prices. Specifically, equation (6) is econometrically equivalent to

$$(15)\quad \frac{1}{4}\left(p_{t+1}^{4}+p_{t+1}^{3}+p_{t+1}^{2}+p_{t+1}^{1}\right)=\gamma_0+\phi_1 p_t^{(1)}+\phi_2 p_t^{(2)}+\ldots+\phi_5 p_t^{(5)}+\bar{\upsilon}_{t+1}.$$

The return-forecasting factor also can be expressed equivalently in terms of bond yields. Specifically,

$$(16)\quad \begin{aligned}\left(y_{t+1}^{4}+.75\,y_{t+1}^{3}+.5\,y_{t+1}^{2}+.25\,y_{t+1}^{1}\right)=-\gamma_0+\psi_1 y_t^{(1)}+\psi_2 y_t^{(2)}+\ldots\\ +\psi_5 y_t^{(5)}-\bar{\upsilon}_{t+1}.\end{aligned}$$

Note that the error terms in equations (6), (15) and (16) are identical except that the sign in (16) is negative. Also note that the equation (16) is econometrically equivalent to equation (6) only for this particular weighted sum of bond yields. For example, it would not hold if the left hand side of equation (16) was the simple average of the four bond yields. However, because of the high correlation among bond yields, the weighted sum of bond yields on the left-hand-side of (16) and the simple average of bond yields are very highly correlated. 0.9988. Hence, there is a correspondingly high degree of correlation between the residuals from equation (16) and the residuals from a model where the left-hand-side of (16) is the simple average of bond yields, 0.9980.

CP's return-forecasting factor is the least squares projection of the average of the future price of the four bonds onto the space spanned by the five bond prices; it is also equivalent to the least squares projection of a particular weighted average of four future bond yields onto the space spanned by the five bond yields. It should also be noted that while these three equations are econometrically equivalent, the return-forecasting factors are expressed in different units of measure: the factor corresponding to equation (6) is expressed in excess returns while the factors corresponding to equations (15) and (16) are expressed in bond prices. This does not negate their econometric equivalence because any of these return-forecasting factors can be express as any other by a simple linear transformation.

Regardless of how the factor is expressed, the encompassing power of the CP factor stems from the fact that the projection of the average of future bond prices is highly correlated with each of the bond prices that make up the average. Of course, the same is true for bond yields. Consequently, models using the factors based on equations (15) or (16) encompass the results given by equations (10) or (11), respectively.

Table 6 shows the estimates of R^2 from equations (10) and (15). As was the case with equations (4) and (7), the estimates of R^2 from the two equations are nearly identical – CP's return-forecasting factor express in bond prices encom-

Table 6

Encompassing Power of the CP Factor in Bond Prices

	Equation (15)		Equation (10)		Equation (15)		Equation (10)	
	coef.	s. e.	coef.	s. e.	coef.	s. e.	coef.	s. e.
	$p_{t+1}^{(1)}$				$p_{t+1}^{(2)}$			
Const.	–1.622	0.275	0.430	0.254	–2.671	0.496	0.505	0.449
$p_t^{(1)}$	0.573	0.374			1.314	0.674		
$p_t^{(2)}$	1.622	0.396			2.541	0.714		
$p_t^{(3)}$	–0.926	0.318			–1.692	0.574		
$p_t^{(4)}$	–1.174	0.223			–2.240	0.402		
$p_t^{(5)}$	0.886	0.136			1.858	0.245		
$CPF^{(p)}$			0.403	0.014			0.811	0.025
$\overline{R}^2$	0.658		0.643		0.702		0.700	
s. e.	1.600		1.627		2.885		2.882	
	$p_{t+1}^{(3)}$				$p_{t+1}^{(4)}$			
Const.	–3.795	0.671	–0.028	0.606	–4.887	0.839	–0.907	0.762
$p_t^{(1)}$	2.438	0.911			3.449	1.140		
$p_t^{(2)}$	2.739	0.965			2.867	1.207		
$p_t^{(3)}$	–2.322	0.776			–2.858	0.970		
$p_t^{(4)}$	–3.013	0.544			–4.081	0.681		
$p_t^{(5)}$	2.729	0.331			3.830	0.414		
$CPF^{(p)}$			1.202	0.033			1.584	0.042
$\overline{R}^2$	0.738		0.738		0.758		0.755	
s. e.	3.902		3.886		4.880		4.886	

passes CP's excess return model expressed in bond prices; CP's return-forecasting factor expressed in bond yields encompasses CP's excess return model expressed expressed in bond yields. This demonstrates that the compassing power of the return-forecasting factor solely due to the serial and cross correlation of bond prices.

