

The Time Variation of Liquidity Risk in US Stock Markets

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Abstract

The influence of liquidity costs and liquidity risk on asset returns has been proven by several empirical studies. This paper analyzes the conditional version of the liquidity-adjusted capital asset pricing model and shows that betas significantly vary over different economic regimes and that liquid portfolios provide diversification benefits compared with illiquid portfolios. The results support the effects of a flight-to-liquidity. The time variation of liquidity betas induces additional risk for investors, which has important implications for investment decisions and asset allocation.

Die Zeitschwankung des Liquiditätsrisikos in US-Aktienmärkten

Zusammenfassung

Der Einfluss von Liquiditätskosten und Liquiditätsrisiko auf die Renditen verschiedener Assets wurde bereits durch einige Studien belegt. Dieses Paper untersucht das conditional liquidity-adjusted capital asset pricing model und zeigt, dass die Liquidity-Betas signifikant über verschiedene Marktphasen hinweg schwanken und dass liquide Portfolios Diversifikationsvorteile gegenüber illiquiden Portfolios bieten. Die Ergebnisse unterstützen den Flight-to-Liquidity-Effekt. Die Schwankungen der Liquidity-Betas erzeugen zusätzliche Risiken für Investoren, die bei Investmententscheidungen und der Portfoliostrukturierung berücksichtigt werden müssen.

Keywords: CAPM, liquidity risk, regime switching model, time variation, liquidity betas

JEL Classification: G11, G12

I. Introduction

Several studies have examined and proven that the influence of liquidity costs and liquidity risk on asset returns are significant. Further, conditional asset pricing

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ing models show that model parameters vary over time and correlations within this time variation induce further risk. Despite many empirical studies on liquidity risk, very little research has been done on conditional models of such risk, which is the focus of this paper. In detail, I analyze the time variation in liquidity betas of the liquidity-adjusted CAPM and find significant differences in the behavior between liquid and illiquid portfolios, which are important for asset allocation and portfolio management. In the following, the most relevant lines of research on liquidity risk and conditional asset pricing models are presented.

Liquidity costs are a central topic in financial markets. Nevertheless, they are completely disregarded in the standard asset pricing that assumes “frictionless (or, perfectly liquid) markets, where every security can be traded at no cost all the time, and agents take prices as given” (Amihud et al. 2005). However, in real markets, several sources of illiquidity exist, such as “exogenous trading costs, private information, inventory risk for market makers, and search problems” (Amihud et al. 2005). Huberman/Halka (2001) provide a general definition of liquidity: “A market is liquid if one can trade a large quantity shortly after the desire to trade arises at a price near the prices of the trades before and after the desired trade” (p. 161). In this manner, one can define liquidity costs as the time and costs associated with the trading of a given quantity of an asset (Gibson/Mougeot 2004). A more detailed overview of high- and low-frequency measures can be found in Goyenko et al. (2009), who also compare the capability of those proxies to measure liquidity.

According to Amihud et al. (2005), these costs of illiquidity and especially the risk of time variation should affect security prices because investors require compensation for bearing them. Amihud/Mendelson (1986) are among the first to investigate the influence of liquidity on asset returns. Since then, a large number of different studies and theoretical models have shown the impact of liquidity on asset returns even after controlling for the Fama and French factors (among others: Amihud/Mendelson 1986, 1989, 1991; Brennan/Subrahmanyam 1996; Chordia et al. 2000; Amihud 2002; Pastor/Stambaugh 2003; Gibson/Mougeot 2004; Acharya/Pedersen 2005; Watanabe/Watanabe 2008; Lee 2011; Rösch/Kaserer 2013; Hagströmer et al. 2013; Amihud et al. 2015). Furthermore, the risk of time-varying liquidity is an additional source of risk that cannot be captured by the market beta (Chordia et al. 2000; Huberman/Halka 2001; Amihud 2002; Pastor/Stambaugh 2003; Acharya/Pedersen 2005; Korajczyk/Sadka 2008). In particular, the so-called “liquidity commonality” – the correlation of asset liquidity with market liquidity – and the “flight-to-liquidity” effect have been examined in several previous studies (Domowitz/Wang 2002; Kamara et al. 2008; Grabowski 2010; Rösch/Kaserer 2013; Mayordomo et al. 2014). Because the number of studies on the influence of liquidity and liquidity risk on asset returns is too large to

be presented to the full extent, I concentrate on the findings most relevant to this study. A detailed literature overview is presented by *Amihud et al. (2005)*.

Another line of research, starting with the work of *Merton (1973)*, focuses on the intertemporal validity of the CAPM and the time variation of the model parameters. *Jagannathan/Wang (1996)*, *Lettau/Ludvigson (2001)*, and *Ang/Cheng (2007)* argue that many of the firm characteristics used in multi-factor models do not directly influence asset returns, but are an instrument for measuring the impact of the time variation within CAPM parameters and show that the influence of such firm characteristics is no longer significant when accounting for the time variation. A large volume of research on conditional models, such as *Abdymomunov/Morley (2011)*, *Fridmann (1994)*, *Huang (2000)*, *Nilsson/Hansson (2004)*, and *French et al. (1987)*, concentrate on the behavior of different versions of the conditional CAPM in periods of low and high market volatility – so-called “volatility regimes.” They find significant time variations in the market beta and its risk premium.

Despite many theoretical models and empirical studies on liquidity risk and its impact on asset returns, few papers combine those two research streams. Thus, the focus of this paper is to investigate the time variation of liquidity risk within a conditional asset pricing model that accounts for liquidity risk.

This paper is derived from the works of *Acharya/Pedersen (2005)*, *Watanabe and Watanabe (2008)*, and *Hagströmer et al. (2013)*. *Acharya/Pedersen (2005)* develop a liquidity-adjusted capital asset pricing model (LCAPM) that enhances the classical pricing formula of the CAPM by the expected level of liquidity and three betas for different dimensions of liquidity risk. Although they also present a conditional specification of the LCAPM, they only empirically test the unconditional version.

