Monetary Policy and Asset Price Bubbles:
A Nonlinear Policy Rule

Alexander Mislin*

Abstract

The recent debate about asset price bubbles and monetary policy in view of "leaning against the wind" is controversial in economic literature. In this paper we argue that depending on the circumstances there is space for the central bank to lean against financial imbalances. With an optimal bounded control problem in continuous time we have developed an augmented nonlinear Taylor Rule. The main advantage of our formulation is its much greater analytical tractability, which produces distinct results. Even under the assumption of nonlinearities the central bank is in a position to move the interest rate above a threshold value to provide financial stability.

Geldpolitik und Vermögenspreisblasen:
Eine nichtlineare Politikregel

Zusammenfassung

Die jüngste Debatte um das Konzept des Gegensteuerns („leaning against the wind")
im Angesicht von Vermögenspreisblasen ist höchst kontrovers in der ökonomischen Literatur. In der vorliegenden Arbeit wird argumentiert, dass die Notenbank unter bestimmten Bedingungen einen Spielraum hat mit einer Politik des Gegensteuerns finanzielle Ungleichgewichte abzubauen. Mit einem optimalen Kontrollproblem in stetiger Zeit wird eine erweiterte nichtlineare Taylorregel ermittelt. Einer der wesentlichen Vorteile dieser Herangehensweise ist die analytische Nutzbarkeit, die differenzierte Ergebnisse erlaubt. Sogar unter der Annahme von Nichtlineartäten ist die Notenbank in der Lage den Zins oberhalb eines Schwellenwertes zu setzen, um Finanzstabilität zu gewährleisten.

Keywords: monetary policy, financial stability

JEL Classification: E52, E58.

---

* Dr. Alexander Mislin, Federal Ministry of Finance, Wilhelmstraße 97, 10117 Berlin, Germany, currently delegated to the German Bundestag, Platz der Republik 1, 11011 Berlin, Germany, E-Mail: mislin@zedat.fu-berlin.de.

I. Introduction

Recent research on asset price bubbles in a monetary policy strategy has fo-cused on the central bank's response to asset price bubbles. This reflects the great importance of misalignments in asset prices, which could end in a bubble. In particular, several experiences in Japan (1985–1992), but also in other coun-tries like Sweden (1984–1994), United Kingdom (1985–1992) and not least the financial crisis of 2008/2009, have shown an inflation of asset prices, because the monetary conditions were too lax and thereby led to a credit boom. In the case of Japan, there was an increase in both asset price inflation and inflation. A burst of a bubble could shatter the stability of the financial system. Moreover, it could sink the economy into deflation combined with sustainable stagnation. The main problem for a monetary authority is to identify bubbles. *Bernanke/Gertler* (1999) and *Meltzer* (2002) consider this problem. The monetary author-ity is not in a position to react to asset price bubbles. *Svensson* (2014) considers a policy of "leaning against the wind" completely inappropriate in Sweden to assure financial stability. The inevitable requirements for such a policy vary widely from country to country. A good and clear study of literature on the re-cent monetary policy and financial stability debate is provided by *Smets* (2014). He makes a valid point when he gives a clear ordering of the objectives of mon-etary policy. A macroprudential policy framework should be the right tool for handling financial stability. Price stability remains the primary objective of monetary policy and financial stability is not merely a by-product but rather the cornerstone for a central bank. *Woodford* (2012), *Semmler/Zhang* (2007), *Bor-do/Jeanne* (2002) and *Borio/Lowe* (2002) provide useful theoretic insights into the relationship between monetary policy, asset prices and bubbles. *Plosser* (2014) emphasizes that policy rules themselves offer a lack of transparency, be-cause the qualitative policy results are based on the underlying model. Never-theless, they are quite helpful instruments insofar as they could be a part of a whole communication strategy. Today, the question is what the best framework for a monetary authority is when handling financial imbalances.