2. *Predicting Bond Yields*

CP note that their return-forecasting factor significantly improves predictability of yields relative to the standard 3-factor term structure model that uses the level, slope, and curvature factors. They note that this occurs despite the fact that the return-forecasting factor "does little to improve the model's fit for yields" (p. 139). Specifically, they note that the five principal components of bond yields "explain in turn 98.6, 1.4, 0.03, 0.02, and 0.01 percent of variance of yields," but explain quite different fractions of the variance of their return-forecasting factor, 9.1, 58.7, 7.6, 24.3, and 0.3 percent, respectively. They suggest that "24.3 means that the fourth factor, which loads heavily on the four- to five-year yield spread and is essentially unimportant for explaining the variation of *yields*, turns out to be very important for explaining *expected returns*" (p. 147, italics in the original). As noted above, these differences in explanatory power are due to the fact that their return-forecasting factor is expressed in excess returns while the principal components are expressed in bond yields, and by the fact that long-term yields are relatively important.

Had CP expressed both in the same units of measure, which they could have easily done because of the econometric equivalence shown above, the reason for the marked increase in explanatory power of the return-forecasting factor would have been obvious. The return-forecasting factor improves the predictability of the standard 3-factor term structure model because the fourth principal component of bond yields is relatively important for the in-sample fit of bond yields across the term structure even though it only accounts for 0.02 percent of the generalized variance of bond yields.

Note that because equation (4) is really a equation for predicting future bond prices or yields, it is equivalent to

(17) $$rx_{t+1}^{(n)} = \kappa_0 + \kappa_1 pc_t^1 + \kappa_2 pc_t^2 + \ldots + \kappa_5 pc_t^5 + \varepsilon_{t+1}^{(n)},$$

where $pc_t^{(i)}$ denotes the i^{th} principal component based on the five bond yields. That is, equation (17) is econometrically equivalent to

(18) $$y_{t+1}^{(n-1)} = \theta_0 + \theta_1 pc_t^1 + \theta_2 pc_t^2 + \ldots + \theta_5 pc_t^5 + (-1/(n-1))\varepsilon_{t+1}^{(n)}.$$

The estimates R^2 from equations (17) and (18) are identical when expressed in terms of excess reserves. However, that the observational equivalence holds only for the unrestricted equations. For example, if the restriction $\kappa_5 = \phi_5 = 0$ is imposed, the R^2 from equation (18), expressed in excess returns, would not be equal to that obtained from equation (17). The reason, of course, is principal components are are not simple linear combinations of the five bond yields. Nevertheless, the estimates are very close even if this restrictions is imposed. With this restriction, the estimate of R^2 from equation (17) is 0.3456, while that based on equation (18) is only a tiny bit smaller, 0.3455. However, if only the first principal component is included, the estimates are 0.0232 and 0.2067, respectively. The marked difference when only the first principal component is included stems from the fact that the level factor is essentially uncorrelated with excess returns, but is highly correlated with future bond yields. This is also why this estimate, 0.2067, is somewhat higher than the estimate based on an AR(1) model reported in Table 3, 0.142.

The return-forecasting factor reflects information in all five bond prices (and correspondingly bond yields), so including it in a standard 3-factor model of bond yields naturally increases the in-sample fit for bond yields and, consequently, the estimates of R^2 expressed in excess returns. But this is an artifact of the results in Table 5; namely, the estimates of R^2 are higher when longer-term prices, $p^{(4)}$ and/or $p^{(5)}$, or long-term yields, are included. This fact also accounts for CP's finding (p. 139) that equation (7) is rejected relative to equation (4) for all values of n, in spite of the fact that the return-forecasting factor encompasses their model.