Gibson/Mougeot (2004) are among the first to approach this idea by investigating the time variation of liquidity and liquidity risk using a bivariate GARCH model. They find that “liquidity risk is indeed priced during the entire as well as over sub-periods in the United States. The sign of the liquidity risk premium is significantly negative and time-varying” (*Gibson/Mougeot 2004*, p. 176). *Watanabe/Watanabe (2008)* also investigate the time variation in liquidity risk using a regime-switching model that allows both time-series and cross-sectional variations in the liquidity beta. They show that “both high liquidity betas and large liquidity risk premium arise simultaneously at times of high preference uncertainty” (*Watanabe/Watanabe 2008*, p. 2458) using trading volume as an indicator of uncertainty. They show that the high liquidity beta state “exhibits high volatility and a wide cross-sectional dispersion in liquidity betas [...] and that coincides with periods of high illiquidity premium” (*Watanabe/Watanabe 2008*, p. 2482). Their conditional liquidity factor also indicates that the cross-sectional pricing of liquidity risk strengthens in these market phases.

While *Watanabe/Watanabe* (2008) and *Gibson/Mougeot* (2004) do not explicitly examine the different components of liquidity risk that were found by *Acharya/Pedersen* (2005), *Hagströmer et al.* (2013) are the first to empirically test the conditional version of the LCAPM using a GARCH model. Nevertheless, both *Gibson/Mougeot* (2004) and *Hagströmer et al.* (2013) concentrate on the risk premiums of the LCAPM and find a significant time varying behavior; however, they do not explicitly investigate liquidity risk.

This paper fills this gap and investigates how and why the betas of the LCAPM vary over time. To do so, I use a Markov-switching approach to identify different economic conditions that lead to time variations of liquidity betas. Thereby, two main questions are addressed.

- How do regimes of high and low market illiquidity relate to the well-known “volatility regimes” and the “volume regimes” of *Watanabe/Watanabe* (2008)?
- How do liquidity betas change across those different regimes? Do liquidity commonality and other risk parameters increase in periods of high volatility and illiquidity?

Regarding the identification and characterization of economic regimes, I find that phases of high illiquidity are characterized by negative average returns, higher return volatility, and higher correlation of asset returns and market returns, as well as lower sensitivity of asset illiquidity and market returns. I demonstrate that high and low illiquidity regimes are persistent and are, to some extent, correlated with both high volatility and trading volume regimes.

The analysis of the time variation of the betas shows that the market beta and the three liquidity betas change significantly across the regimes. In detail, two important effects become visible. First, the flight-to-liquidity effect leads to higher liquidity sensitivity to market returns when uncertainty is high, as indicated by higher trading volume or higher return volatility. The results indicate that in high uncertainty phases, traders tend to buy more liquid assets such that these assets are less influenced by decreasing overall returns. At the same time, already illiquid assets are sold and, therefore, strongly affected by overall return shocks. Further, betas 2 and 4 increase with illiquidity, supporting the finding of *Rösch/Kaserer* (2013) of an increase in liquidity commonality. Thus, single asset liquidity is especially fragile when market illiquidity is already high.

Further, I find several important differences between liquid and illiquid portfolios, which should be considered in portfolio management and asset allocation. Whereas liquid portfolios overall have lower betas and are, therefore, less risky, they are also less sensitive to regime changes and even provide diversification advantages to investors in several regimes. In contrast, illiquid portfolios have higher betas and tend to be more sensitive to regime changes, which further increases the risk for investors.

The remainder of this paper is structured as follows. Chapter II. describes the LCAPM and the research methodology, as well as the data set used. The empirical results are presented in chapter III., and chapter IV. discusses the results and conclusions.

II. Research Design

1. Theoretical Background

Several papers present models that attempt to capture and quantify the effect of illiquidity risk on asset returns. Whereas most of these models concentrate on one dimension of illiquidity, Acharya/Pedersen (2005) develop the liquidity-adjusted CAPM, which includes the level of liquidity, the market beta, and three liquidity betas. Thereby, they can consider different ways in which liquidity risk influences asset returns. The conditional version of the LCAPM is defined by the following formula:

$$(1) \quad E_t(r_{t+1}^i) - r_{t+1}^f = E_t(c_{t+1}^i) + \lambda_t \beta_t^{1i} + \lambda_t \beta_t^{2i} - \lambda_t \beta_t^{3i} - \lambda_t \beta_t^{4i}$$

where

$$(2) \quad \beta_t^{1i} = \frac{\text{cov}_t(r_{t+1}^i, r_{t+1}^M)}{\text{var}_t(r_{t+1}^M - c_{t+1}^M)}$$

$$(3) \quad \beta_t^{2i} = \frac{\text{cov}_t(c_{t+1}^i, c_{t+1}^M)}{\text{var}_t(r_{t+1}^M - c_{t+1}^M)}$$

$$(4) \quad \beta_t^{3i} = \frac{\text{cov}_t(r_{t+1}^i, c_{t+1}^M)}{\text{var}_t(r_{t+1}^M - c_{t+1}^M)}$$

$$(5) \quad \beta_t^{4i} = \frac{\text{cov}_t(c_{t+1}^i, r_{t+1}^M)}{\text{var}_t(r_{t+1}^M - c_{t+1}^M)}$$

$E(c_t^i)$ represents the expected illiquidity of asset i at time t and β^{1i} is the equivalent to the classical CAPM beta.

β^{2i} describes the correlation of asset illiquidity to market illiquidity, also known as liquidity commonality.

β^{3i} represents the correlation of asset returns and market illiquidity. If assets show a positive beta 3, returns increase when market liquidity decreases and

vice versa. In other words, such assets give investors a risk diversification advantage, which should decrease the expected returns represented by the negative λ .