The conventional wisdom of "leaning against the wind" is controversial in economic literature. *Galí* (2014) found no evidence in his study supporting the effects of monetary policy on rational asset price bubbles when taking the "con-ventional view". He develops a simple partial equilibrium model and shows that the connection between the interest rate and the component which drives the bubble innovation is based on an assumption. The basis for our argumentation is the relationship between aggregate demand and asset prices which is suffi-ciently discussed in economic theory. For example, *Mishkin* (2001) analyses sev-eral channels (including the balance-sheet channel), which underpin the rela-tionship between monetary policy and asset prices. *Angeloni/Faia* (2013) ana-lyze optimal policy in a model with a risk-taking channel. *Adrian/Shin* (2009)

assign monetary policy a prominent role in conducting financial stability. The balance-sheet channel offers sufficient space for the monetary authority to coordinate both policies, i. e. monetary policy and financial stability. A tightening in interest rates may affect risk taking behavior. Naturally, these remain assumptions when considering monetary policy in terms of asset price bubbles, but we think this is justified by the economic literature mentioned above.

In this paper we argue that depending on the circumstances there is space for the central bank to lean against financial imbalances. The point is to examine, whether the central bank is able to respond to extraordinary financial imbalances, e. g. asset price bubbles, in special circumstances when macroprudential policy fails. With an optimal bounded control problem in continuous time it is possible to develop an augmented nonlinear Taylor Rule. The main advantage of our formulation is its much greater analytical tractability, which produces distinct results. Even under the assumption of nonlinearities the central bank is in a position to move the interest rate above a threshold value to provide financial stability. The study of monetary policy in view of financial imbalances is best addressed with continuous-time modeling, which corresponds closely to reality on this issue. It is also, incidentally, a powerful mathematical tool. The paper is organized as follows. Section II. introduces the New Keynesian model and section III. presents the results. Section IV. concludes.

II. The Model

We develop a simple New Keynesian model which is described by the following three equations,

(1) $$\dot{y}_t = \sigma^{-1}\left(i_t - r_t - \pi_t + b_t\right),$$

(2) $$\dot{\pi}_t = \rho\pi_t - \varphi y_t - \xi b_t,$$

(3) $$b_t = \theta\left(i_t, y_t, k_t\right),$$

where the coefficients satisfy $\rho, \varphi, \sigma, \xi > 0$ and the path $\{r(t)\}$ is exogenous and given. Eqn (1) stands for the standard consumer's Euler Equation, where i_t represents the nominal interest rate and r_t stands for the *"natural rate of interest"* in terms of *Wicksell* (1936). σ^{-1} is the intertemporal elasticity of substitution. We can underpin this relationship with the wealth channel, which is explicitly discussed in *Mishkin* (2001). Eqn (2) represents the forward-looking Phillips Curve, where φ controls the degree of price stickiness. If we regard $\varphi \to \infty$, we have the situation with perfectly flexible prices. In eqn (3) we pick up the idea from *Woodford* (2012), who introduce a state variable introduce a measure of financial misalignments, i. e. the asset price bubble which depends on an endo-

genous growth rate θ, which depends itself on the interest rate i_t, on the output gap y_t and a combined variable k_t. The basic idea of eqn (3) is in line with *Mishkin* (2001), *Bordo/Jeanne* (2002), *Semmler/Zhang* (2007) and *Woodford* (2012). The growth rate θ is fueled by other variables (political environment, expectations of the participants, debt-to-equity ratio, psychological components, such as behavioral errors or irrational exuberance), which are combined by k_t. An example whereof k_t depends on is the financial risk-taking, respectively the leverage L_t. To illustrate this formally we assume a simple AR(1), written here in differential form

$$(4) \qquad dL_t = -\vartheta L_t dt + \varsigma dv_t, \text{ where } 0 < \vartheta < 1.$$