Whether at least four of the five bond yields are important for predicting future bond yields can investigated by estimating

$$y_{t+1}^{(n)} = \varsigma_0 + \varsigma_1 y_t^{(1)} + \varsigma_1 y_t^{(2)} \ldots + \varsigma_1 y_t^{(5)} + \omega_{t+1}^{(n)}, \quad n = 1, 2, \ldots 5 \tag{19}$$

and testing the restriction $\varsigma_j = 0$ for each value of n for each maturity.

The chi-square statistics and corresponding p-values are reported in Table 7. The column headings denote the maturity of the dependent variable and the rows denote the omitted yield. With exception of $y_t^{(3)}$ (where all of the tests are

Table 7

Tests of Bond Yield Restrictions

	$y_t^{(1)}$		$y_t^{(2)}$		$y_t^{(3)}$		$y_t^{(4)}$		$y_t^{(5)}$	
	χ^2	p-value	χ^2	p-value	χ^2	p-value	χ^2	p-value	χ^2	p-value
$y_{t+1}^{(1)}$	1.272	0.259	2.162	0.141	4.189	0.041	5.405	0.020	6.470	0.011
$y_{t+1}^{(2)}$	8.568	0.003	6.901	0.009	4.608	0.032	3.297	0.069	2.510	0.113
$y_{t+1}^{(3)}$	4.760	0.029	4.690	0.030	4.578	0.032	4.494	0.034	5.095	0.024
$y_{t+1}^{(4)}$	12.157	0.000	12.665	0.000	12.227	0.001	14.048	0.000	12.906	0.000
$y_{t+1}^{(5)}$	17.877	0.000	21.962	0.000	24.565	0.000	30.354	0.000	31.179	0.000

rejected at least the 5 percent significance level), four the five bond yields are necessary. Moreover, for the remaining four bond yields, the results are consistent with those presented in Table 5. Specifically, long-term yields are more important than short-term yields. It is always the case that the restriction on 1-year or 2-year yields is not rejected. Furthermore, the 4- and 5-year yields are relatively important for predicting all five yields. Indeed, this accounts for PC's finding that long-term forward rates add significantly to the predictability of excess returns on short-term bonds. The critical question is not why is the return factor important for predicting bond yields across the term structure. The critical questions are: Why are four of the five bond yields (or nearly equivalently, the first four of the five principal components) necessary for the in-sample fit of future bond yields across the term structure and why are long-term yields relatively more important than short-term yields?

3. *Robustness Check*

This section preforms a robustness check on the results presented in the previous section. Specifically, the sample period is extended to June 2007 and zero coupon bond yields from 1 to 10 years are used.[7] The ten principal components were calculated and the equation

$$(20) \qquad y_{t+1}^{(n)} = \vartheta_0 + \vartheta_1 pc_t^{(1)} + \vartheta_2 pc_t^{(2)} + \ldots + \vartheta_{10} pc_t^{(10)} + \xi_{t+1}^{(n)}, \quad n = 1, 2, \ldots, 10$$

was estimated. The restrictions $\vartheta_{10} = 0$, $\vartheta_{10} = \vartheta_9 = 0$, $\vartheta_{10} = \vartheta_9 = \vartheta_8 = 0$, and so on, are tested sequentially until the null hypothesis was rejected at the 5 percent significance level. For bond yields with maturities from 1 to 4 years, the null hypothesis was rejected when the last four principal components were deleted – six principal components were necessary. For bonds yields with maturities of 5-years or longer the null hypothesis was rejected when the last 6 principal components were deleted – four principal components were necessary. Hence, the previously reported results appear to be robust. Despite the widespread use of 3-factor models, at least four factors are required for predicting bond yields in sample for longer-term yields and more than four factors appear to be necessary for the in-sample predictability of shorter-term yields. Also, long-term yields are relatively more important than short-term yields.

[7] These data are available on FRED and are due to Gurkaynak et al., (2006).

