

## A Two-Agent Model of Inflation

Frank Browne and David Cronin\*

### Abstract

Models of inflation usually have monetary policy affecting the economy through either an interest rate channel or a monetary/credit quantity channel but not through both simultaneously. It is argued here that policy is transmitted via two distinct types of agents – those that are and that are not liquidity-constrained. The implication is that both interest rate and monetary channels must be seen as complementary, joint indicators of inflation and must both be incorporated into models of inflation. A formal representation of price level determination and behaviour in this two-agent framework is provided and evaluated econometrically using US data.

### Ein Zwei-Agenten-Inflationsmodell

### Zusammenfassung

In Inflationsmodellen wird üblicherweise angenommen, dass die Geldpolitik auf die Wirtschaftsleistung entweder durch den Zinskanal oder durch die Geldmenge/Kreditmenge wirkt, jedoch nicht simultan durch beide Kanäle. Hier wird argumentiert, dass die Geldpolitik auf zwei verschiedene Typen von Agenten wirkt – liquiditätsbeschränkte und nicht-liquiditätsbeschränkte. Dies hat zur Folge, dass sowohl der Zinskanal, als auch der Geldmengenkanal als komplementäre Indikatoren der Inflation angesehen werden können und gemeinsam in einem Inflationsmodell berücksichtigt werden müssen. Es wird eine formale Darstellung des Preislevels sowie des Verhaltens der Agenten in einem solchen zwei-Agenten Framework dargestellt und ökonometrisch mit US Daten untersucht.

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## I. Introduction

This article puts forward a two-agent model of inflation. It builds on two propositions, that inflation is the outcome of monetary policy actions and that inflation is transmitted to the economy by the central bank via two distinct channels. Each channel reflects the behaviour of one of two distinct types of agents in the economy, namely those that are liquidity-constrained and those that are not liquidity-constrained. A monetary policy action has its effects on economic activity, and ultimately on inflation, by disturbing the portfolio equilibrium, and hence the expenditure behaviour, of both types of agent. When portfolio equilibrium is restored for both sets of agents, the inflation generated from the monetary-policy-induced perturbation ceases and price stability is re-established.

All major central banks have price stability as their primary, if not exclusive, macroeconomic objective. Given the consensus on the long and variable lags in the transmission of monetary policy to economic activity, and ultimately to inflation, it is crucially important for these central banks to have a view on how the stance of monetary policy is transmitted to these variables. If central banks do not have a good handle on at least the broad contours of this process, they risk misestimating the extent of inflationary pressures already in the system and, accordingly, the appropriate current stance of monetary policy to attain, or maintain, price stability.

The novelty of this article stems from its identifying two main generic channels of monetary policy transmission and that it views them as operating simultaneously. The model provides a richer representation of price level determination than the more standard approaches based on either the Quantity Theory or Wicksellian price theory. Inflation studies have, invariably, only one or other of these channels but not both simultaneously. In other words, monetary policy is modelled as being channelled to the economy either through a financial price (i.e., an interest rate, to which, for example, adherents of the Taylor rule would subscribe) or a financial quantity (i.e., a credit or a monetary aggregate, to which adherents of the P-star model of inflation would subscribe<sup>1</sup>), but not through both simultaneously.<sup>2</sup> We depart from the consensus in arguing that both channels do operate at the same time. We build a theoretical model to reflect this. The model is then evaluated using US data. The econometric results

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<sup>1</sup> See Hallman et al. (1991).

<sup>2</sup> For example, models put forward by Woodford (2003, 2010) and Neiss/Nelson (2003) clearly indicate the “real interest rate gap” channel as an alternative to indices using financial quantity variables such as monetary or credit aggregates. Woodford (2008) contends that models without money can explain inflation, while, in contrast, Favara/Giordani (2009) see money shocks having effect on inflation.

reveal that the model's two gap variables compare favourably to estimations using the output gap and that their explanatory power is strongest when the monetary policy stance is either extremely loose or tight.

The article is organised as follows. In section 2, the dichotomy among participants in the loans market is discussed and is used to motivate the two-agent model of inflation. Section 3 provides a formal representation of price level determination and behaviour in the segmented markets framework. In section 4, the results of an econometric evaluation of the model, using US data, are given. Section 5 concludes.

## II. The Two-Agent Approach and the Impact of Monetary Policy

In arguing that inflation operates through two channels, it is necessary to think of an economy in which agents are segmented into two groups with contrasting degrees of participation in financial markets. This kind of distinction has proved very useful in empirical applications, most familiarly in examining aggregate consumption and investment. In aggregate consumption research, the two types of agents have been described variously as maximising and rule-of-thumb agents (see *Campbell/Mankiw* 1989, 1991) and non-liquidity-constrained and liquidity-constrained agents (*Zeldes* 1989). The segmented markets conceptual framework is not just a convenient heuristic device but is a sensible description of reality. The article on inflation by *Alvarez et al.* (2001) is also in this vein. It is referred to further below.

A description of the two agents begins with liquidity-constrained (*L*) agents who have inadequate access to financial markets. This is because they cannot, for example, easily mobilise their non-human assets as collateral in the loan market or cannot leverage their human capital (future labour income). Therefore, they cannot always gain access to the money balances needed for consumption purposes. They are unable to participate fully in financial markets and could be said to experience “portfolio stickiness”. The binding constraint that is relevant to these *L* agents in undertaking spending, and that is amenable to control by the central bank, is the amount of the nominal money stock held by them. Although they do hold some fraction of the money stock, their holdings are not easily adjusted and so these agents are frequently constrained relative to their desired expenditure plans. The binding constraint they face is the amount of liquidity rather than its price. Consequently, the expenditures of liquidity-constrained agents are not affected by the rate of interest.