β^{4i} is defined as the correlation of asset illiquidity and market returns. Equivalent to beta 3, a positive correlation of asset illiquidity and market returns provides a diversification advantage for investors because the liquidity costs decrease when markets decrease, and vice versa. Thus, beta 4 can also be interpreted as a measure of the flight-to-liquidity or flight-to-quality effect.

I use a Markov-switching model to account for time variation in the parameters because it provides deeper insights into the possible economic reasons for this time variation compared with GARCH models. I use market volatility, market illiquidity, and trading volume as instrumental variables for the Markov-switching model. The intuition behind these variables is as follows. *Watanabe/Watanabe* (2008) use trading volume as an indicator of uncertainty. The volatility of asset returns is an intuitive alternative measure of uncertainty. Further, as previously described, several empirical studies prove that market participants change their trading behavior with different levels of market volatility (*Abdymonov/Morley* 2011; *Fridmann* 1994; *Huang* 2000; *Nilsson/Hansson* 2004; *French et al.* 1987). Lastly, *Watanabe/Watanabe* (2008) find indications that volatility, illiquidity, illiquidity risk, and risk premiums are connected to each other. Therefore, a comparative analysis of these different effects provides new insights into the time variation of liquidity risk and the possible causes.

2. Theoretical Hypothesis

Using theoretical expectations and previous findings of existing papers, hypotheses about the behavior of the betas of the LCAPM are developed. In the first step, expectations on volume are derived.

Because *Watanabe/Watanabe* (2008) only analyze the liquidity beta and not the classical CAPM beta, there are no existing findings on the behavior of beta 1 within volume regimes. Existing studies of volatility regimes, which also can be interpreted as indicators for uncertainty, find beta 1 to decrease with volatility. Thus, beta 1 can be expected to decrease with market trading volume.

H1.1: Beta 1 decreases in the high-volume regime.

Rösch/Kaserer (2013) find higher liquidity commonality in periods of higher uncertainty, although they measure uncertainty using return volatility. When trading volume is interpreted as an indicator of uncertainty, liquidity commonality should behave similarly.

H1.2: Beta 2 is higher in the high-volume regime than in the low volume regime.

Regarding the analysis of volume regimes, *Watanabe/Watanabe* (2008) argue that increasing future illiquidity costs representing higher preference uncertainty translates to higher returns required by investors. Therefore, “return sensitivity to illiquidity shocks is negative and larger in magnitude” (p. 2450) and, thus, beta 3 should be negative and larger in absolute terms in phases of high preference uncertainty. Their empirical results support this hypothesis. Further, they show that illiquid stocks have more sensitive betas than liquid stocks.

H1.3: Beta 3 is higher in the high-volume regime than in the low volume regime, and the increase is stronger for illiquid portfolios.

Rösch/Kaserer (2013) find that, in line with the flight-to-liquidity-effect, liquid stocks become more liquid in market downturns, whereas illiquid stocks become more illiquid. Therefore, beta 4 should change differently for liquid and illiquid stocks. Liquid stocks should have positive betas and illiquid stocks negative betas, but both should increase in absolute terms.

H1.4: Beta 4 increases in absolute terms in the high-volume regime; moreover, liquid portfolios show positive betas and illiquid portfolios show negative betas.

Given the previous findings of empirical studies on volatility regimes, the following expectations can be derived. As described in *Abdymomunov/Morley* (2011), the CAPM beta can change differently across volatility regimes. In particular, value and growth stocks change differently when volatility increases. Using the flight-to-liquidity effect, one also assumes that liquidity portfolios behave differently because liquid stocks might be more stable in market downturns, whereas illiquid stocks face higher price declines.

H2.1 Beta 1 is lower in the high-volatility regime.

Rösch/Kaserer (2013) find higher liquidity commonality in periods of high volatility. The flight-to-liquidity effect should lead to different magnitudes for liquid and illiquid stocks.

H2.2 Beta 2 is higher in the high-volatility regime and stronger for illiquid portfolios.

Using the argumentation of *Watanabe/Watanabe* (2008) as previously described, the return sensitivity to liquidity shocks should be stronger when uncertainty is higher.

H2.3 Beta 3 is higher in the high-volatility regime.

As described above, *Rösch/Kaserer* (2013) find that, in line with the flight-to-liquidity-effect, liquid stocks become more liquid in market downturns, whereas illiquid stocks become more illiquid. Therefore, beta 4 should change differently for liquid and illiquid stocks. Liquid stocks should have positive betas and illiquid stocks negative betas, but both should increase in absolute terms.

H2.4: Beta 4 increases in absolute terms in the high-volatility regime, and liquid portfolios show positive betas and illiquid portfolios show negative betas.

Lastly, theoretical expectations based on liquidity regimes are derived. Because liquidity regimes have yet to be investigated in detail, no existing empirical results can be used. Based on the known effects previously described, the following hypotheses are derived.

Few previous findings exist on the connection between the market beta and market liquidity. Assuming that investors react to liquidity shocks and to increasing market uncertainties similarly, beta 1 should decrease with market illiquidity.

H3.1 Beta 1 is lower in the high-illiquidity regime.

Regarding the liquidity commonality, *Rösch/Kaserer* (2013) find increasing commonality when illiquidity increases in the market.

H3.2 Beta 2 is higher in the high-illiquidity regime.

Given the flight-to-liquidity effect in market downturns, liquid portfolios are bought by investors, whereas illiquid stocks are sold. Therefore, liquid portfolio returns should increase and illiquid portfolio returns should decrease. Thus, beta 3 should change differently for liquid and illiquid stocks.

H3.3 Beta 3 has smaller negative or positive values for liquid portfolios and higher negative values for illiquid portfolios when illiquidity is high.