In integral form taking now the form

$$(5) \qquad L_t = \int_{\tau=0}^{\infty} e^{-\vartheta\tau} \varsigma dv_{t-\tau},$$

where dv_t denotes increments to standard Brownian motion. Therefore it makes sense to regard $\theta(i_t, y_t, k_t)$ in the further procedure as a nonlinear function, with the properties $\partial\theta(i_t, y_t, k_t)/\partial i_t < 0$, $\partial\theta(i_t, y_t, k_t)/\partial y_t < 0$ and $\partial\theta(i_t, y_t, k_t)/\partial k_t > 0$. This is in line with the conventional wisdom of "leaning against the wind". We know this is purely an assumption. We are conscious of this point. *Galí* (2014) shows in a very simple partial equilibrium model that this connection is extreme fragile. Usually the central bank is not in a position to know, which of these two effects predominate. It is like fishing in murky waters without additional information. The qualitative novelty in our analysis is to regard an interval $\left[\underline{i}_t ; \overline{i}_t\right]$ with $\underline{i}_t \geq 0$, in which the positive effect of the variable k_t is bigger than the negative effect of the interest rate i_t in terms of

$$(6) \qquad \left|\frac{\partial\theta(i_t, y_t, k_t)}{\partial k_t}\right| > \left|\frac{\partial\theta(i_t, y_t, k_t)}{\partial i_t}\right| \text{ for } i \in \left[\underline{i}_t ; \overline{i}_t\right].$$

In this scenario the central bank is committed to set the interest rate above $\overline{i}_t$ to provide financial stability. Such an assumption should be lined with an empirical analysis. This is not quite simple because asset price bubbles are not observable. For the following theoretical framework we will satisfy with this thought experiment and incorporate this constraint in the remainder of the study.

Furthermore, we will assume that the central bank minimizes a quadratic loss function [see *Ball* (1999), *Clarida* et al. (1999) and *Woodford* (2003, 2012)]. We regard here a continuous-time version, which can be written in integral form

(7)
$$\frac{1}{2}\int_0^\infty (y_t^2 + \lambda\pi_t^2)e^{-\rho t}\, dt,$$

where π describes deviations of the inflation rate from the inflation target and y denotes the output gap: the log difference between the actual output of an economy and its potential output. We assume the inflation target is zero. Here λ is a positive weight which represents the preference of the central bank and $0 < \rho < 1$ is a discount factor.

For simplification we get (3) in (1) and (3) in (2)

(1*)
$$\dot{y}_t = \sigma^{-1}(i_t - r_t - \pi_t + \theta(i_t, y_t, k_t)).$$

(2*)
$$\dot{\pi}_t = \rho\pi_t - \varphi y_t - \xi\theta(i_t, y_t, k_t).$$

For now the problem is formulated as an intertemporal optimization problem in continuous time with the objective function (7) subject to (1*)–(2*) and (6), with both initial values of the states, π_0, y_0 free. Additionally, we have a constraint of the nominal interest rate in the way of $i \geq \overline{i_t}$.

III. Results

We examine the optimal reaction of the central bank in the face of asset price bubbles. The optimization can be analyzed as a conventional bounded optimal control problem. The associated Current-Value Hamiltonian is given by

(8)
$$H \equiv \frac{1}{2}y^2 + \frac{1}{2}\lambda\pi^2 + \mu_y\sigma^{-1}(i - r - \pi + \theta) + \mu_\pi(\rho\pi - \varphi y - \xi\theta) + \mu_i i.$$

Here μ_y, μ_π, μ_i are co-state variables, associated with the dynamic constraints (1*), (2*) and (6). The pontryagin's minimum principle means that the co-state for y must be non-negative throughout and zero whenever the nominal interest rate is strictly positive

$$\mu_y \geq 0,$$

$$i\mu_y = 0.$$

The first order conditions (FOCs) of this problem are as follows

(9)
$$\dot{\mu}_y = -y_t + \left(\varphi + \xi\frac{\partial\theta_t}{\partial y_t}\right)\mu_\pi + \left(\rho + \frac{\partial\theta_t}{\partial y_t}\right)\mu_y,$$

(10)
$$\dot{\mu}_\pi = -\lambda\pi_t + \sigma^{-1}\mu_y,$$

(11)
$$\dot{\mu}_i = -\sigma^{-1}\mu_y.$$

Besides both initial values are free, we have

(12) $\mu_y(0) = 0$,

(13) $\mu_\pi(0) = 0$.