The binding constraint on non-liquidity-constrained (*N*) agents is the real rate of interest. Although these agents also hold a certain proportion of the money stock, their holdings do not constitute a constraint on their spending plans because they can always borrow from banks at the prevailing loan rate. To

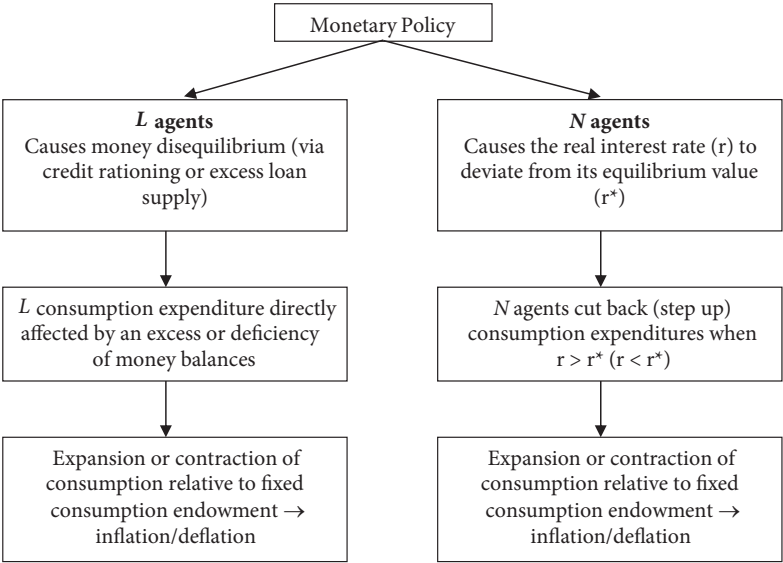


Figure 1: Flow Diagram of Monetary Policy Transmission

have an impact on the expenditures of these agents, the central bank must raise or lower the actual real rate of interest, which it is able to do over a short-to-medium term horizon, relative to the corresponding natural or equilibrium rate, which it cannot control. These agents are only concerned about the price of liquidity and not its quantity, since they can always obtain whatever amounts of liquidity they want at the going rate of interest.

Before turning to a formal model in section 3 below, these two channels are illustrated in the flow diagram in Figure 1, which is largely self-explanatory. The existence of two channels does not mean that the central bank controls two things at the same time but merely that its operations affect both types of agents differently, determined by their contrasting levels of success in raising funds in the bank loan market. Figure 1 shows, to paraphrase *Stiglitz/Greenwald* (2003), that with credit rationing, monetary policy exerts its effects not only through interest rates, but also through credit availability. This twin effect could be generalised to refer to loan market disequilibrium, with the effects of a monetary policy change on the consumption spending of *L* and *N* agents reinforcing each other.

All funds raised in the loan market by *L* and *N* agents are assumed to be credited instantaneously to their overnight deposit accounts at their lending banks and become immediately available as liquidity (money) to be used for consumption purposes. The funds are added to whatever money balances the agents have

already accumulated. This, conveniently, allows us consider liquidity constraints in terms of money balances even though the source of those constraints originates in the bank loan market.

### III. Price Level Determination and Adjustment in the Two-Agent Economy

#### 1. Description of the Economy

Both types of agent, or household, are assumed to receive the same endowment of goods,  $y$ , each period. The economy's resource constraint is written as:

$$(1) \quad y = \lambda c_{Lt} + (1 - \lambda) c_{Nt}$$

The parameters  $\lambda$  and  $(1 - \lambda)$  represent the fractions of households (where  $\lambda$  is less than or equal to one) that are liquidity-constrained ( $L$ ) and non-liquidity-constrained ( $N$ ), respectively, and  $c_{Lt}$  and  $c_{Nt}$  their respective real consumption bundles in period  $t$ .<sup>3</sup>

The money stock is denoted as  $M$ . The funds available to  $L$  households for consumption purposes can come from three sources: a variable fraction ( $v$ ) of current period labour income or sales receipts (i.e.,  $v_t P_t y$ ); unspent receipts from labour income or sales in the previous period (i.e.,  $(1 - v_{t-1}) P_{t-1} y$  or, equivalently,  $M_{Lt-1}$ ); additional loans (money) from banks following expansionary monetary policy measures. With the fraction of the current endowment consumed this period denoted by  $v$  and the goods endowment ( $y$ ) and the price level ( $P$ ) common to all agents, consumption expenditures differ between  $N$  and  $L$  agents depending only on their relative success or otherwise in raising loans from the banking system.  $N$  households have unlimited access to bank loans and do not encounter any funding difficulties in the sense that they can obtain as much as desired provided they are willing to pay the going interest rate. They need funding to bridge the gap between the consumption that can be paid for from the first two sources just noted and that required to fund their desired consumption bundle in the current period,  $c_{Nt}^*$ . But since this required funding is always forthcoming, it is never a binding constraint on their consumption.

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<sup>3</sup> The formal resource constraint and two-agent representation is in the vein of Alvarez et al. (2001) but the model specified here is quite different otherwise. One important difference in assumption is that in the Alvarez et al. article, all agents are liquidity-constrained at particular times. In the model presented here, only  $L$  agents are sometimes liquidity-constrained while  $N$  agents are never liquidity-constrained.

## 2. Price Level Determination

The total amount of bank lending arising following a monetary policy action is denoted by  $\Delta \bar{M}$ . The bar indicates that  $M$  is exogenously determined to the private non-banking sector of the economy by monetary policy and the portfolio decisions of commercial banks. A  $\Delta M$  variable without a bar means that it is endogenously determined by the relevant sector of the economy. The differences between the two sets of households are modelled by assuming that, following a monetary policy-driven expansion of the money stock,  $N$  households have first call on the change in the money stock (taking  $\Delta M_N$  of it, which is assumed to be always sufficient to satisfy  $N$ 's consumption needs), with the remaining amount,  $(\Delta \bar{M} - \Delta M_N)$ , accruing to  $L$  households.<sup>4</sup>

Nominal consumption expenditure by  $L$  households in period  $t$  is then determined as follows:

$$\begin{aligned}
 P_t c_{Lt} &= \bar{M}_{Lt-1} + v_t P_t y + [\Delta \bar{M}_t - \Delta M_{Nt}] \\
 (2) \quad &= [\bar{M}_{t-1} - M_{Nt-1}] + v_t P_t y + [\bar{M}_t - \bar{M}_{t-1}] - [M_{Nt} - M_{Nt-1}] \\
 &= [\bar{M}_t - M_{Nt}] + v_t P_t y
 \end{aligned}$$

This says that  $L$  households' consumption spending in the current period is *constrained* by the amount of money available to them. This, in turn, is equal to the exogenous amount which can be borrowed from banks following any central bank monetary policy operation in the current period after the loan demand of  $N$  agents is satisfied; plus the varying amount that may become available from the efficiency or productivity of money as reflected in velocity ( $v$ ), which is proportional to current sales (i.e.,  $v_t P_t y$  in total).