Rösch/Kaserer (2013) find that, in line with the flight-to-liquidity effect, liquid stocks become more liquid in market downturns, whereas illiquid stocks become more illiquid. One would assume to behave beta 4 similarly when the flight-to-liquidity effect is triggered by increasing market illiquidity.

H3.4: In the high-illiquidity regime, beta 4 increases in absolute terms, liquid portfolios show positive betas, and illiquid portfolios show negative betas.

3. Data and Empirical Methodology

The dataset includes all companies listed on the NYSE, AMEX, and NASDAQ between 1973 and 2012, and that have total return, price, and volume data available on DATASTREAM (12,784 companies). First, non-trading days are removed from the dataset (leaving 10,094 days).

For the analysis, I follow the established procedure of sorting the single stocks into 25 test portfolios and a market portfolio. At the beginning of each year, all companies with a price between \$5.00 and \$1,000.00 and more than 100 observations during the year are used in the market and 25 test portfolios.

The companies are sorted into 25 test portfolios by their average illiquidity, as described in formula 6. I use the liquidity measure presented by *Amihud* (2002),

which was applied in several recent studies, including the work of *Watanabe/Watanabe* (2008).

$$(6) \quad ILLIQ_{i,t} = \frac{1}{D_t} \sum_{d=1}^{D_t} \frac{|r_{i,d}|}{VOL_{i,d}}$$

Here, $r_{i,d}$ is the continuously compounded return on asset i on day d and $VOL_{i,d}$ is the trading volume (in millions of USD) on day d calculated as the volume by turnover from DATASTREAM on that day multiplied by the quoted price of the stock on this day. D_t is the number of trading days in month t . Within the market and test portfolios, equal weights and not value weights are used to ensure that the portfolios are not dominated by large liquid stocks (see *Hagströmer et al.* 2013). Because the illiquidity measure described in formula 6 has a strong positive trend, I use an adjusted version for the time series analysis. I adjust the illiquidity measure for inflation using the monthly CPI values. Because the LCAPM further requires a measure for the cost of a trade, I follow *Acharya/Pedersen* (2005) and define the normalized illiquidity measure as:

$$(7) \quad ILLIQ_{i,t,norm} = \min \left(0.25 + \frac{1}{200} \left(\frac{1}{D_t} \sum_{d=1}^{D_t} \frac{|r_{i,d}|}{VOL_{i,d}} \right) \frac{CPI_t}{CPI_0}, 30.00 \right)$$

Thereby, the illiquidity measure is capped at 30.00 % because higher values are not reasonable. The parameters 0.25 and 1/200 are chosen in a way that the illiquidity measure for this study has comparable characteristics as that reported in *Acharya/Pedersen* (2005). Their illiquidity measure ranges from 0.25 % to 8.83 % and the standard deviation ranges from 0.00 % to 1.46 % across the portfolios. The illiquidity measure constructed as in formula 7 ranges from 0.251 % to 7.559 % and has a standard deviation ranging from 0.001 % to 3.060 %.

The market portfolio is then used to identify the economic regimes. *Hagströmer et al.* (2013) argue that the GARCH approach allows for more time variation in the parameters than the Markov-switching approach of *Watanabe/Watanabe* (2008) and, therefore, is more suitable for the test of a conditional model. However, the disadvantage of the GARCH model relative to the Markov-switching approach is that the former only provides limited insights into how the parameters vary over time and the underlying factors that might be the reason for this time variation. Therefore, in this study, a Markov-switching approach is used to investigate and separate the effects of volatility, volume, and liquidity regimes.

The theory and application of Markov-switching models are presented by *Hamilton* (1990) and several subsequent works. The basic idea is that an unobservable process jumps between two or more distinct regimes. *Watanabe/Watanabe* (2008) use trading volume as an instrumental variable for the unob-

servable regimes of high/low preference uncertainty. Their model formulation is extended by additionally using market illiquidity and return volatility.

For the market returns, the equal weighted monthly returns for the market portfolio for the complete time series are calculated. This time series is then divided into two separate regimes with two different normal distributions: one with high volatility and one with low volatility of monthly returns. I follow the standard procedure of the EM-Algorithm based on the work of *Baum et al.* (1970), which iteratively estimates the distributions within the regimes and optimizes the transition probabilities between the regimes. The resulting filtered probabilities provide information on how likely a specific point in time does or does not correspond to the “high” regime. If the probability of being in the “high” regime is greater than 0.5, this point is assigned to the “high” regime and, otherwise, to the “low” regime. Thereby, the time series of monthly returns is divided into two distinct regimes. The differences between these two regimes are presented in table 2.

Analogously, I calculate the time series for market trading volume and market liquidity and then divide the sample period into two distinct regimes for each instrumental variable. For the market trading volume, I calculate the STOV measure of *Watanabe/Watanabe* (2008) as:

$$(8) \quad STOV_t = \frac{volume_t^M}{MATOV_t} volume_t^M$$

where $volume_t^M$ is the volume of the equal weighted market portfolio and $MATOV_t$ is the last 24-month moving average of $volume_t^M$.

4. Descriptive Statistics

Table 1 presents the descriptive statistics for the market and illiquidity portfolios. In accordance with *Acharya/Pedersen* (2005) and *Hagströmer et al.* (2013), I find that the standard deviation of the monthly illiquidity measure increases with the level of illiquidity, meaning that more illiquid assets also have a higher risk of changing illiquidity. Beta 1 varies around 100 %, which is the expected corridor. The betas for the most liquid and most illiquid portfolios are smaller than for the “middle” portfolios, which is similar to the characteristics reported by *Acharya/Pedersen* (2005). Beta 2 significantly increases with illiquidity, meaning that more illiquid assets are more sensible to market illiquidity shocks. Betas 3 and 4 both increase with illiquidity and are negative for all portfolios, which is in accordance with the findings of *Acharya/Pedersen* (2005).