V. Conclusions and Implications

The paper contributes to the literature on forecasting excess returns by showing that all of the estimates of R^2 from FB's excess return model and about half of the estimates of R^2 from CP's model are accounted for by the high correlation between excess returns and the change in bond prices. In the case of CP's model, the estimates of R^2 are also due to the high degree of cross correlation among bond prices. The high degree of cross correlation of bond prices also explains why CP's return-forecasting factor encompasses their model, why their return-forecasting factor significantly improves the in-sample fit relative to a standard three-factor term structure model, and why long-term forward rates add significantly to the predictability of excess returns on short-term bonds.

However, answers to these questions raises others: Specifically, why are four bond yields (or nearly equivalently, the first four principal components) necessary for the in-sample predictability of bond yields? Why do long-term bond yields improve the in-sample fit for both short- and long-term bond yields more than short-term bond yields? Are the answers to these questions related? Thornton (2006) has shown that correlation between future short-term rates and current long-term rates is a necessary, but not sufficient condition, for the expectation hypothesis to hold. Hence, the expectations hypothesis could account for the importance of long-term yields for the in-sample predictability of short-term yields. But the expectations hypothesis is massively rejected using a variety of rates, sample periods, and tests (e.g., Mankiw and Miron, 1986; Campbell and Shiller, 1991; Roberds et al., 1996; Kool and Thornton, 2004; Thornton, 2005; Sarno et al., 2007; and Della Corte et al., 2008). Moreover, the importance of long-term yields is also consistent with the standard classical theory of interest rate, which asserted that the long-term rate is determined by economic fundamentals and that short-term rate is anchored to the long-term rate (Thornton, 2014, 2016).

These results have implications for other excess return models, e.g., Huang and Shi, 2012; Ludvigson, and Ng, 2009; Wright and Zhou, 2009; Radwanski, 2010; Greenwood and Vayanos, 2014; and Hamilton and Wu, 2012. Given that $rx_{t+1}^{(n)} \equiv p_{t+1}^{(n-1)} - p_t^{(n)} - y_t^{(1)} \approx p_{t+1}^{(n-1)} - p_t^{(n-1)}$, the results presented here raise questions about how much of the estimates of R^2 in these models is due to the high correlation between the independent variables in these models and bond prices rather than excess returns *per se*.

The answer to this question is relatively unimportant becasue it is well known that in-sample fit is a poor indicator of out-of-sample forecasting performance (Inoue and Kilian 2004, 2006). But it is impossible to forecast excesss returns *per se* out-of-sample because forecasts of excess returns are necessarily forecasts of future bond prices (or equivalently bond yields). The reason is $p_{t+1}^{(n-1)}$ is the