Under the assumption of convexity and appropriate transversality conditions are these conditions (9)–(11) necessary and sufficient for an optimum. To elaborate the policy rule, we neglect the zero lower bound of the nominal interest rate [see eqn (6)] and set $\mu_y = \dot{\mu}_y = 0$, so we get $y_t = \left(\varphi + \xi \dfrac{\partial \theta_t}{\partial y_t}\right)\mu_\pi$ and $\dot{\mu}_\pi = -\lambda \pi_t$. This gives:

(14) $\dot{y}_t = \left(\varphi + \xi \dfrac{\partial \theta_t}{\partial y_t}\right)\dot{\mu}_\pi.$

Further, notice that

(15) $\dot{y}_t = -\lambda\left(\varphi + \xi \dfrac{\partial \theta_t}{\partial y_t}\right)\pi_t.$

In conjunction with eqn (1*), this yields the following term, which can be written as a function of the policy instrument i,

(16) $i_t = r_t + \left(1 - \lambda\sigma\left(\varphi + \xi \dfrac{\partial \theta_t}{\partial y_t}\right)\right)\pi_t + \theta_t\,(i_t\,,y_t\,,k_t).$

In general (16) is a nontrivial, nonlinear problem. Some remarks are in order regarding the structure of the policy rule. The form is an augmented nonlinear Taylor Rule. The response of the central bank in face of financial misalignments depends on the nonlinear growth rate of the bubble. It depends further on the size of the effect of the interest rate i_t and the combined variable k_t on θ. If $\left|\partial\theta(i_t\,,y_t\,,k_t)\,/\,\partial i_t\right| > \partial\theta(i_t\,,y_t\,,k_t)\,/\,\partial k_t$, there is theoretical evidence for a "leaning against the wind" policy. The constraint is not binding. Thus, if $\left|\partial\theta(i_t\,,y_t\,,k_t)\,/\,\partial i_t\right| < \partial\theta(i_t\,,y_t\,,k_t)\,/\,\partial k_t$, the central bank is in a position to react exceptionally restrictive namely to set the interest rate above $\bar{i}_t$ to provide financial stability. The best way is a one-size-fits-all solution, which is surely preferable, but at least extremely difficult to evolve into a closed form. The results are based on assumptions, which are occasionally controversial discussed in economic literature. *Smets* (2014), *Woodford* (2012) and *Borio/Lowe* (2002) are in a way proponents of a policy which bear in mind that asset prices, respectively factors (i. e. leverage, debt-to-equity ratio) which could drive to bubbles should be regarded in a monetary policy strategy. *Galí* (2014), *Galí/Gambetti* (2015) and *Svensson* (2014) for Sweden on the other hand have showed that the con-

ventional wisdom of a policy of "leaning against the wind" depends on assumptions which are fragile. Certainly they did not preclude such a policy by hook or by crook because a monetary policy which tightens up the interest rate is a powerful tool to counter speculative bubbles.

Finally, the point here is to show that depending on special circumstances a policy of "leaning against the wind" could be a useful tool to provide financial stability besides other instruments such as macroprudential threshold value. The task for the central bank is particularly given when macroprudential policy fails. Incentive and coordination problems of these policies are provided by *Smets* (2014). Surely our result is possible in an environment that allows the central bank to move the interest rate above a threshold value of $\bar{i_t}$.