Regarding the average returns of the 25 illiquidity portfolios, the results are contrary to previous findings of *Hagströmer et al.* (2013) and *Acharya/Pedersen*

(2005). The average gross returns decrease slightly with increasing illiquidity. The standard deviation of monthly gross returns is nearly similar for all 25 test portfolios and about two times higher than the standard deviation reported by Acharya/Pedersen (2005), but close to those reported by Hagströmer et al. (2013). The reasons for this finding might be the differences in the data set (different period and the inclusion of NASDAQ stocks).¹ Butt et al. (2016) show that the choice of an illiquidity measure can have a significant influence on the estimated premium. Gibson/Mougeot (2004), who test a conditional model for liquidity risk and use S&P500 stocks from 1973 to 1997, also finds significant negative risk premiums. Because the aim of this paper is to more deeply investigate the time varying behavior of the liquidity betas in different economic regimes, the differences in the premiums are not further analyzed.

Table 1
Descriptive Statistics – Conditional Model – *Amihud*

<i>PF</i>	<i>E(r) %</i>	<i>Sd(r) %</i>	<i>E(illiq)</i>	<i>Sd(illiq)</i>	<i>Beta 1</i> *100	<i>Beta 2</i> *100	<i>Beta 3</i> *100	<i>Beta 4</i> *100
Market	0.521	3.968	2.295	0.980	–	–	–	–
1	0.414	4.983	0.251	0.001	83.934	0.001	–1.158	–0.009
3	0.427	5.221	0.255	0.006	95.845	0.008	–0.895	–0.047
5	0.419	5.531	0.261	0.010	102.161	0.023	–1.449	–0.131
7	0.379	5.413	0.271	0.021	103.341	0.062	–1.370	–0.206
9	0.355	5.673	0.289	0.045	110.593	0.147	–1.557	–0.569
11	0.376	5.684	0.324	0.083	110.962	0.304	–1.349	–0.841
13	0.280	5.566	0.384	0.168	109.102	0.662	–2.166	–1.445
15	0.281	5.640	0.493	0.280	111.971	1.177	–1.759	–2.563
17	0.190	5.756	0.649	0.413	112.471	1.767	–2.199	–4.122
19	0.267	5.825	0.932	0.690	116.887	3.035	–1.978	–5.426
21	0.178	5.308	1.525	1.081	106.666	4.892	–2.298	–10.293
23	0.381	5.047	2.669	1.690	105.223	7.923	–1.871	–11.864
25	0.307	4.998	7.559	3.060	102.386	14.01	–2.381	–36.083

Note: Descriptive statistics for the market portfolio and the illiq sorted equal weighted test portfolios. The average monthly return $E(r)$ in % is given for each portfolio in the first column. The standard deviation of monthly returns $Sd(r)$ in % is given in column 2. The mean and the standard deviation of the illiquidity measure $E(illiq)$ and $Sd(illiq)$ are shown in column 3 and 4. The columns 5–8 show the 4 betas of the conditional LCAPM calculated according to the formulas 2–5.

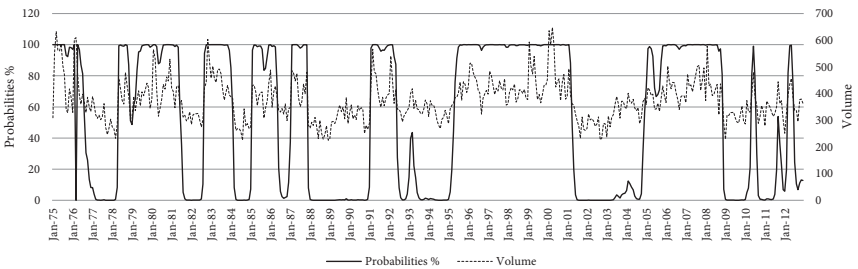
¹ These results are also robust to several sub-sample analyses.

III. Empirical Results

1. Identification of Economic Regimes

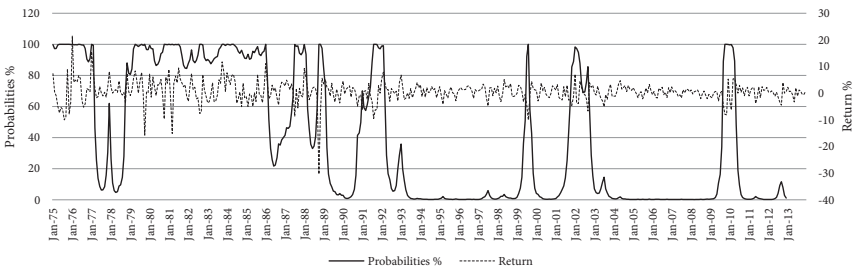
In the first step, I use the Markov-switching model to identify the different regimes on a monthly basis.

The results for the market volume regimes can be seen in Figure 1. It can be noted that the trend-adjusted volume measure is quite volatile during the entire sample period. Figure 2 presents the results of the market return volatility regimes. The high and low volatility phases are less persistent than the high trading volume phases during the second half of the sample period. The model only stays in the high volatility state for a few months before returning to the calm regime. Second, volume and volatility regimes do not exactly correspond to each other. Therefore, it is at least questionable whether the use of trading volume alone as a proxy for investors' uncertainty, as in *Watanabe/Watanabe* (2008), is sufficient. Lastly, an analysis of the normalized illiquidity measure as



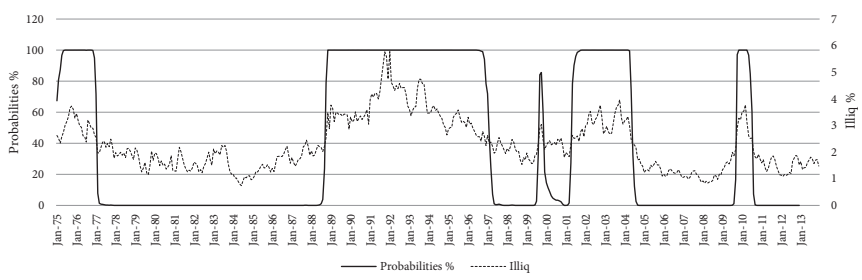
Note: This graph shows the monthly values for the equal weighted market portfolio trading volume measure of *Watanabe/Watanabe* (2008) (black line). The filtered probabilities for the regimes are estimated using the EM-algorithm (blue line).