In contrast, the consumption of  $N$  agents is the outcome of an optimising framework. The  $N$  consumers are assumed to have identical preferences as encapsulated in the following utility function:

$$(3) \quad E_t \sum_{i=0}^{\infty} (1 + \delta)^{-i} U(c_{Nt+i})$$

Where  $c$  is consumption,  $\delta$  is the subjective rate of discount, and  $E_t$  is the expectation conditional on information available at time  $t$ . If the representative

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<sup>4</sup> In response to a point made by a referee, we assume that  $\Delta \bar{M} > \Delta M_N$ . This leaves some liquidity available to the  $L$  sector following monetary policy measures. If  $N$ 's new demand for money were always greater than  $\Delta \bar{M}$  then there would be no scope for any variation in the loan/money transmission channel.

consumer can borrow and lend at the real interest rate,  $r$ , then the first-order condition necessary for an optimum is:

$$(4) \quad E_t U'(c_{Nt+1}) = \left( \frac{1+\delta}{1+r} \right) U'(c_{Nt})$$

This implies that, given the interest rate and the discount rate, each  $N$  consumer seeks to consume a particular utility-maximising bundle of goods in the current period, which is denoted as  $c_{Nt}$ . The nominal expenditure of  $N$  consumers in period  $t$  is denoted  $P_t c_{Nt}$ , or  $C_{Nt}$ . The consumption spending of  $N$  households is independent of the level of funding.

Multiplying equation (1) by  $y$ , and substituting in for the nominal expenditures of  $L$  and  $N$  households, the following is obtained:

$$\begin{aligned} P_t y &= \lambda P_t c_{Lt} + (1-\lambda) C_{Nt} \\ &= \lambda (\bar{M}_t - M_{Nt}) + \lambda v_t P_t y + (1-\lambda) C_{Nt} \end{aligned}$$

Therefore,

$$P_t = \left[ \frac{\lambda}{(1-\lambda v_t)y} \right] (\bar{M}_t - M_{Nt}) + \left[ \frac{1-\lambda}{(1-\lambda v_t)y} \right] C_{Nt},$$

And

$$(5) \quad P_t = \left[ \frac{\lambda}{(1-\lambda v_t)y} \right] \bar{M}_{Lt} + \left[ \frac{1-\lambda}{(1-\lambda v_t)y} \right] C_{Nt}(r_t)$$

Equation (5) indicates the determination of the price level in this segmented markets model. In this representation, the price level is determined at any point in time by that part of the money stock held by  $L$  households,  $\bar{M}_L$ , which is exogenously determined by the monetary policy actions of the central bank and the loan supply behaviour of commercial banks, and by the (realised) consumption plans of  $N$  households which, according to the optimising framework in equations (3) and (4), is a function of the current real rate of interest (i.e.,  $r_t$ ) as shown in equation (5).<sup>5</sup>

Equation (5) can be described as a modified quantity theory equation. In taking account of the difference between  $L$  and  $N$  households, it says that the strict version of the quantity theory only holds when  $\lambda = 1$ , i.e., in a financially repressed financial system where liquidity constraints are pervasive.<sup>6</sup> When  $\lambda$  is

<sup>5</sup> For convenience, it is only written explicitly as a function of  $r$  in the last equation.

<sup>6</sup> If the segmentation assumption is dropped and all agents are assumed to be liquidity-constrained (i.e.,  $\lambda$  set equal to 1) in equation (5), we get a standard quantity theory equation.

less than one, only that part of the money stock held by  $L$  agents has an impact on the price level.  $N$  agents are able to adjust their money balances passively and smoothly to whatever level is needed to fund their desired level of expenditure. It is monetary policy's effect on the real interest rate that allows it affect  $N$ 's optimal level of consumption spending relative to the fixed consumption endowment and which, in turn, has effect on the price level.

### 3. Price Level Disequilibrium

Equation (5) indicates the determination of the price level at any given time. It is not necessarily, however, a price level consistent with long-run equilibrium. For  $L$  consumers, their money holding,  $\bar{M}_{Lt}$ , may exceed or fall short of their demand for money. For  $N$  consumers, the real interest rate,  $r$ , may deviate from its long run equilibrium value. Price stability (denoted here by  $P^*$ ) only occurs in the (possibly rare) event of both agents being in portfolio equilibrium simultaneously. This happens when  $L$  households have their demand for money fulfilled exactly and  $N$  households' consumption is not subject to a gap between the actual and equilibrium real rates of interest. The equilibrium version of equation (5) is written as follows:

$$(6) \quad P_t^* = \left[ \frac{\lambda}{(1 - \lambda v_t)y} \right] M_{Lt} + \left[ \frac{1 - \lambda}{(1 - \lambda v_t)y} \right] C_{Nt}(r_t^*)$$

Price stability occurs when  $L$  households have their demand for money satisfied exactly (i.e.,  $\bar{M}_{Lt} = M_{Lt}$ ) and the actual real interest rate is equal to its natural or equilibrium level (i.e.,  $r = r^*$ ), obviating any incentive for either  $L$  or  $N$  households to alter their level of consumption.

Subtracting equation (6) from (5) then gives the following:

$$(7) \quad P_t - P_t^* = \left[ \frac{\lambda}{(1 - \lambda v_t)y} \right] (\bar{M}_{Lt} - M_{Lt}) + \left[ \frac{1 - \lambda}{(1 - \lambda v_t)y} \right] [C_{Nt}(r_t) - C_{Nt}(r_t^*)]$$

Equation (7) indicates that deviations of the price level,  $P$ , from its equilibrium value,  $P^*$ , are owing to actual money balances deviating from desired levels and the real interest rate differing to the equilibrium real rate. These deviations occur as a result of monetary policy actions upsetting the portfolio equilibrium of both  $L$  and  $N$  households at the same time. These actions leave  $L$  agents with either a deficiency of money balances (forcing them to cut consumption expenditure) or a surplus (encouraging them to spend more than they had planned) and  $N$  agents facing a real rate of interest which is either above the equilibrium rate (thereby causing a retrenchment in consumption) or below it



(inducing  $N$  agents to increase consumption spending). As portfolio equilibrium is restored, consumption spending is driven above or below the fixed endowment causing the price level to adjust towards  $P^*$ . As can be seen from equation (7), only when portfolio equilibrium is fully restored (i.e.,  $\bar{M}_{Lt} = M_{Lt}$  and  $r_t = r_t^*$ ) is price stability re-established (i.e.,  $P_t = P_t^*$ ).

#### 4. Price Level Adjustment

The price disequilibrium embodied in equation (7) is resolved through the price level adjusting to the equilibrium level. The inflation (or deflation) of the price level required to resolve the disequilibrium, in turn, must be generated by the nominal money gap and real interest rate gap on the right-hand-side of (7). The implication for empirical work is that inflation can be modelled as a function of these two gaps.