IV. Concluding Remarks

The present paper should be regarded as an effort to bring an impetus in view of the "leaning against the wind" policy debate. With an optimal bounded control problem we have developed an augmented nonlinear Taylor Rule. Even under the assumption of nonlinearities the central bank is in a position to move the interest rate above a threshold value to provide financial stability. It is clear that the results should be accompanied by further empirical research to improve the understanding of the diversified relationships between interest rates and asset price bubbles respectively the main factors which drive the bubble innovation process. To give a central bank a useful set of instrument rules is to know what are the factors for emerging bubbles and in what relationship these factors are linked each other.

References

Adrian, T./*Shin,* H. S. (2009): Money, Liquidity and Monetary Policy. American Economic Review: Papers and Proceedings, Vol. 99(2), pp. 600–605.

Angeloni, I./*Faia,* E. (2013): Capital Regulation and Monetary Policy with Fragile Banks. Journal of Monetary Economics, Vol. 60(3), pp. 311–324.

Ball, L. (1999): Efficient Rules for Monetary Policy. International Finance, Vol. 2(1), pp. 63–83.

Bernanke, B./*Gertler,* M. (1999): Monetary Policy and Asset Price Volatility. Economic Review Vol. 84(4), Federal Reserve Bank of Kansas City, pp. 17–51.

Bordo, M./*Jeanne,* O. (2002): Monetary Policy and Asset Prices: Does 'Benign Neglect' Make Sense? International Finance, Vol. 5(2), pp. 139–164.

Borio, C./*Lowe,* P. (2002): Asset Prices, Financial and Monetary Stability: Exploring the Nexus. BIS Working Paper No. 114.

Clarida, R./*Gali,* J./*Gertler,* M. (1999): The science of monetary policy: A new Keynesian perspective. Journal of Economic Literature, Vol. 37(4), pp. 1661–1707.

Galí, J. (2014): Monetary Policy and Rational Asset Price Bubbles. American Economic Review, Vol. 104(3), pp. 721–752.

Galí, J./*Gambetti,* L. (2015): The Effects of Monetary Policy on Stock Market Bubbles: Some Evidence. American Economic Journal: Macroeonomics, Vol. 7(1), pp. 233–257.

Meltzer, A. (2002): Rational and Irrational Bubbles. Keynote address for the Federal Reserve Bank of Chicago – World Bank Conference on Asset Price Bubbles, Chicago, April, 23, 2002. Available via the internet: https://pdfs.semanticscholar.org/724a/20e4426b0dcbf46a750f61018deb1d1eb684.pdf.

Mishkin, F. (2001): The Transmission Mechanism and the Role of Asset Prices in Monetary Policy. NBER Working Paper No. 8617.

Plosser, C. I. (2014): Monetary Rules: Theory and Practice. Frameworks for Central Banking in the Next Century Policy Conference. Hoover Institution, Stanford, CA. Available via the internet: https://www.philadelphiafed.org/-/media/publications/speeches/plosser/2014/05-30-14-hoover.pdf?la=en.

Semmler, W./*Zhang,* W. (2007): Asset Price Volatility and Monetary Policy Rules: Dynamic Model and Empirical Evidence. Economic Modelling, Vol. 24(3), pp. 411–430.

Smets, F. (2014): Financial Stability and Monetary Policy: How Closely Interlinked? International Journal of Central Banking, Vol. 10(2), pp. 263–300.

Svensson, L. (2014): Why Leaning Against the Wind Is the Wrong Monetary Policy for Sweden. Stockholm School of Economics Working Paper, June. Available via the internet: http://larseosvensson.se/files/papers/why-leaning-against-the-wind-is-the-wrong-monetary-policy-for-sweden.pdf.

Wicksell, K. (1936): Interest and Prices (tr. of 1898 edition by R. F. Kahn). London: Macmillan.

Woodford, M. (2003): Interest and Prices: Foundations of a Theory of Monetary Policy. Princeton: Princeton University Press.

– (2012): Inflation Targeting and Financial Stability. Sveriges Riksbank Economic Review, 2012:1, pp. 7–32.