Figure 1: Volume Regimes



Note: This graph shows the monthly values for the equal weighted market portfolio returns (black line) and the filtered probabilities for the regimes calculated using the EM-algorithm (blue line).

Figure 2: Volatility Regimes



Note: This graph shows the monthly values for the equal weighted market portfolio normalized illiquidity measure (black line). The filtered probabilities for the regimes are calculated using the EM-algorithm (blue line).

Figure 3: Illiquidity Regimes

described in formula 7 – as shown in Figure 3 – is performed. Market liquidity changes significantly over time and high values of illiquidity are persistent during longer periods, in accordance with the findings of Acharya/Pedersen (2005).

Table 2 shows descriptive statistics for the different regimes. From the analysis of high and low trading volume, as in Watanabe/Watanabe (2008), one can see that high trading volume is accompanied by higher average returns and higher liquidity. The results for the volatility regimes show little differences in the average returns, but the standard deviation is approximately three times as high in the high volatility regime. Further, one can see that the average illiquidity in the high illiquidity regime is nearly 1.5 times the average illiquidity in the low illiquidity regime with a standard deviation, which is about the same level.

Table 2
Market Portfolio – Regimes Analysis

Regime	$E(r)$ %	$Sd(r)$ %	$E(illiq)$	$Sd(illiq)$
Whole time series	0.521	3.968	2.295	0.980
High volume	1.005	4.463	1.982	0.857
Low volume	-0.007	3.277	2.635	0.994
High volatility	0.508	5.983	2.302	1.073
Low volatility	0.528	1.991	2.291	0.922
High Illiquidity	-0.513	4.135	3.166	0.983
Low illiquidity	0.696	3.918	2.147	0.900

Note: The average monthly return $E(r)$ is given in % for the market portfolio in the first column. The standard deviation of the monthly returns $Sd(r)$ in % is given in column 2. The mean and the standard deviation of the illiquidity measure $E(illiq)$ and $Sd(illiq)$ are shown in column 3 and 4. The Volume Regimes are identified based on the volume measure of Watanabe/Watanabe (2008).

2. Time Variation of Betas

As is seen in section 3.1, there is a broad time variation in the return volatility and the market volume and illiquidity. In the next step, the betas are calculated within the distinct economic regimes. The calculations are based on the monthly returns and the illiquidity measures as described in formula 7. These monthly time series are also the basis for the beta calculations in the two distinct regimes. Thus, the returns are not recalculated after identifying the regimes. Instead, the data points that do not correspond to the regime are dropped. Because the chronological order of return observations has no influence on the calculation of betas, the fact that the regime time series contain data points that did not occur in subsequent months does not violate the theoretical assumptions of the model.

First, the differences in betas between high and low volume regimes are analyzed and presented in table 3.

According to H1.1, beta 1 should decrease in the high-volume regime. The empirical results confirm this hypothesis because, in particular, liquid portfolios

Table 3
Betas in Volume Regimes for Conditional LCAPM

PF	Beta 1 *100		Beta 2 *100		Beta 3 *100		Beta 4 *100	
	high	low	high	low	high	low	high	low
1	75.634	105.469	0.001	0.001	0.360	-1.889	0.001	-0,006
3	86.903	121.320	0.004	0.013	0.921	-3.025	0.006	-0,052
5	92.669	129.863	0.012	0.037	0.287	-3.813	0.003	-0,111
7	94.199	129.343	0.025	0.112	0.407	-3.020	-0.010	-0,181
9	103.777	133.407	0.046	0.290	0.962	-4.684	-0.013	-0,329
11	108.806	125.805	0.130	0.553	0.981	-4.382	-0.056	-0,588
13	105.027	126.883	0.301	1.184	0.128	-5.025	-0.069	-0,977
15	109.398	127.846	0.438	2.222	0.393	-4.622	-0.116	-1,463
17	108.933	129.144	0.694	3.275	0.604	-5.641	-0.450	-2,455
19	118.535	123.416	1.200	5.546	0.675	-4.301	-0.483	-3,700
21	106.637	116.922	2.364	8.577	0.302	-5.783	-1.798	-5,957
23	111.313	103.957	4.054	13.457	0.295	-4.343	-3.700	-4,909
25	111.440	95.668	9.151	20.188	-0.908	-3.817	-9.499	-8,847
p-value	0.000		0.059		0.000		0.003	

Note: For 25 illiquidity sorted (from low illiquidity to high illiquidity) equal weighted test portfolios the Betas 1–4 are calculated within the two distinct Regimes of high and low market trading volume, calculated according to the volume measure of Watanabe/Watanabe (2008). All calculations are performed using monthly data and an equal weighted market portfolio. The betas are calculated using the conditional model formulation of the LCAPM according to the formulas 2–5. A two-sided Wilcoxon rank sum test on the differences between the two distinct regimes is performed for each beta 1–4. The corresponding p-values of this test are given in the last row.

have significantly lower betas in the high-volume regime. The results support the existing findings of volatility regimes, where beta 1 was found to decrease with market volatility, which can be interpreted as a different indicator of uncertainty. *Rösch/Kaserer* (2013) find higher liquidity commonality in periods of greater uncertainty, although they measure uncertainty using return volatility. The results for beta 2 are different. Liquidity commonality seems to decrease with market trading volume for all portfolios. H1.3 states that beta 3 is higher in the high-volume regime than in the low volume regime. The empirical results draw a different picture. Most portfolios have positive betas in the high-volume regime, but were negative in the low-volume regime. Thus, in times of high trading volume, most portfolios show increasing returns when illiquidity rises and, therefore, provide a risk diversification advantage to investors. In line with the flight-to-liquidity effect, *Rösch/Kaserer* (2013) find that liquid stocks become more liquid in market downturns, whereas illiquid stocks become more illiquid. The results for beta 4 partly confirm this view because liquid portfolios show smaller negative or even positive betas. In contrast, illiquid portfolios do not show increasing negative betas. Instead, the betas also decrease with trading volume, although the magnitude is smaller than for liquid portfolios.