The money gap can be expressed as follows: since,

$$\bar{M}_{Lt} = \bar{M}_t - M_{Nt},$$

then

$$\begin{aligned} \bar{M}_{Lt} - M_{Lt} &= \bar{M}_t - (M_{Lt} + M_{Nt}) \\ (8) \qquad \qquad &= \bar{M}_t - M_t \end{aligned}$$

This, intuitively and conveniently, allows  $L$  household money disequilibrium to be replaced by economy-wide money disequilibrium since the  $N$  households are always in equilibrium with respect to money holdings.<sup>7</sup>

All the causal effect running from aggregate monetary disequilibrium is coming exclusively from monetary disequilibrium affecting  $L$  households. The remaining part, i.e., money balances held by  $N$  households, are reacting passively to  $N$ 's demand. The mean-value theorem can be invoked to rewrite part of the second term on the right-hand side of equation (7), as follows:<sup>8</sup>

<sup>7</sup> Equation (8) shows that  $L$  sector money disequilibrium is the same as economy-wide money disequilibrium since the  $N$  sector is always in equilibrium. We don't observe the  $L$  sector's money constraint nor its money demand, but we can observe the overall money stock and, in principle, we can estimate overall money demand. This is the rationale for equation (8) in the text.

<sup>8</sup> The mean-value theorem (see *Chiang* 1984) states that the difference between the value of a function  $\phi$  evaluated at  $x_0$  and at any other  $x$  value can be expressed as the product of the difference  $(x - x_0)$  and the first derivative,  $\phi'$ , of the function evaluated at some point,  $\rho$ , between points  $x$  and  $x_0$ , i.e.,  $\phi(x) - \phi(x_0) = \phi'(\rho)(x - x_0)$ . Proceeding analogously here gives us the right-hand-side of equation (9) in the text.

$$(9) \quad C_{Nt}(r_t) - C_{Nt}(r_t^*) = C'_{Nt}(\rho)(r_t - r_t^*)$$

It is assumed that adjustment of the price level to its equilibrium value in the next period takes place, at a fraction,  $\theta$ , of the current period discrepancy.

Accordingly, inflation in period  $t + 1$ , can be expressed as:

$$(10) \quad \Delta P_{t+1} = P_{t+1} - P_t = \theta(P_t - P_t^*) \\ = \left[ \frac{\theta\lambda}{(1 - \lambda v_t)y} \right] (\bar{M}_t - M_t) + \left[ \frac{\theta(1 - \lambda)C'_{Nt}(\rho)}{(1 - \lambda v_t)y} \right] (r_t - r_t^*)$$

It is notable in equation (10) that the inflation rate depends on a nominal variable (variations in the liquidity constraints experienced by the  $L$  agent) and a real variable (variations in a real interest rate differential experienced by the  $N$  agent). This is because  $N$  agents' decisions are not affected by any kind of nominal frictions. Those depend only on real variables (such as the real rate of interest and the equilibrium real rate of interest) which allows us to write  $N$ 's nominal consumption, and  $N$ 's equilibrium nominal consumption, as a function of the actual and equilibrium real rates of interest respectively, as in equation (10). The  $N$  sector is, therefore, never constrained by a nominal rigidity as might be proxied by some nominal quantity variable. On the other hand, market imperfections, or frictions, prevent  $L$  agents from smoothing their consumption over time. Accordingly, consumption for  $L$  agents is a function of a nominal variable, i.e., money disequilibrium, as in equation (10).

Indeed, if there were only real variables in equation (10), then the price level would be indeterminate. In the prevailing monetary world of unbacked fiat currencies, there must be some entity (a central bank, for example) that places the bar on  $M$  and thereby pins down the price level. This nominal rigidity, which arises from agent behaviour in the bank loan market, does not disappear in the steady state and so the price level is pinned down. Accordingly,  $(\bar{M}_t - M_t)$  in equation (10) must be nominal, while  $N$ 's nominal consumption remains a function of the real interest rate.

## IV. An Empirical Assessment of the Two-Agent Model

### *I. Methodological Approach and Data*

The two-agent model outlined in the previous section points to two gap variables being used in the empirical modelling of inflation: a real interest rate gap and a measure of actual money balances' deviation from equilibrium holdings (a money gap, or velocity gap). In this section, an econometric framework,

drawing on *Diebold/Yilmaz* (DY) (2009, 2012), is employed to examine how these two gap variables can help explain US inflation developments over time. The approach uses vector-autoregressive regressions (VARs) to provide measures of the extent to which the velocity gap and real interest rate gap account for inflation's forecast error variance decomposition. In other words, it uses the "innovation accounting" that arises from VAR estimations to show the proportion of inflation's shocks that are accounted for by past innovations, or shocks, to the two gap variables, as well as that proportion accounted for by past inflation shocks. This "spillover" approach is at its most informative when used as part of a rolling window estimation procedure, as one can then see at which time the gap variables' influence on inflation is stronger or weaker and whether that coincides with particular monetary policy actions by the Federal Reserve.

A measure of the US real interest gap over time is calculated with data provided on a Federal Reserve of San Francisco website and based on *Holston et al.* (2017).<sup>9</sup> The gap is the difference between the real interest rate (the real Federal Funds rate for post-1965 data; the Federal Reserve Bank's discount rate for pre-1965 data) and the authors' measure of the natural rate of interest.

Choosing a money gap measure is complicated by the acknowledged breakdown of the previously stable relationship between the broad money measure, M2, and nominal economic activity during the 1980s and 1990s (*Friedman/Kuttner* 1992; *Estrella/Mishkin* 1997). The approach taken here to this issue is that of *Cronin* (2018), with measures of the money velocity gap (that is, the deviation of the velocity of money from its equilibrium value) being used to capture disequilibrium in money holdings. The velocity of money is measured as nominal economic activity divided by the nominal money stock. Two measures of money velocity are employed. The first uses the M2 money stock in the velocity denominator, while the second uses the MZM (Money Zero Maturity) money stock. The latter, like M2, is a broad money aggregate and had a stable money demand function in the early 1990s when the stability of its M2 counterpart was undermined (*Carlson et al.* 2000).

In estimating the velocity gaps, the equilibrium velocity of money is not directly observed. The approach then taken (as in *Cronin* 2018) is to apply a Hodrick-Prescott filter, with a lambda value of 1600, to a quarterly series of the natural log of M2 velocity and MZM velocity series provided on the Federal Reserve Bank of St. Louis's FRED database. The difference between the natural log series and the fitted value, multiplied by 100, from the filter application provides a measure of the velocity gap, i.e., a measure of money market disequilibrium.