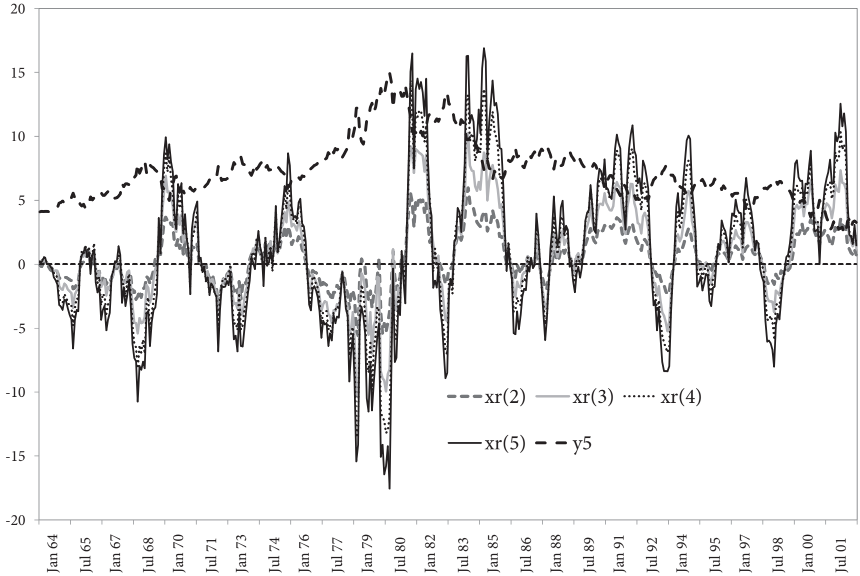

Figure 2: Excess Returns and the 5-Year Bond Yield

only variable at in unknown when the forecast is made. $p_t^{(n)}$ and $y_t^{(1)}$ are known, so they cannot be forecasted. Hence, excess return models are only useful for forecasting excess returns if they can forecast future bond prices. None of these models is useful unless they can forecast future bond prices out-of-sample. Moreover, the out-of-sample forecasting performance of excess return models can and should be compared with models that are specifically designed to forecast bond yields (e.g., Diebold and Rudebusch, 2013).[8]

The equivalence between forecasting excess returns and bond prices (or, equivalently, bond yields) has two other important implications. First, it creates a corresponding equivalence between forecasting excess returns and market efficiency. If markets are efficient (or market participants have rational expectations) past information will be useless or forecasting future bond prices; future bond prices, and, consequently, excess returns, will be unforecastable.

If bond yields are unpredictable, we should expect to find that excess returns will be negative when bond yields are rising and positive when bond yields are falling. Figure 2 shows the four excess returns used here and the corresponding 5-year bond yield, $y_{t+1}^{(5)}$. Consistent with a large body of evidence that bonds

[8] Thornton and Valente (2012) find that neither CP's nor FB's models provide forecasts that can generate economic value.

yields are very difficult to predict beyond their current level, the figure shows a strong inverse relationship between bond yields and excess returns. During the period for January 1964 to August 1981, a period when bond yields were generally rising, excess returns were negative. During the period from the mid-1970s to August 1981, when bond yields rose rapidly and by a large amount, excess returns were decidedly negative. During the period after August 1981, when bond yields were generally falling, excess returns were more often than not positive; they were particularly large during periods when yields fell quickly and by large amounts. They were negative only during short periods when bond yields were rising. This fact largely explains FB's observation that excess returns were procyclical "mostly positive during good time and mostly negative during recessions."

The second implication is related to CP's claim (pp. 144–45) that their finding were a refutation of the expectations hypothesis. If the expectations hypothesis is valid, excess returns cannot be predictable, but this does not mean that bond prices cannot be predictable. It only means they cannot be predictable beyond $p_t^{(n-1)} + y_t^{(1)}$. However, if bond markets are efficient, bond yields will be unpredictable and, so too, will be excess returns. But this does not unnecessarily imply that the expectations hypothesis is valid since this is merely a consequence of market efficiency. On the contrary, if markets are efficient, it is difficult to believe that expectations hypothesis is valid because it is difficult to understand why rational market participants would price long-term bonds based on their expectation (forecast) of the future yield on short-term bonds that they know they cannot forecast. Hence, at a minimum, the unforecastability of bond yields should raise concern about the validity of the expectations hypothesis. But it most likely won't because the expectations hypothesis is strongly entrenched in economics and finance in spite of the fact that it has been massively rejected over different sample periods, using different interest rates, and over different monetary policy regimes.

References

Buraschi, A./*Whelan*, P. (2012): "Term Structure Models with Differences in Beliefs," unpublished manuscript.

Campbell, J./*Shiller*, R. (1991): "Yield Spreads and Interest Rate Movements: A Bird's Eye View," *Review of Economic Studies*, 58, 495–514.

Cieslak, A/*Povala*, P. (2011): "Understanding Bond Risk Premia," unpublished manuscript.

Cochrane, J. H./*Piazzesi*, M. (2005): "Bond Risk *Premia*," *American Economic Review*, 95, 138–60.

– (2004): "Appendix to "Bond Risk Premia," unpublished manuscript.

Della Corte, P./*Sarno*, L./*Thornton*, D. (2008): “The Expectations Hypothesis of the Term Structure of Very Short-Term Rates: Statistical Tests and Economic Value,” *Journal of Financial Economics,* 89, 158–174.