Table 4 presents the results for betas calculated within the two volatility regimes. Based on the existing findings and theoretical expectations, beta 1 should decrease with market volatility, which is strongly supported by the empirical results. Thus, beta 1 decreases significantly across all portfolios in periods of higher market uncertainty as indicated by both trading volume and return volatility. Regarding liquidity commonality, the results again do not confirm the findings of *Rösch/Kaserer* (2013) because beta 2 seems lower in high-volatility regimes for all portfolios, as was the case for the volume regimes. These differences in the results might be explained by the different markets and the different liquidity measure examined by *Rösch/Kaserer* (2013). Based on the argumentation of *Watanabe/Watanabe* (2008), the return sensitivity to liquidity shocks should be greater when uncertainty is higher. The results for beta 3 support this hypothesis because the betas are more negative for most portfolios in the high volatility regime. Nevertheless, the differences are not consistent for all portfolios and not statistically significant based on the Wilcoxon-test. Beta 4 is expected to increase in absolute terms, where liquid portfolios should have positive or small negative betas and illiquid portfolios should have higher negative betas. The results partly support this hypothesis because beta 4 seems to increase with market volatility, especially for illiquid portfolios. Nevertheless, liquid portfolios do not show positive or smaller negative betas, as was found for volume regimes.

Table 5 presents the results for the four betas calculated within the two distinct illiquidity regimes. When investors react to liquidity shocks and rising market uncertainty in a similar way, beta 1 should decrease with market illiquidity. The empirical results do not confirm this expectation because most

Table 4
Betas in Volatility Regimes for the Conditional LCAPM

PF	Beta 1 *100		Beta 2 *100		Beta 3 *100		Beta 4 *100	
	high	low	high	low	high	low	high	low
1	79.298	105.772	0.000	0.004	-1.556	0.819	-0.001	-0,002
3	90.000	123.416	0.004	0.026	-1.263	0.940	-0.017	-0,009
5	94.442	138.645	0.011	0.079	-1.425	-1.483	-0.040	-0,038
7	95.395	140.915	0.037	0.179	-1.079	-2.680	-0.096	-0,010
9	103.051	146.245	0.090	0.414	-1.609	-1.235	-0.167	-0,069
11	103.331	147.085	0.172	0.929	-1.367	-1.217	-0.358	0,036
13	100.046	151.961	0.382	1.984	-1.835	-3.676	-0.557	-0,142
15	103.357	152.751	0.608	3.870	-1.709	-1.947	-0.850	-0,552
17	103.542	154.710	0.994	5.423	-2.147	-2.380	-1.740	-0,095
19	110.868	145.355	1.647	9.607	-2.140	-1.158	-2.465	-0,810
21	101.201	132.454	2.732	15.115	-2.540	-1.073	-4.792	0,242
23	103.131	115.084	4.082	26.153	-2.255	-0.011	-6.552	0,996
25	100.474	111.323	9.019	37.732	-2.546	-1.520	-12.094	-6,764
p-value	0.000		0.029		0.271		0.005	

Note: For 25 illiquidity sorted (from low illiquidity to high illiquidity) equal weighted test portfolios the Betas 1–4 are calculated within the two distinct Volatility Regimes (high) and (low) using monthly data and an equal weighted market portfolio. The betas are calculated using the conditional model formulation of the LCAPM according to the formulas 2–5. A two-sided Wilcoxon rank sum test on the differences between the two distinct regimes is performed for each beta 1–4. The corresponding p-values of this test are given in the last row.

portfolios show increasing betas. Only illiquid portfolios have smaller betas in the high illiquidity regime. Thus, investors react differently in periods of increasing illiquidity than they do when market volatility or trading volume increases. Regarding beta 2, Rösch/Kaserer (2013) find increasing commonality when illiquidity increases in the market. The empirical results support this finding, except for the most liquid portfolios. Further, the increase in beta 2 is not very strong. Therefore, the Wilcoxon test does not confirm statistically significant differences. As previously described, the flight-to-liquidity effect should affect liquid and illiquid portfolios in different ways. Whereas liquid portfolios are strongly bought by investors, illiquid stocks are sold. Thus, beta 3 should have smaller negative or positive values for liquid portfolios and higher negative values for illiquid portfolios. The results partly support this finding given that liquid portfolios show positive betas, whereas illiquid portfolios still have negative betas. Nevertheless, the betas for illiquid portfolios do not increase with market illiquidity. Similar behavior is expected for beta 4. The absolute values of beta 4 are more negative when illiquidity is high, at a 1 % confidence level, meaning that asset illiquidity seems more sensitive to negative market returns when market illiquidity is already high. Only the most illiquid portfolio shows a smaller