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<sup>9</sup> The data are available at: [http://www.frbsf.org/economic-research/economists/Holston\\_Laubach\\_Williams\\_estimates.xlsx](http://www.frbsf.org/economic-research/economists/Holston_Laubach_Williams_estimates.xlsx).

Two other series are employed, both sourced from FRED. The first is the measure of inflation, which comprises the quarter-to-quarter percentage change in the GDP implicit deflator (GDPD). The other series is the percentage rate of change in oil prices (West Texas Crude spot price). It is included as a cost-push variable in the VARs, which may capture a fraction of the short-term variability in inflation. The data series extend from 1961Q1 to 2016Q4.<sup>10</sup> The various series are plotted in Figure 2. The shaded areas in the charts, and in some of the subsequent Figures, are those quarters where the NBER adjudges the US economy to have been in recession.

## 2. Econometric Results

Each VAR estimation includes four variables, which are ordered as follows: inflation, the interest rate gap, the velocity gap, oil price inflation. This ordering is chosen as the forecast error variance decomposition is based on orthogonalised shocks, so that the relative positioning of the variables matters to the decomposition.<sup>11</sup> With the ordering chosen here, shocks to the gap variables and oil inflation will not have an effect on inflation until the following quarter, a standard feature of inflation modelling where the explanatory variables affect final prices with a lag. The forecast error variance decomposition for inflation provides a measure of the proportions of inflation shocks that are accounted for by past innovations, or shocks, in the variable itself (its own-variance share) and in the other variables (the cross-variance shares). Rolling-window VAR estimations are employed so that the share accounted for by each variable can vary over time. The main interest is in seeing how shocks to the two gap variables contribute to inflation's decomposition. Their relative share of the decomposition can also indicate whether inflationary pressure is emanating more from money disequilibrium (i.e. from the *L* agents) or the real interest rate channel (i.e. from the *N* agents).

In relation to the modelling choices made, the VAR lag length is four, the forecast horizon is ten quarters ahead, and the window size is 60 quarters.<sup>12, 13</sup> With

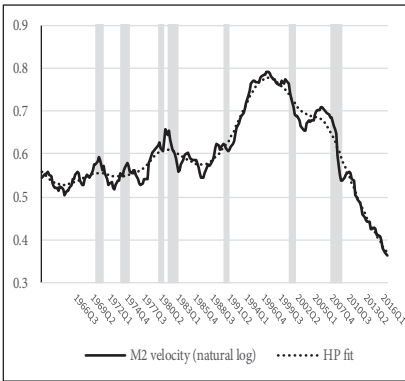
<sup>10</sup> The M2 velocity and MZM velocity quarterly data collection was extended back to 1959Q3, while an autoregressive forecasting process allowed projections of velocity to 2018Q2 to be made. The Hodrick-Prescott filter was then applied to the natural logs of these extended series and the initial six and final six observations were discarded to address the end-point problem associated with the filter.

<sup>11</sup> Reversing the order of the two gap variables in the VAR has little effect on the results.

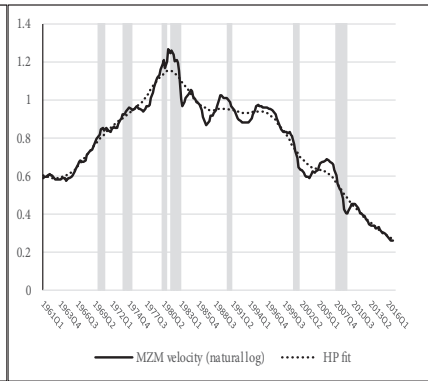
<sup>12</sup> The effects of each of these modelling choices, as well as the choice of orthogonalised over generalised decompositions, is the focus of the appendix to the article where the sensitivity of the decompositions to alternative lag lengths etc. is considered.

<sup>13</sup> A ten-quarter-ahead forecast horizon is broadly in line with empirical findings as to when monetary policy has its peak effect on US inflation. See *Friedman* (1961, 1972), *Batini/Nelson* (2001).

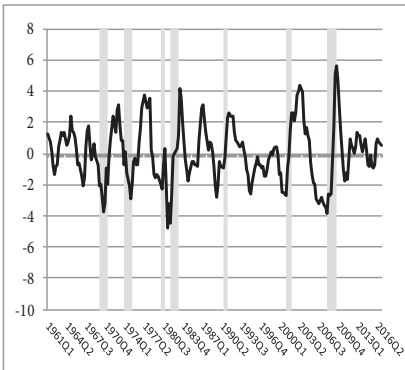
i. M2 velocity and fitted series



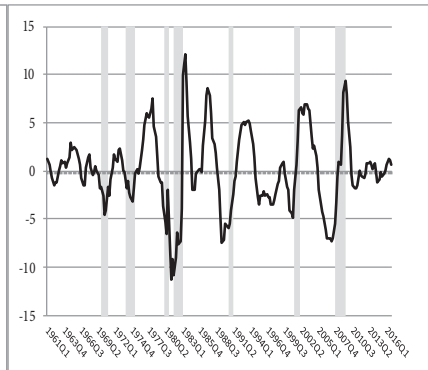
ii. MZM velocity and fitted series



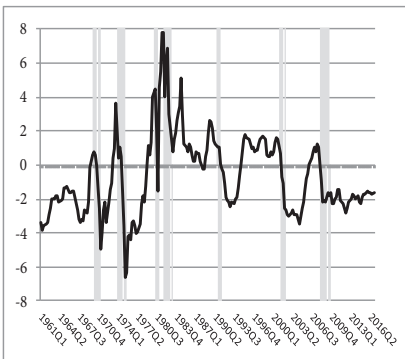
iii. M2 velocity Gap (%)



iv. MZM velocity Gap (%)



v. Real interest rate gap (%)



vi. Oil price inflation (%)

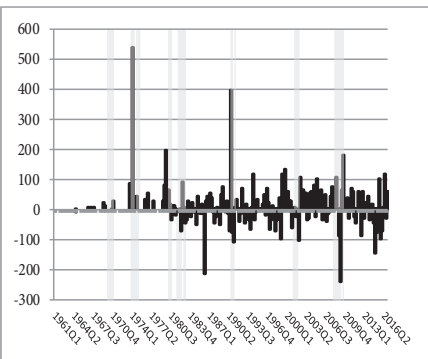
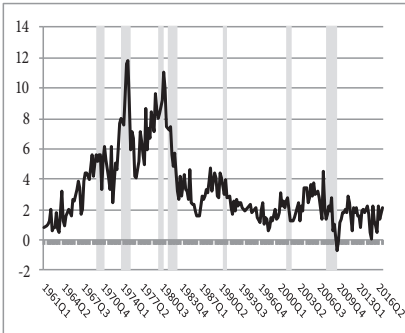