Diebold, F. X./*Rudebusch*, G. D. (2013): *Yield Curve Modeling and Forecasting: The Dynamic Nelson-Siegel Approach,* Princeton University Press: Princeton, N. J.

Duffee, G. (2007): “Are variations in term premia related to the macroeconomy?” unpublished manuscript.

– (2011): “Information in (and not in) the term structure,” *Review of Financial Studies,* 24, 2895–2934.

Fama, E. F. (1984): “Term Premiums in Bond Returns,” *Journal of Financial Economics,* 13, 529–46.

– (1986): “Term Premiums and Default Premiums in Money Markets,” *Journal of Financial Economics,* 17, 175–96.

Fama, E. F./*Bliss*, R. R. (1987): “The Information in Long-Maturity Forward Rates,” *American Economic Review, 7,* 680–92.

Fichtner, L. P./*Santa-Clara*, P. (2012): “Out-of-Sample Predictability of Bond Returns,” unpublished manuscript.

Greenwood, R./*Vayanos*, D. (2014): “Bond Supply and Excess Bond Returns,” *Review of Financial Studies* 27(3), 663–713.

Gurkaynak, R. S./*Sack*, B./*Wright*, J. H. (2006): “The U.S. Tresuary Yield Curve: 1961 to the Present,” Finance and Economics Discussion Series, Board of Governors of the Federal Reserve System, 2006-28.

Hamilton, J. D./*Wu*, C. (2012): “The Effectiveness of Alternative Monetary Policy Tools in a Zero Lower Bound Environment,” *Journal of Money, Credit, and Banking, 44*(1), 3–46.

Huang, J-z/*Shi*, Z.: “Determinants of Bond Risk Premia,” unpublished manuscript.

Inoue, A./*Kilian*, L. (2004): “In-Sample or Out-of-Sample Tests of Predictability: Which One Should We Use?” *Econometric Reviews 23,* 371–402.

– (2006): “On the Selection of Forecasting Models,” *Journal of Econometrics 130*, 273–306.

Kool, C. J. M./*Thornton*, D. L. (2004): “A Note On The Expectations Hypothesis At The Founding Of The Fed,” *Journal of Banking and Finance,* 28(12), pp. 3055–68.

Ludvigson, S. C./*Ng*, S. (2009): “Macro Factors in Bond Risk Premia,” *Review of Financial Studies, 29*(12), 5027-67.

Mankiw, N. G./*Miron*, J. A. (1986): “The changing behavior of the term structure of interest rates,” *Quarterly Journal of Economics,* 101, 211–228.

Roberds, W./*Runkle*, D./*Whiteman*, C. (1996): “A Daily View of Yield Spreads and Short-Term Interest Rate Movements,” *Journal of Money, Credit and Banking,* 28, 34–53.

Sarno, L./*Thornton*, D./*Valente*, G. (2007): “The Empirical Failure of the Expectations Hypothesis of the Term Structure of Bond Yields,” *Journal of Financial and Quantitative Analysis,* 42, 81–100.

Thornton, D. L. (2005): "Test Of The Expectations Hypothesis: Resolving The Anomalies When The Short-Term Rate Is The Federal Funds Rate," *Journal of Banking and Finance,* 29(10), pp. 2541–56.

– (2006): "Tests of the Expectations Hypothesis: Resolving the Campbell-Shiller Paradox," *Journal of Money, Credit and Banking,* 38(2), 511-42.

– (2014): "Monetary Policy: Why Money Matters and Interest Rates Don't," *Journal of Macroeconomics,* 40 (June), 202-13.

– (2015). "Greenspan's Conundrum and the Fed's Ability to Affect Long-Term Yields," unpublished manuscript.

Thornton, D. L./*Valente*, G. (2012): "Out-of-Sample Predictions Of Bond Excess Returns With Forward Rates: An Asset-Allocation Perspective," *Review of Financial Studies,* 25 (10), pp. 3141–3168.

Wright, J. H./*Zhou*, H. (2009): "Bond risk premia and realized jump risk," *Journal of Banking and Finance, 33*, 2333–2345.