Table 5

Betas in Illiquidity Regimes for the Conditional LCAPM

PF	Beta 1 *100		Beta 2 *100		Beta 3 *100		Beta 4 *100	
	high	low	high	low	high	low	high	low
1	99.959	78.397	0.000	0.001	0.734	-0.183	-0.003	0.000
3	111.284	93.061	0.002	0.003	0.623	-0.110	-0.029	0.001
5	118.697	98.830	0.005	0.007	0.827	-0.605	-0.069	-0.001
7	118.338	101.802	0.028	0.017	0.856	-0.169	-0.125	-0.015
9	128.558	107.212	0.093	0.029	-0.341	-0.132	-0.242	-0.006
11	125.489	111.426	0.200	0.054	0.237	-0.247	-0.379	-0.098
13	118.441	115.077	0.491	0.114	-1.100	-0.815	-0.637	-0.111
15	121.321	118.374	0.791	0.201	-0.432	-0.591	-1.109	-0.045
17	119.483	120.766	0.979	0.352	0.289	-0.102	-1.989	-0.227
19	124.604	125.578	1.732	0.545	-0.190	-0.476	-2.645	-0.514
21	113.710	114.535	2.293	1.068	-0.217	-1.328	-4.872	-0.957
23	106.889	119.470	3.237	1.888	-1.343	-1.051	-4.382	-2.778
25	96.418	124.063	5.786	4.030	-0.459	-1.506	-6.701	-10.623
p-value	0.095		0.263		0.049		0.007	

Note: For 25 illiquidity sorted (from low illiquidity to high illiquidity) equal weighted test portfolios the Betas 1–4 are calculated within the two distinct Regimes of high and low absolute normalized illiquidity using monthly data and an equal weighted market portfolio. The betas are calculated using the conditional model formulation of the LCAPM according to the formulas 2–5. A two-sided Wilcoxon rank sum test on the differences between the two distinct regimes is performed for each beta 1–4. The corresponding p-values of this test are given in the last row.

value in the high illiquidity regime. Decreasing or positive betas are not found for liquid portfolios in the high illiquidity regime.

Generally, significant differences exist between liquid and illiquid portfolios. Whereas liquid portfolios overall have lower betas and, therefore, are less risky, they are also less sensitive to regime changes and provide diversification advantages to investors in several regimes. In contrast, illiquid portfolios have higher betas and tend to be more sensitive to regime changes, further increasing the risk for investors. Overall, the analyses show significant time variation of the four betas. Further, betas change differently across volume, volatility, and illiquidity regimes. In detail, I find beta 1 to decrease with trading volume and return volatility, which is in line with existing findings, but increases with illiquidity. Further, betas 2 and 4 increase with illiquidity, supporting the finding of Rösch/Kaserer (2013) of an increase in liquidity commonality. Thus, single asset liquidity is especially fragile when market illiquidity is already high. Interestingly, I cannot find similar behavior for volume and volatility regimes. Liquidity commonality decreases in high-trading volume and high-return volatility phases, which contrasts with the findings of Rösch/Kaserer (2013). I find very different results for beta 3 within the three analyses. Whereas beta 3 is

positive for most portfolios in high-trading volume phases, the betas become more negative when volatility increases, which supports the theory of *Watanabe/Watanabe* (2008) that higher uncertainty leads to greater reactions to liquidity shocks. Lastly, I find significant signs supporting the theory of a flight-to-liquidity because beta 4 in particular shows different results for liquid and illiquid portfolios.

IV. Conclusion

The aim of this paper is to investigate the behavior of liquidity risk across different economic regimes. To do so, I implement the liquidity-adjusted capital asset pricing model (LCAPM) presented by *Acharya/Pedersen* (2005), which includes the expected level of illiquidity as well as three illiquidity betas in addition to the classical beta. Using the Markov-switching approach, I identify regimes of high and low market trading volume, volatility, and market illiquidity. I test the LCAPM within those different regimes and analyze the variation of the betas. Thereby, I can answer the following questions and contribute to the existing literature on liquidity risk and conditional asset pricing models.

- How do regimes of high and low market illiquidity relate to the well-known “volatility regimes” and the “volume regimes” of *Watanabe/Watanabe* (2008)?
- How do liquidity betas change across those different regimes? Does liquidity commonality increase in periods of low market liquidity when it is needed the most?

Regarding the identification and characterization of economic regimes, I find that high illiquidity phases are characterized by negative average returns, higher return volatility, and higher correlation of asset returns and market returns, as well as lower sensitivity of asset illiquidity and market returns. I demonstrate that high and low illiquidity regimes are persistent and, to some extent, correlated with both high volatility and trading volume regimes.

The analysis of the time variation of the betas shows that the market beta and the three liquidity betas change significantly across the regimes. In detail, two important effects become visible. First, the flight-to-liquidity effect leads to higher liquidity sensitivity to market returns when uncertainty is high, indicated by higher trading volume or higher return volatility. The results indicate that in high uncertainty phases, traders tend to buy more liquid assets such that these are less influenced by decreasing overall returns. At the same time, already illiquid assets are sold and, thus, strongly affected by overall return shocks. Further, betas 2 and 4 increase with illiquidity, supporting the finding of *Rösch/Kaserer* (2013) of increased liquidity commonality. Thus, single asset liquidity is especially fragile when market illiquidity is already high.

Further, I find several important differences between liquid and illiquid portfolios, which should be considered in portfolio management and asset allocation. Whereas liquid portfolios overall have lower betas and are, therefore, less risky, they are also less sensitive to regime changes and, in several regimes, even provide diversification advantages to investors. In contrast, illiquid portfolios have higher betas and tend to be more sensitive to regime changes, which further increases the risk for investors.

The results of this paper also have important implications for further research. First, the analysis of the 25 test portfolios shows that returns are decreasing with illiquidity, which is different from the results of several previous studies. The main reason for this difference might be the inclusion of NASDAQ stocks, as in *Gibson/Mougeot* (2004), who analyze S&P 500 stocks and find negative liquidity risk premiums. Therefore, it would be interesting to separately analyze CRSP data and NASDAQ stocks. A second important starting point for further research concerns the influence of the time variation of liquidity risk on traditional trading strategies, for example, a “buy-and-hold-strategy” or “rebalancing-strategies” with different rebalancing periods. It would be interesting to observe how the liquidity characteristics of such portfolios vary over time and how they are connected to the asset selection and weighting scheme of the trading strategy.

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