Figure 2: Data

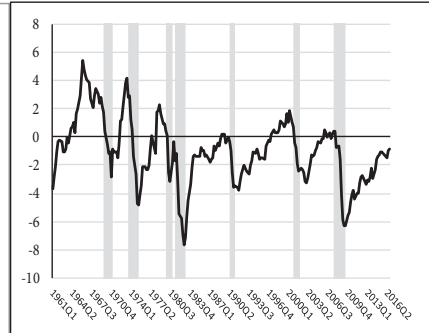
(Continue next page)

(Figure 2: Continued)

vii. GDPD inflation (%)



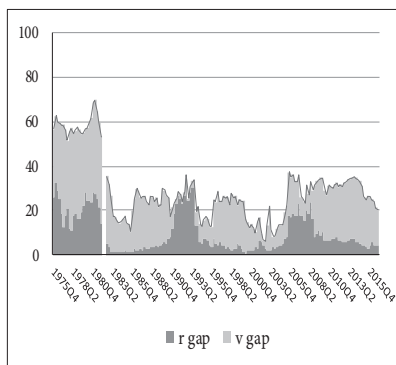
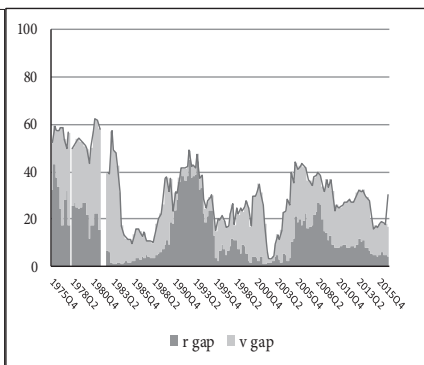
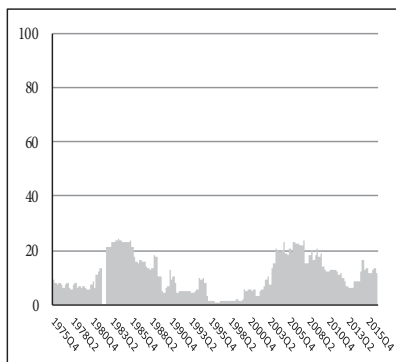
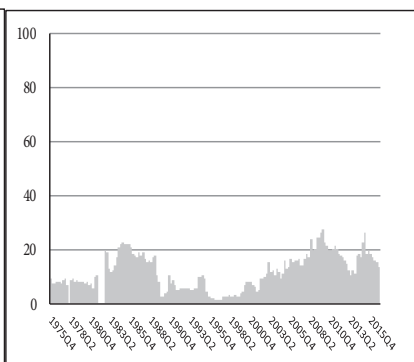
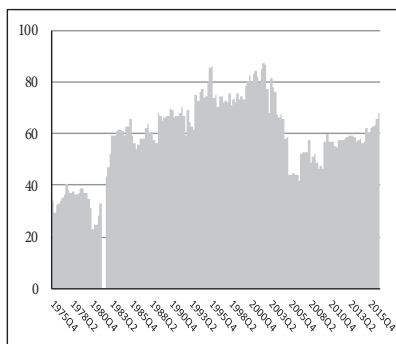
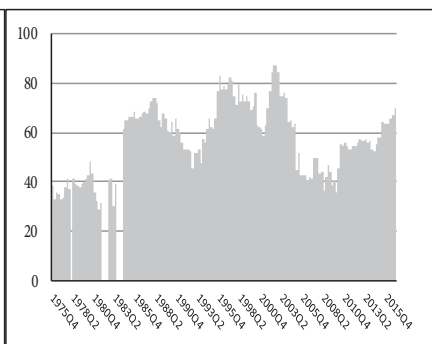
viii. Output gap



a 60-quarter window, the initial window ends in 1975Q4, while the final one ends in 2016Q4, rendering 165 windows estimated. The horizontal axis labelling in Figures 3 and 4 mark those end-dates. Figure 3 then shows the shares of the forecast error variance decomposition for inflation accounted for by each of the four variables in the VARs (with those shares summing up to 100 % in each window). The left-hand-side charts are those where the M2 velocity gap is used in the VARs, while those where the MZM velocity gap is the monetary variable are on the right-hand-side. In the left-hand panel, decomposition values are recorded for every window bar two (those ending in 1982Q1 and 1982Q2), while in the right-hand panel, there are four windows where no decomposition values arise (ending 1978Q1 and 1981Q4 to 1982Q2).<sup>14</sup>

The entries in panels (i) and (ii) of Figure 3 show the spillover of shocks from the real interest rate gap (r gap) and velocity gap (v gap) to inflation, stacked on one another to show the combined monetary policy impulse occurring through both channels. Panels (i) and (ii) show a considerable monetary policy impulse from the two gap variables (ranging from 51 % to 69 % in panel (i) and 43 % to 62 % in panel (ii)) to inflation from 1975Q4 into the 1981–1982 recession. This was a period when inflation rates were high against a background of low interest rates and strong money growth and when a tighter monetary policy stance was eventually adopted by the Federal Reserve to address this price environment, resulting in a sharp drop in the inflation rate from 11 % in 1980Q4 to 2.7 % in 1983Q2 (the so-called Volcker disinflation). Most of the impulse to inflation

<sup>14</sup> No decomposition values will be reported where one or more variables exhibit explosive behaviour within the estimation window. A rolling estimation where only two or four windows out of 165 do not provide decompositions is quite satisfactory relative to other applications of the DY approach.

**i. Impulse from gap variables****ii. Impulse from gap variables****iii. from oil inflation shocks****iv. from oil inflation shocks****v. from own shocks****vi. from own shocks***Figure 3: Spillover to GDPD inflation: Two-agent Model (%)*

was coming through the money channel, although the interest rate channel does account in some windows for over 30 % of the decomposition.

After the 1981–2 recession, the monetary impulse declines but remains above 20 % for most of the windows that follow, and for many windows well in excess of that value. It increases sharply in the mid-2000s, with both gap variables contributing more strongly to the inflation decomposition. The share of the forecast decomposition accounted for by the velocity gap remained elevated after the 2007–9 recession, a period when the Federal Reserve engaged in three programmes of quantitative easing. These programmes sought to stimulate the economy through increasing the money supply. The real interest rate gap's share of the decomposition declined from 2008 onwards.<sup>15</sup>

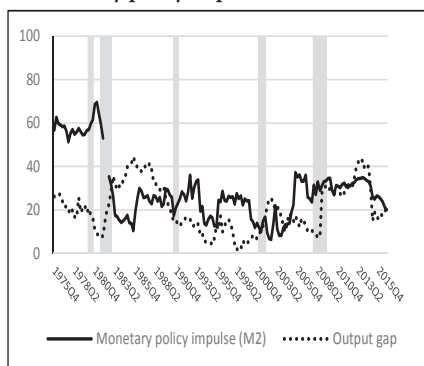
The monetary policy share of the forecast error variance decomposition of inflation is then at its strongest when policy has been at its most active. One period when this holds is the 1970s and the early 1980s (when the Federal Reserve went from a period of accommodating high inflation rates to reducing them through an aggressive, contractionary monetary policy). The second period is from the mid-2000s onwards when a loose monetary policy (contributing to the difficulties experienced in the 2008–9 financial crisis) was followed by further reductions in official interest rates and quantitative easing programmes that increased the monetary base four-fold from 2008 to 2014 and coincided with large increases in broader money aggregates as well. That the monetary impulse is at its strongest when monetary policy is either extremely loose or tight is consistent with the basic premise of the two-agent model that a sharp dosage of monetary policy, such as occurred in the 1970s and early 1980s and in the 2000s, creates greater disequilibrium in the money and interest rate channels.

Panels (iii) and (iv) of Figure 3 show the share of the forecast error variance decomposition of inflation owing to oil price shocks. Those shares are much lower in most windows than the monetary policy impulse in panels (i) and (ii). Like the monetary policy impulse, oil price shocks' share is larger in the earlier and later parts of the sample (if not quite at the tail-ends of the charts). The greater share of the inflation decomposition in the second half of the 2000s accounted for by oil price shocks may have a monetary origin, as has been posited in recent studies. *Gattini et al.* (2015) emphasise the relevance of money demand shocks to both inflation and commodity price developments. *Browne/Cronin* (2008, 2010) indicate that changes in both commodity prices and consumer prices arise from money supply shocks and that commodity prices leading final goods prices is a monetary phenomenon. Through the employment of a model of the interest rate channel, *Frankel* (2008) makes a similar connection

<sup>15</sup> There is a third period when the monetary policy impulse's share is high: the late-1980s and early-1990s, although only in panel (ii) where the MZM gap is used.



## i. Monetary policy impulse



## ii. Monetary policy impulse

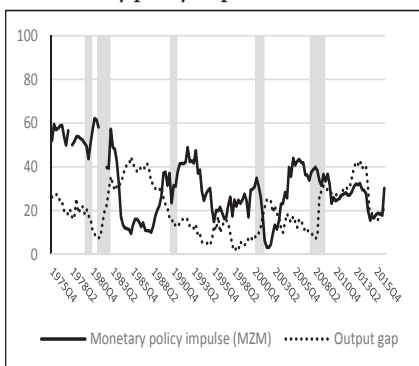


Figure 4: Spillover to GDP Inflation: Two-agent Model Versus Output Gap (%)<sup>16</sup>

between commodity prices and final goods prices. The final two panels of Figure 3 (i.e. panels (v) and (vi)) show the relative share of own inflation shocks to the decomposition. Its share increases in the wake of the Volcker disinflation and dominates the decomposition through to the mid-2000s at which stage the other variables exercise renewed influence.

As a final exercise, the output gap is substituted into the VAR specification in place of the two gap variables.<sup>17</sup> The (orthogonalised) forecast error variance decompositions from the resulting three-variable VARs (inflation, output gap, oil inflation) are estimated over the same windows and with the same modelling choices (lag length set to four, etc.) as those in Figure 3. The three-variable VAR mimics a Phillips curve model of inflation, where inflation is determined by an output gap and its own past values. (The oil inflation variable is retained in the VAR specification given its inclusion in the two-gap-variable VARs.) In Figure 4, the forecast error variance share of the output gap in the inflation decomposition of the three-variable VAR is compared to the monetary policy impulse values replicated from panels (i) and (ii) of Figure 3.<sup>18</sup> The two-gap representation accounts for a higher average share of the decomposition across all window estimates than the output gap does in the three-variable, Phillips curve case. Its

<sup>16</sup> The “monetary policy impulse” entry in panel (i) is that found in panel (i) of Figure 3, and, likewise, its entry in panel (ii) corresponds to panel (ii) of Figure 3.

<sup>17</sup> The output gap is calculated using two series from FRED: real GDP (billions of chained 2009 dollars) and real potential GDP (billions of chained 2009 dollars). The natural log of the latter is subtracted from the natural log of the former, with the difference multiplied by 100 to give the series shown in panel (viii) of Figure 2.

<sup>18</sup> The decomposition shares of oil inflation and GDP inflation from the three-variable VAR are not shown but are available on request.

explanatory power is at its strongest relative to the output gap when inflation rates are high (i.e. from 1975Q4 up to 1983/4). After the early 1980s, when a relatively benign inflationary environment arose, the gap variables maintain a larger average share of the inflation decomposition than the output gap.<sup>19</sup>

## V. Conclusion

In this article, a theory of inflation based on the distinction between two types of agent who populate the non-financial sector of the economy has been put forward. Agents are either liquidity-constrained or not liquidity-constrained. This implies that monetary policy is transmitted to the economy and affects the price level exclusively via two separate channels, which correspond to the distinct behaviour of these two types of agents.

The model provides a richer representation of price level determination than standard Quantity Theory or Wicksellian explanations. It shows the price level being determined by the actions of both agents and, accordingly, monetary policy affecting the price level through the money stock available to liquidity-constrained agents and the real interest rate which matters to the intertemporal allocation of consumption expenditures by non-liquidity-constrained agents. The first channel is similar to the monetarist explanation of the process of inflation while the second is in line with a Wicksellian description. Yet, because each channel relates only to one of the two agents, neither on its own gives a complete account of the inflation process. An implication of the model then is that there is no choice to be made between modelling inflation as occurring either through a financial price (i.e., an interest rate) or a financial quantity (i.e., a credit or monetary aggregate) as both channels operate simultaneously. This is because any monetary policy action affects the two agents differently arising from their contrasting experiences in the bank loans market. A complete picture of the inflation generated by the central bank then requires that both channels be accounted for.

An empirical application of the two-agent model was also provided. It shows two gap variables (a velocity-of-money gap and a real interest rate gap) comparing favourably to the output gap in explaining inflation shocks over time. The gap variables account for larger shares of inflation's forecast error variance decomposition when monetary policy is either extremely loose or tight. This is consistent with the basic premise of the theoretical model that the monetary policy stance dictates the degree of disequilibrium in the loan market and that this then takes effect on final prices through the money and interest rate channels.

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<sup>19</sup> After 1982Q3, when the inflation rate was 2.7%, the two gap variables have a higher average share of the decomposition (24.1% in the case where the M2 velocity gap is used and 27% where the MZM gap is used) than the output gap (an average of 21%).

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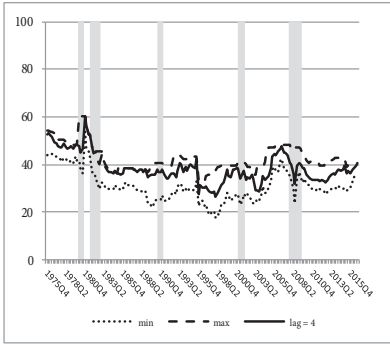
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### Appendix: Robustness Tests

In estimating the VARs reported above, several modelling choices were made. The VAR lag length was set at 4, the forecasting horizon at 10 quarters, the rolling window at 60 quarters, and an orthogonalised forecast error variance decomposition was preferred. Tests of the sensitivity of the total spillover index (TSI) (see Diebold and Yilmaz, 2012) to these four modelling choices are reported in turn in Figures A1–A4. In each case, the other original three modelling assumptions are maintained while the fourth varies.

In Figure A1, the lag length varies between 3 and 6 and the minimum and maximum TSI values across these lag lengths for each window are reported in the charts alongside the default lag length (of 4). While there are differences in these values over the various windows, the qualitative pattern is similar between the three series in both charts. Next, the forecasting horizon was allowed vary between 8 and 12 quarters. The minimum and maximum index values are shown in Figure A2. The spillover index is not sensitive to the choice of horizon, which indicates that the decompositions tend to settle down at longer forecasting horizons. In Figure A3, the window size varies between 40 quarters, 60 quarters, and 80 quarters. The values for the 60-quarter and 80-quarter options are closer in value to one another than the 40-quarter and 60-quarter options. Moreover, for many of the 40-quarter windows, there is no spillover index value reported, which supports the selection of the longer, 60-quarter window for the estimations shown in Figure 3. Finally, in Figure A4, there is little difference in spillover values between the orthogonalised and generalised decompositions.

*i. VAR with M2 velocity gap*



*ii. VAR with MZM velocity gap*

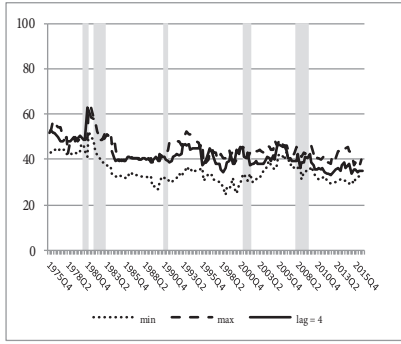
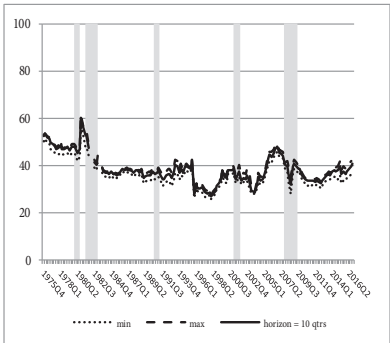


Figure A1: Total Spillover Index (%) – Different Lag Lengths

*i. VAR with M2 velocity gap*



*ii. VAR with MZM velocity gap*

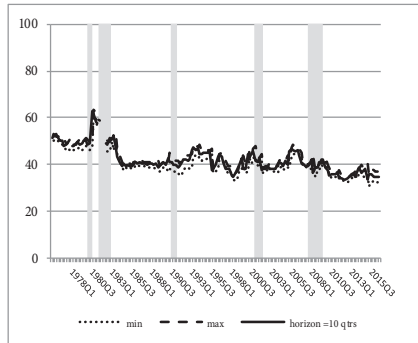
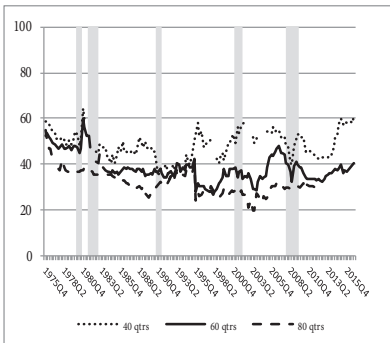


Figure A2: Total Spillover Index (%) – Different Forecast Horizons

*i. VAR with M2 velocity gap*



*ii. VAR with MZM velocity gap*

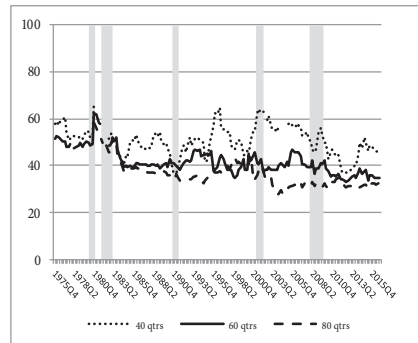


Figure A3: Total Spillover Index (%) – Different Window Sizes

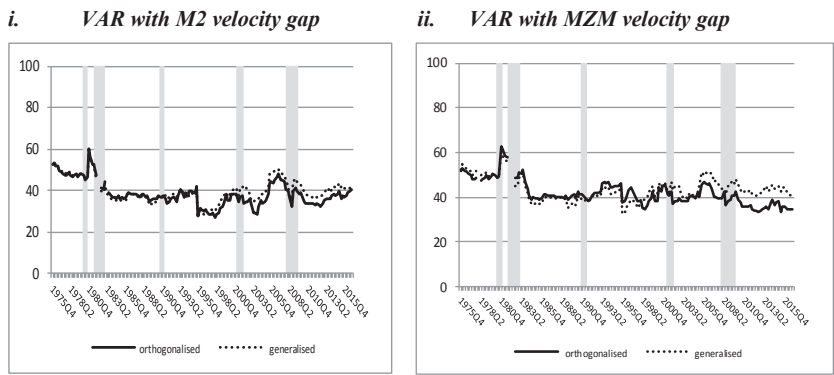


Figure A4: Total Spillover Index (%) – Orthogonalised vs. Generalised Decomposition